Studii şi Cercetări Ştiinţifice
Chimie şi Inginerie Chimică, Biotehnologii, Industrie Alimentară

2021, **22** (1), pp. 089 – 105

Scientific Study & Research
Chemistry & Chemical Engineering, Biotechnology, Food Industry

ISSN 1582-540X

ORIGINAL RESEARCH PAPER

STUDY CONCERNING THE DAMPING OF PRESSURE WAVE PROPAGATION IN ADIABATIC EVOLUTION

Liliana Topliceanu[1], Carmen-Violeta Muraru[2], Iulian Furdu[2], Petru Gabriel Puiu[1*]

[1]Vasile Alecsandri University of Bacau, Engineering Faculty, Calea Marasesti, no. 157, 600115, Bacau, Romania
[2]Vasile Alecsandri University of Bacau, Science Faculty, Calea Marasesti, no. 157, 600115, Bacau, Romania

*Corresponding author: ppgabriel@ub.ro

Received: December, 14, 2020
Accepted: March, 22, 2021

Abstract: The propagation of acoustic waves affects people's daily life but especially the working conditions in certain industrial areas, including in the chemical and food sectors. In order to dampen noise and to improve the quality of the working environment, the phenomenon of pressure wave propagation must be studied. The paper analyzes the unidirectional pressure waves produced by a pneumatic machine operating at a constant speed. The hyperbolic equations with non-linear derivatives were studied using the method of finite difference and an original program was created to obtain the visualisation of the speed and pressure oscillations along the pipeline. The graphical representations of the oscillation speed and the pressure wave at different sections along the pipe have led to interesting conclusions that could be used to develop methodologies and systems for attenuating the pressure pulsation and therefore the noise.

Keywords: *attenuation of noise, finite difference method, pneumatics, wave propagation.*

INTRODUCTION

The phenomenon of pressure wave propagation often takes place naturally in the environment and is also present in many technical applications. The most illustrative example from nature, which influences the comfort of daily life, is the propagation of sonorous waves, of noise. In industry, the use of pneumatic systems have the main advantage of being non-polluting and for this reason, being preferred in sectors which imposed this condition, such as pharmaceutical or food industry. Depending on the design of the systems, the repetitive processes that the pneumatic cylinders execute can cause pressure waves propagating on the pipes. This this phenomenon produces noise, solicitation of pipes, connections and sealing elements, to mention only some of the most obvious effects. In fact any unsteady process in liquid and gas is accompanied by acoustic waves and therefore by noise.

The waves propagation, of different frequencies and pressures, depending of the source, have complex effect on living organisms. The problems of noise and its effect on human body, physiological and psychological, was studied starting from the middle of twenty century [1 – 3] and continued until today [4, 5].The issue has grown in importance with the increasing number of sources of noise induced by modern society, such as the road and air traffic [6, 7].

The diminishing of the noise could be realized in three ways: at source, on the route of transmission and to the receiver. It is obvious that the best way is to combat noise in the place where it occurs, followed by the second way. The third solution is still applied in industrial halls, in construction, but it is not convenient in everyday life. Considering the first to ways, it is important to identify and to solve the equations which describe the noise propagation. Afterwards, the amortization solutions could be found, depending on the type and source of the waves and ingenious dispositive could be designed.

The phenomenon of waves propagation is described by equations with non-linear partial derivatives of a hyperbolic type, which create difficulties to found analytical solutions and had led to a reconsideration of numerical methods of integration.

From a chronological point of view, the method of finite difference was among the first numerical methods developed, but, as compared to other more recent techniques (finite elements, spectral methods etc.), it maintains its advantages: a high level of efficiency, simplicity of use, low cost, no important calculation resources being needed.

The application of numerical methods in meteorology strated in 1922 with „Weather Prediction by Numerical Process", by Richardson L. F. and continues until today [8]. The problem of the network step dimension and of the convergence of the results obtained by numerical and analytical means was studied by Altford in 1974 [9] in the field of oil explorations. It was demonstrated that the finite difference method offers correct solutions to a sufficiently fine network step. The same conclusion was reached by other authors who have used the parallel between pressure wave propagation and electric current propagation, a parallel which generated a series of studies [10, 11]. Starting with 1960 s the use of the finite difference method for the calculation of pressure wave propagation was also the subject of many papers [12 – 14].

Numerical methods have been used throughout time in order to find solutions for the propagation of waves in various situations. They have been used in geography, in meteorology, in seismology, in medicine, the flowing of blood through the arteries being a form of pressure wave propagation [15 – 18].

One of the problems that come with the use of numerical methods is that of the stability of the obtained solutions. In fact, numerical methods offer an approximation of the real solution and thus involve errors. Some consistent studies of this subject were developed [19, 20], emphasizing the importance to correctly identify the limits of the stability domain for the approximate solutions obtained in the case of wave propagation.
Despite the enthusiasm manifested by a part of the researchers in the application of newer numerical methods, the finite difference method hasn't lost its usefulness and attractiveness and important works, which systematizes the steps that are necessary in the application of the finite difference method in the case of wave propagation were published in the last years [21 – 23].
The use of the finite difference method initially requires the establishing of the system of differential equations or of partial derivates appliable to any point of the field and its transformation into a system appliable only to certain points of the field which define the discretization network of the field. The main disadvantage of this method is represented by the use of the discretization rectangular network, which creates difficulties when it is used for fields with irregular forms or curved surfaces. In the analysis we propose, these problems do not exist, and the analysis of the stableness field of the solutions has been successfully solved, which leads to interesting conclusions regarding this phenomenon.
The stability of numerical solutions, essential in this situation, was studied through the construction of the error propagation diagram, by which it has been proved that in this case, too, the stableness of an equation with partial derivates is contained in its very form, and that, apart from this, it also depends on the value of the characteristic parameters (Strouhal and Reyolds's number), the fineness of the network (the ratio of the adimensional space and time steps) and its way of working.

MATERIALS AND METHODS

Defining the analyzed case

The researches which are the subject of the presented paper had as starting point the noise produced by reciprocating compressors that provides compressed air in different industrial branches and also those produced by lobe blowers used in wastewater treatment plants.
For the purpose of these researches it was considered the case of pressure waves produced in pipes as a result of the action of a piston activated by a crank rod mechanism. In fact this device simulated the functioning of any pneumatic machine working at a constant speed, so the conclusions obtained could be extended in other similar cases.
We consider the pipe of constant section and infinite length (to avoid the problems produced by the reflected wave), and the processes involving the gas atrest as being of an adiabatic type.
Given these hypotheses, the phenomenon is defined by the equations of: fluid movement, mass conservation and the adiabatic evolution of the gas:

$$V_t' + VV_X' + \frac{1}{\rho} P_X' = 0 \tag{1}$$

$$\rho'_t + \rho V'_X + V\rho'_X = 0 \tag{2}$$

$$P = K_p \rho^{\kappa} \tag{3}$$

In these equations κ is the adiabatic exponent, and K_p is a coefficient which establishes the ratio between pressure and density.
It can be observed that the expression of the pressure wave propagation speed is obtained from the relation (3).

$$P'_\rho = K_p \kappa \rho^{\kappa-1} = c^2 = \kappa RT \approx const. \tag{4}$$

Given the initial hypotheses, the variations of the ***c*** wave propagation speed due to density and temperature, which are valid in the real case, are neglected, and this speed is considered as being constant.
The boundary conditions that can be used are related to speed, as they are established by the movement of the piston which produces the pressure waves. Therefore the equations will be worked on in order to eliminate the other unknown items and to obtain only a speed relation V(X,t) for which solutions will be looked for in the field. Since we started from the hypothesis of the gas at rest (V=0), the equations of the system will be re-written by taking this into account and by replacing the pressure derivate according to x in the (relation 1) with the corresponding expression according to density. By deriving the (5) equation according to the time, and the (3) equation according to the space, and by also using the continuity equation, the movement equation becomes

$$V''_{t^2} + V'_t \frac{\rho'_t}{\rho} + K_p \kappa \left[(\kappa - 1)\rho^{\kappa-1} \frac{\rho'_t}{\rho} \frac{\rho'_X}{\rho} + \rho^{\kappa-1} \frac{\rho''_{Xt}}{\rho} \right] = 0 . \tag{5}$$

Going further, the $\rho(X,t)$ function being uniform and limited, it is eliminated on the basis of Schwartz's commutative relation of the second order mixt derivate $\rho''_{Xt} = \rho''_{tX}$, and thus from the system of two equations (1) and (2) it can be obtaine a sigle relation, having the following expresion:

$$V''_{X^2} - \frac{1}{c^2} V''_{t^2} - \frac{\kappa - 1}{c^2} V'_X V'_t = 0 \tag{6}$$

A non-linear equation with second order partial derivates of the hyperbolic type has been obtained, for whose solution a numerical integration method will be used, since the function is continuous, uniform and limited, and the initial and boundary conditions ensure the uniqueness of the solution.
For a more general relevance of the solution, the (4) relation is adimensionalized by using the new variables and adimensional functions:

$$x = \frac{X}{R}; \qquad v = \frac{V}{c}; \qquad \tau = \frac{t}{t_0} \ ,$$

resulting from the use of characteristic measures: R – the radius of the crank in the mechanism; c – the sound speed, that is, the speed with which the oressure wave is propagated; t_0 – the period of piston oscillation.
By using these notations, the (6) relation will take the form:

$$v''_{x^2} - \frac{R^2}{c^2 t_0^2} v''_{\tau^2} - \frac{\kappa - 1}{c^2} \frac{c^2}{R t_0} v'_x v'_\tau = 0 . \tag{7}$$

We observe that the $\frac{R}{ct_0} = Sh$ ratio is Strouhal's similitude criterion, which represents the homochronicity criterion and the (7) relation then becomes:

$$\mathrm{v}''_{x^2} - \mathrm{Sh}^2 \mathrm{v}''_{\tau^2} - (\kappa - 1)\mathrm{Sh}\mathrm{v}'_x \mathrm{v}'_\tau = 0 \tag{8}$$

Taking into account that Strouhal's criterion is influenced by the relatively great value of the ***c*** speed, which is one of the chosen characteristic measures, we notice the very small value of Strouhal's number, due to the great difference between the R radius and the $\lambda = ct_0$ wave length

$$Sh = \frac{R}{ct_0} = \frac{R}{\lambda} << 1. \tag{9}$$

The application of the finite difference method

The iterative algorithm for the approximation of the numerical solution of an equation with partial derivates consists in a procedure of replacement of the respective equation by an algebric equation constituting a system of equations with finite difference in the nodes of a network that discretizes the considered field. The closer the network nodes are, the better the precision of the method will be. Also, since equations with partial derivates generally admit infinity of solutions, in order to uniquely characterize the process one must add to the equation a sufficient number of specific conditions [22, 23]. Consequently, a rectangular network is constructed with different admînesional steps $\delta x \neq \delta\tau$, through which the phenomenon is studied. The developments of the speed function are noted in the network nodes, from which the expressions of the partial derivates are extracted which are necessary for the obtaining of the algebraic relation associated to the equation with partial derivates of a hyperbolic type.
By replacing the expression of these derivates in the adimensional (8) equation, there follows, in an initial form, the relation:

$$\frac{v_1 - 2v_0 + v_3}{\delta x^2} - Sh^2 \frac{v_2 - 2v_0 + v_4}{\delta\tau^2} - (\kappa - 1) Sh \frac{(v_1 - v_3)(v_2 - v_4)}{4\delta x \delta\tau} = 0 \tag{10}$$

Since movement in the network is done by always advancing with point 2, it is useful to remove the speed expression in this node according to the other measures (v_0, v_1, v_3, v_4, Sh, $\frac{\delta x}{\delta\tau}$). An associated algebraic relation is obtained:

$$v_2 = \frac{v_1 + v_3 - 2v_0 + Sh^2 \frac{\delta x^2}{\delta\tau^2}(2v_0 - v_4) + \frac{\kappa - 1}{4} Sh \frac{\delta x}{\delta\tau}(v_1 - v_3) v_4}{Sh^2 \left(\frac{\delta\kappa}{\delta\tau}\right)^2 + \frac{\kappa - 1}{4} Sh \frac{\delta x}{\delta\tau}(v_1 - v_3)} \tag{11}$$

The analysis of the speed evolution in time, in sections situated at different lengths of the pipe, requires the establishing of certain numerical values. Starting from the real case analysed, a speed is adopted at first: n = 1500 rot·min^{-1} (N_0 = 25 rot·sec^{-1})
The dimensional time step δt and the adimensional step δτ of the network are established according tot he relations:

$$t_0 = \frac{1}{N_0} = \frac{1}{25} = 0{,}04s; \quad \delta t = \frac{t_0}{10} = 0{,}004s$$
$$\delta\tau = \frac{\delta t}{t_0} = \frac{0{,}004}{0{,}04} = 0{,}1 \quad - a\,dim\,nsional\ step \tag{12}$$

For the radius of the crank in the mechanism the value R = 0.08 m is adopted, and for the wave propagation speed, c = 343m·s^{-1}. Taking in consideration these values, it follows that $Sh = \frac{R}{ct_0} = \frac{25 \cdot 0{,}08}{343 \cdot 1} = 0{,}0058309$.

Boundary and initial conditions

The study of the movement is performed by considering the starting point at a value of π/2, so for t=0, the piston is in the middle of the return stroke. This leads to the following expression for the X stroke and V speed (in dimensional terms) of the piston:

$$X = R\cos\left(\varphi + \frac{\pi}{2}\right) = R\cos\left(\omega t + \frac{\pi}{2}\right)$$
$$V = X = X_t' = -R\omega\sin\left(\omega t + \frac{\pi}{2}\right) = -R\omega\sin\left(\frac{2\pi t}{t_0} + \frac{\pi}{2}\right) = -R\omega\sin\left(2\pi\delta t + \frac{\pi}{2}\right) \tag{13}$$
$$\omega_0 = \frac{2\pi n_0}{30} = 2\pi N_0 = \frac{2\pi}{t_0} = 50\pi$$

There follows the only boundary condition which we impose for the adimensional equation:

$$v(0,\tau) = \frac{V}{c} = -\frac{R\omega}{c}\sin\left(2\pi\delta\tau + \frac{\pi}{2}\right) \tag{14}$$

which, for the initial moment τ = 0, becomes:

$$v(0,0) = -\frac{R\omega}{c} \tag{15}$$

For the calculation of speed in the considered network nodes, we adopt the value in the initial point v (0,0) and on the whole (C) characteristic (Figure 1), and, for (C^{-1}), we adopt the values obtained for a δτ step before the initial moment (down the 0τ axis), corresponding to an anterior time, when the piston has not yet reached the π/2 position.

The stability of the numerical solution

Following the analysis of the initial and boundary conditions, and of the characteristic on which the wave is propagated, we admit that the network can be covered in the first phase in the τ direction. For this way of network covering, we will study the appearance and propagation of a calculation error.

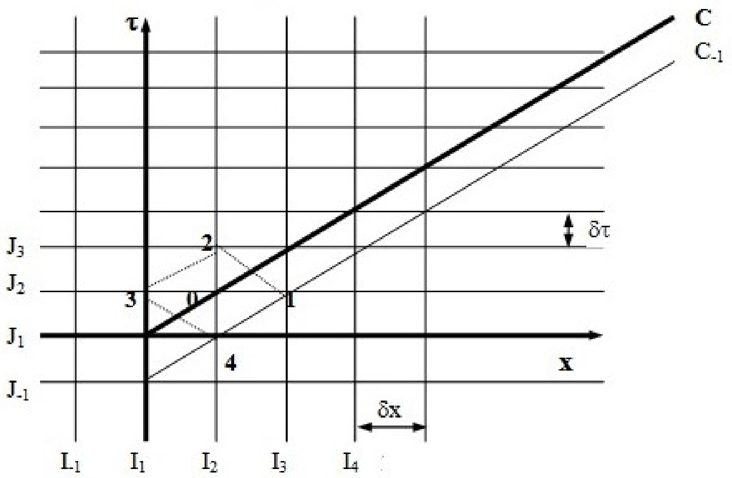

***Figure 1**. Network built in the area*

The relation of error propagation on the time axis

It can be observed that the error recurrence relation will have a double trace due to the calculation errors in the 0 and 4 points.

By taking this thing into account and by calculating the difference

$$\delta v_2 = (v_2 + \delta v_2) - v_2,$$

we can obtain the generalized expression of the error recurrence formula for the movement on the axis of time:

$$\delta v_{n+1}^{+\tau} = \frac{2\left[-1 + Sh^2\left(\frac{\delta x}{\delta \tau}\right)^2\right]}{Sh^2\left(\frac{\delta x}{\delta \tau}\right)^2 + \frac{\kappa-1}{4} Sh \frac{\delta x}{\delta \tau}(v_1 - v_3)} \delta v_n + \frac{\left[-Sh^2\left(\frac{\delta x}{\delta \tau}\right)^2 + \frac{\kappa-1}{4} Sh \frac{\delta x}{\delta \tau}(v_1 - v_3)\right]}{Sh^2\left(\frac{\delta x}{\delta \tau}\right)^2 + \frac{\kappa-1}{4} Sh \frac{\delta x}{\delta \tau}(v_1 - v_3)} \delta v_{n-1} \tag{16}$$

In a compressed notation, the (16) relation can be written in this form:

$$\delta v_{n+1}^{+\tau} = C_n \delta v_n + C_{n-1} \delta v_{n-1} \tag{17}$$

By analysing the error propagation relation we observe that its relaxation depends on the fineness of the network, therefore on the step ratio, present in the $Sh\frac{\delta x}{\delta \tau} = S_R$ term, which we have called the Strouhal number of the network and noted by S_R. The (15) relation will now have the simplified form:

$$\delta v_{n+1}^{+\tau} = \frac{2\left[-1 + S_R{}^2\right]}{S_R{}^2 + \frac{\kappa-1}{4} S_R(v_1 - v_3)} \delta v_n + \frac{\left[-S_R{}^2 + \frac{\kappa-1}{4} S_R(v_1 - v_3)\right]}{S_R{}^2 + \frac{\kappa-1}{4} S_R(v_1 - v_3)} \delta v_{n-1} \tag{18}$$

In order to determine the conditions for the reduction of the approximate calculation error is resulting from the used method, and to find a stable solution. Two main conditions have been identified, which are presented below.

The boundary case of cancelling the denominator

The main condition for the existence of a solution to the (17) relation is that the denominator should be other than zero.There follows from here a condition for speed which dependent on Strouhal's number:

$$v_3 \neq v_1 + \frac{4S_R}{\kappa - 1} = v_1 + \frac{S_R}{0{,}1}. \tag{19}$$

Coefficients C_n şi C_{n-1} should be less than 1

Considering that the spatial step is very small ($\delta x \langle\langle$) and the frequency of oscillations is very high, in order to simplify calculations, the v_1-$v_3 \approx 0$ approximation can be made, only for the denominator term, which needs to be other than zero. It follows that:

$$\begin{cases} C_n < 1 \\ -2 + 2S_R^2 < S_R^2 + \dfrac{\kappa - 1}{4} S_R (v_1 - v_3) \end{cases} \tag{20}$$

and, with the simplification imposed above, this becomes

$$S_R^2 < 2 \text{ so } Sh\frac{\delta x}{\delta \tau} < \sqrt{2}, \qquad \delta x < \frac{\sqrt{2}}{Sh} \cdot \delta \tau$$

The condition obtained for the network step involved again the Strouhal's number.

The second $C_{n-1} < 1$ coefficient

$$-S_R^2 + \frac{\kappa - 1}{4} S_R (v_1 - v_3) < S_R^2 + \frac{\kappa - 1}{4} S_R (v_1 - v_3). \tag{21}$$

It follows that the second coefficient does not generate any stability problems.

Since all stability conditions also involve the term noted as S_R, we go back to it and also search for the stability condition.

$$S_R = Sh\frac{\delta x}{\delta \tau} = \frac{R}{ct_0} \frac{\delta X}{R} \frac{t_0}{\delta t} = \frac{\delta X}{c\delta t} \tag{22}$$

The relation we have obtained represents a (local) network Sh, therefore the ratio between the spatial step and a chosen time step – the fineness of the network – also influences the stability of the solution.

Drawing the diagrams of calculation error propagation

The following methodology for the drawing of the error relaxation diagram is used: an initial error $\delta v_i = 1$ is adopted and its propagation is followed graphically in a network supposed to be perfectly stabilized, considering the simplifying hypothesis $v_1 \cong v_3$, and using the iterative procedure.

In this case, the error propagation relation will be

$$\delta v_{n+1}^{+\tau} = C_n(S_R)\delta v_n + C_{n-1}(S_R)\delta v_{n-1}, \tag{23}$$

considering at the first step

$$\delta v_n = 1; \qquad \delta v_{n-1} = 0$$

The two coefficients from the (23) relation have these expressions:

$$C_{n-1} = -1; \qquad C_n = \frac{2(-1+S_R^2)}{S_R^2} \tag{24}$$

Different values are given to S_R and the way in which the error propagates, is graphically drawn (Figure 2), searching for those S_R values through which a stable solution is obtained. The S_R boundary stability values are obtained from the condition

$$\delta v_{n+1} = \delta v_n = \delta v_{n-1}, \tag{25}$$

a relation which is valid when the error is maintained the same. In the present case, the boundary values for which the error is equally transmitted are: $S_R = 0.7072$, $S_R = 1.4140$ The conclusion is that, for this way of covering the network, in parrallel with time axis, the error is not diminished and a stable solution cannot be obtained. The graphic representation (Figure 2) contains also other values of S_R (Strouhal of the network).

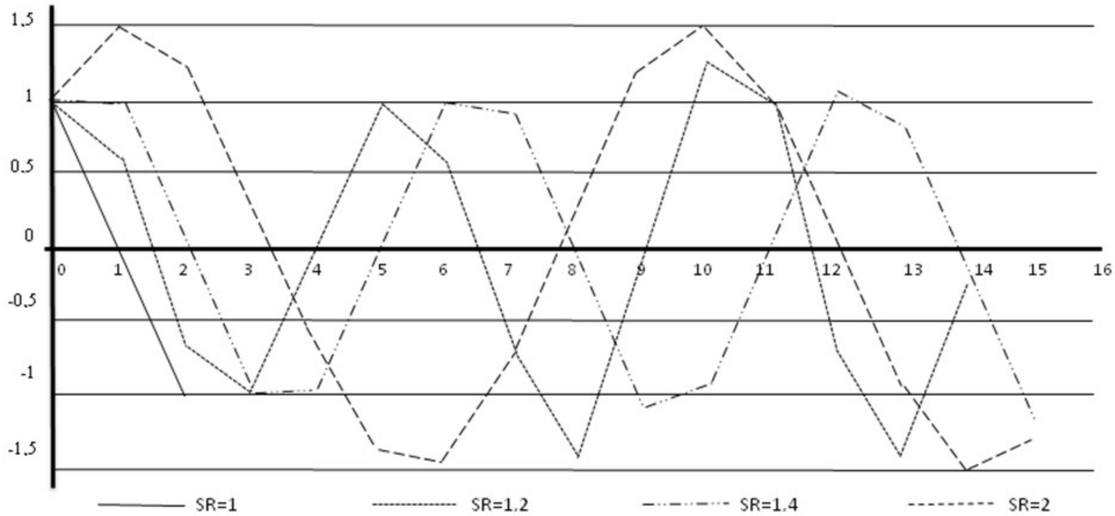

Figure 2*. Chart error propagation for network browsed parallel with time axis*

We will now consider the covering of the space-time network according to the direction of the diagonal to the grid that is, going parallel with the direction of wave propagation. For this way of covering the network, the error propagation relation will have a simple trace, therefore the calculation error will be transmitted to the network from a single point, point 3, situated behind in relation to the direction of network covering.

We will use the same reasoning and we will follow the same steps as in the previous case. This time the conditions obtained are: $S_{R1} > -1.0018318$ and $S_{R2} > 0.998168$.

It follows that, to reduce the approximate calculation error and to obtain a stable solution, the S_R values should be greater than 0.998168.

Drawing the diagrams of calculation error propagation

In order to draw the error relaxation diagram we use the same methodology presented above: an initial error $\delta v_i=1$ is supposed and the error convergence is studied according to the relation $\delta v_{n+1}=C_n(S_R)\,\delta v_n$, for different S_R values (Figure 3).

The boundary values for error relaxation, that is, the values for which the error is maintained at a constant level, are obtained from the $\delta v_{n+1}= \delta v_n$ condition, from which it follows that $S_R = 0.998168$ and we adopt $S_R = 1$.

To exemplify, several values are given and it is observed that the error tends to zero (Table 1, for S_R=1.1 and Table 2, for S_R=1.5).

S_R=1.1
$$C_n = \frac{1-1.1\cdot 3.66\cdot 10^{-3}}{1.1^2} = \frac{0.9959}{1.21} = 0.823$$

Table1.*The error value for S_R=1.1*

δv_1=0.8232	δv_5=0.3775	δv_9=0.1732	δv_{13}=0.07946	δv_{17}=0.03645
δv_2=0.6773	δv_6=0.3107	δ_{v10}=0.1425	δv_{14}=0.0654	δv_{18}=0.0300
δv_3=0.5574	δv_7=0.2557	δv_{11}=0.11732	δv_{15}=0.0538	δv_{19}=0.02469
δv_4=0.4587	δv_8=0.2104	δv_{12}=0.09656	δv_{16}=0.04429	δv_{20}=0.02032

S_R=1.5
$$C_n = \frac{1-1.5\cdot 3.66\cdot 10^{-3}}{1.5^2} = 0.442$$

Table 2. *The error value for S_R=1.5*

$\delta v_{1=}$ 0.442	δv_4=0.0380	δv_7=0.00382	δ_{v10}=0.00028	δv_{13}=0.00002446
δv_2=0.1952	δv_5=0.0168	δv_8=0.00145	δv_{11}=0.0001252	δv_{14}=0.0000108
δv_3=0.0860	δv_6=0.007426	δv_9=0.000641	δv_{12}=0.0000553	δv_{15}=0.00000477

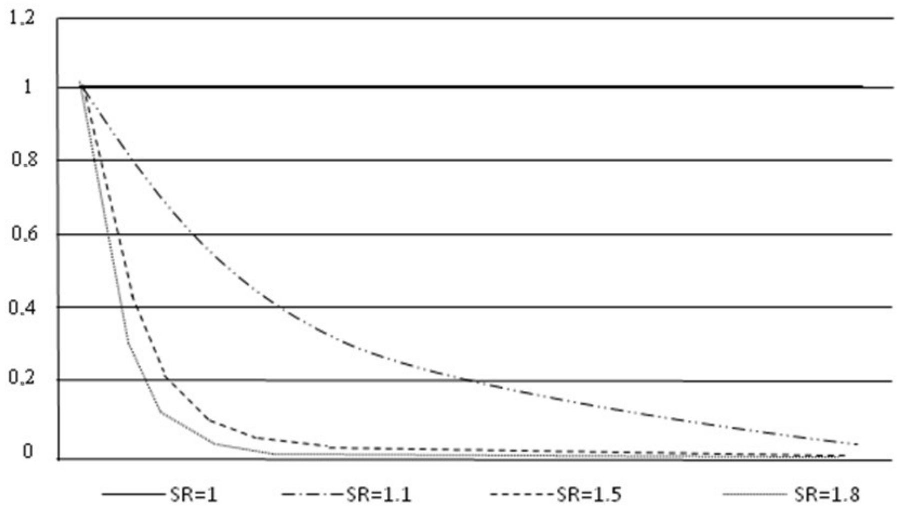

Figure 3*. Chart error propagation on the 3-2 direction*

The error relaxes (decreases) only when the network is covered diagonally, in the 3-2 direction, therefore in parallel with the movement characteristic and, the error is more or less attenuated, according to the value given to SR (Figure 3).

In order to cover the network of $\delta x \neq \delta\tau$ steps and to determine the speed values in nodes of the grid, a program was built, which allows the calculation of speeds in the domain and, subsequently, of pressures. In this respect, the Matlab medium has been used.

Determining the pressure oscialltion values and making the graphic representation

We search for the pressure expression and then for its solution also by means of numerical methods, benefiting from the already determined speed values in the network nodes.
Since we know the working hypotheses from the previuos section, we re-write the equations that describe the phenomenon:

$$V_t' + \frac{1}{\rho} P_X' = 0 \tag{26}$$

$$\rho_t' + \rho V_X' = 0 \tag{27}$$

$$P = K_p \rho^{\kappa} \tag{28}$$

The equations are worked on in a manner similar to that which has been used to obtain the speed expression and the same variables and adimensional functions presented above are used.
Given that the only boundary condition we can use is in the piston section, the initial section, we must determine the pressure variation according to time at the level of this section, after which we will be able to cover the network.
It must also be added that, for the determination of pressure values in the network nodes the half step is used, and, consequently, the expression used for the pressure calculation through the numerical method will have this form:

$$p_2 = p_0 - \kappa \frac{1}{Sh} p_0 \delta\tau \frac{v_1 - v_0}{\delta x} = p_0 - \kappa p_0 \frac{1}{S_R}(v_1 - v_0) \tag{29}$$

By means of this relation, if we know the speed values determined previously, we can calculate the pressure values for the whole domain.
The adimensional $p(0,0) = 1$ value is imposed for the initial section, the piston section and the initial moment.

RESULTS AND DISCUSSIONS

The use of the (11) and (26) relations for the determination of speed and pressure values in the nodes of the considered network, of (τ,x) coordinates, required the development of a calculation programme, as was toldbefore. The programming medium Matlab was used, in order to obtain the graphic representations of pressure and velocity.
At first, the values on the time axis (0,τ) were calculated by using the (13) boundary condition for speed, which is determined by the movement of the piston actuating mechanism (Figure 4), after which, by using these values as initial working data for the programme, the speed and pressure values for the entire network were obtained, and thus the oscillation speed and the pressure in the pipe were established.

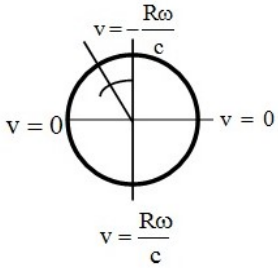

Figure 4. *The movement of the piston actuating mechanism*

The graphic transposition of the results obtained mathematically is presented in figures 5 ÷ 9, which foreground the success of the numerical integration method that was used, and the information obtained as a result of the graphic visualization allow us to draw interesting coherent conclusions on the studied phenomenon.

The graphic representations have been made for various values of the S_R number, which also leads to various values of the $\delta x/\delta\tau$ ratio. This will permit to analysis the influence of the finesse of the network on the results obtained.

For all S_R used, in the initial (x = 0) section, corresponding to the piston section, the speed chart has a permanent character, as it represents the movement produced by the crank rod mechanism in fact, whereas the average pressure in this section has a decreasing variation (Figure 5).

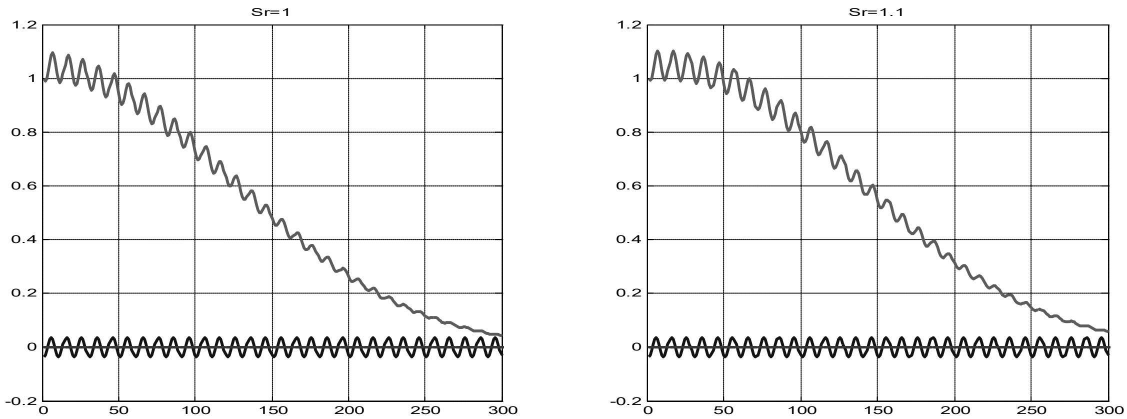

Figure 5. *The variation of pressure (in up) and velocity (in down) in the initial section for SR=1 and SR =1.1*

At the initial moment considered to be $\pi/2$, the piston is at the return stroke, which causes a small decrease of pressure below the initial value of $p_0=1$, after which, at the forward stroke of the piston (a movement in the positive sense of the ox axis), there appears an increase of pressure that is much higher than the initial value in the pipe, because, at each advancing sequence of the piston, an ever denser mass is pushed, due to the previous compressions.

On the other hand, at the return stroke, the movement of the piston to the initial position is done through a pressure mass that is already decreased, so the fluid quantity which it causes to move is much smaller. The repetition of the piston movement leads to a drastic decrease of the average pressure value at the level of the considered section.

The same process is manifested in the other sections of the pipe, the decrease of the average pressure value being accompanied by an increase in the local particle oscillation speed (Figures 6 and 7).

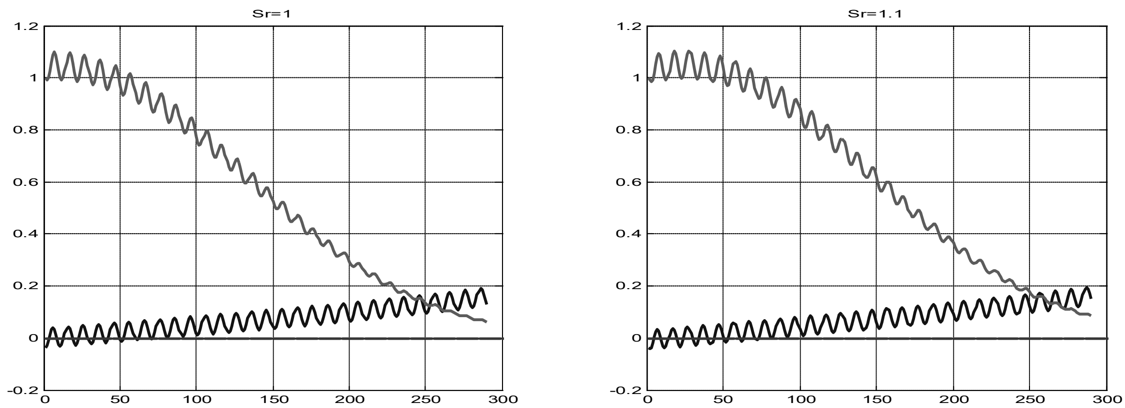

Figure 6. *The variation of pressure (in up) and velocity (in down) at 10 adimensional space steps (x=10δx) for SR=1 and SR =1.1*

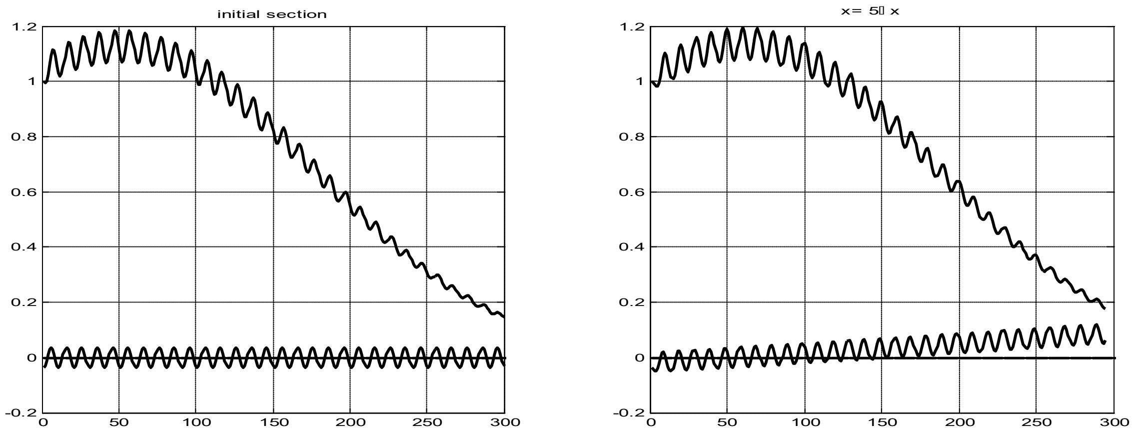

Figure 7. *The variation of pressure (in up) and velocity (in down) in the initial section for SR = 1.5 and x=5δx*

The behavior described above, can be also observed for other S_R numbers, in the piston section and in the other sections of the pipe, so for different lengths of pipe.

It is important to note that different S_R numbers mean different dimensional space steps, taking in consideration that the time step remains constant, which explain the modifications in the graphics allure, modification which will be explained in the next paragraphs.

The decrease of the average pressure value in time can be interpreted from the physical point of view as a pipe emptying phenomenon due to a gas movement caused by the compression waves. Therefore, although the fluid was at rest at the initial moment, the wave propagation generated by the piston actuated by the crank rod mechanism leads to a movement or circulation of the fluid in the pipe.

The decrease of the average pressure value is simultaneous with a process of decrease in the pressure wave amplitude which is affected by an attenuation phenomenon.

The wave attenuation is due both to the fact that in a low pressure environment the action of the piston creates waves of ever smaller amplitude, and to the fact that a relative flowing of the fluid appears on both sides of the wave front, from the maximum to the minimum, as a natural pressure equalizing phenomenon, which thus reduces the oscillation amplitude value.

The oscillatory speed transmitted to the fluid by the piston in the initial section increases to positive values in the next sections along the pipe, while the average pressure value decreases, which emphasizes the same fluid flowing phenomenon. The speed oscillation is no longer related to the rest value (v = 0), but to ever higher positive values.

There appears a metamorphosis of the pressure wave as a result of the variation in the local wave propagation speed due to the local modification of density.

The analysis of the graphic representations for various sections and various Strouhal numbers S_R network foregrounds the appearance of a temporary mass accumulation phenomenon, manifested more significantly at bigger distances along the pipe ($S_R = 1.1$; $S_R = 1.5$, step 50, (Figure 8) and for higher S_R numbers ($S_R = 2$; step 5 and 10, Figure 9). We remind that a higher number means a higher dimensional space step.

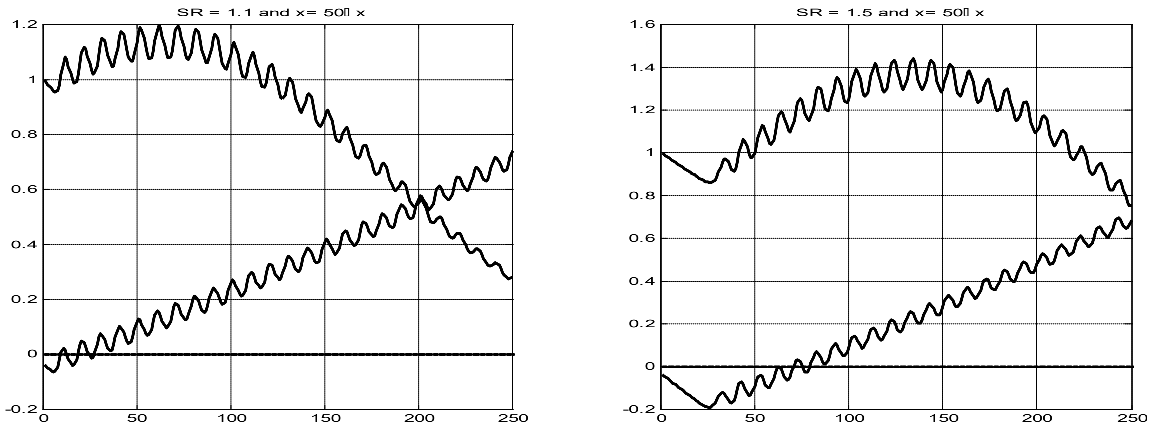

Figure 8. *Mass accumulation phenomenon (pressure in up and velocity in down)*

This phenomenon is caused by the fact that the fluid masses moved by the pressure wave propagation meet a fluid mass at rest which reduces their movement. Consequently, a mass accumulation is produced, a higher pressure than in the rest of the pipe, which eventually determines, according to the sanctioned physical phenomenon, a flowing of the gas towards the lower pressure end.

The graphic representations obtained for various S_R numbers foregrounds the importance of the value of this parameter, and, implicitly, of the fineness of the network, for the aspect and stability of the solution.

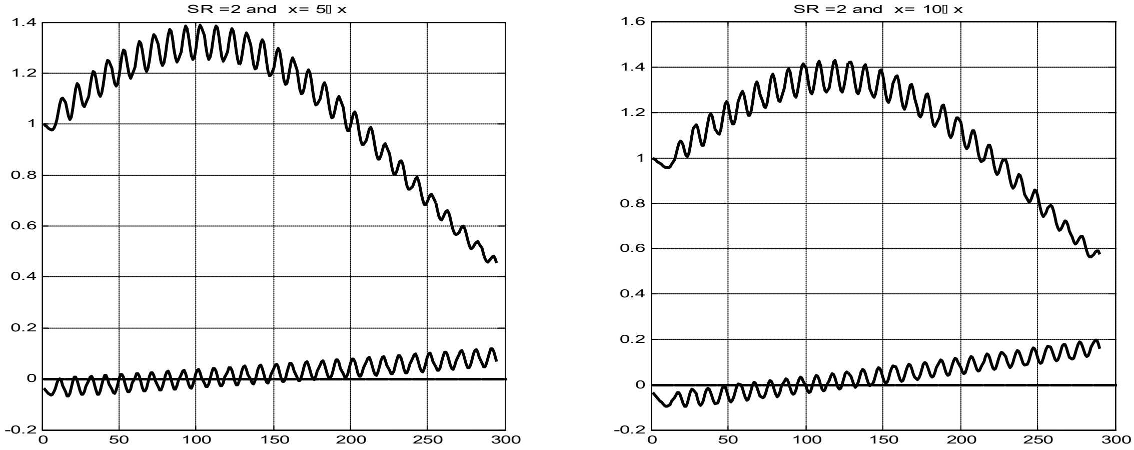

Figure 9. *Mass accumulation phenomenon (pressure in up and velocity in down)*

Summarizing, the study about the propagation the pressure waves generated by a pneumatic machine working at a constant speed, emphasized the followings aspects:

- a non-permanent rapid phenomenon of pressure wave propagation based on the impenetrability of matter;
- a non-permanent slow phenomenon which asymptotically tends to a permanent state and which consists in the decrease of the average pressure in time
- a pressure wave attenuation consisting in the decrease of the pressure oscillation amplitude;
- a metamorphosis of the pressure wave shape due to the local variation of the wave propagation speed, caused in its turn by the local modifications of density;
- the change of the fluid state from rest to movement, which eventually causes the emptying of the pipe

CONCLUSIONS

Using numerical integration methods allows the numerical solving of the phenomena defined by non-linear hyperbolic equations and a pertinent analysis of the evolution of these phenomena in time and space.
The finite difference method can be successfully applied in the case of pressure waves propagated in a pipe, allowing us to study the internal variations at the level of oscillation speed and pressure and on this basis there can be developed methodologies and systems for the attenuation of pressure pulsation effects such as noise.
The correctness of the results obtained by means of the finite difference method largely depends on the fineness of the network built in the field. The limits of the method are generated by the error reduction conditions, the stability conditions, which are in fact generated by any numerical solution.
Having as starting point the conclusions obtained, different technical solutions can be developed for an effectively controlled dampening of the pressure pulsation. Taking in consideration the natural attenuation of the pressure which appears after a number of space steps, it can be concluded that a system which will force the wave to go through a device until the attenuation process takes place could be a good way to diminish the

noise. Other solutions which take in consideration the pulsating character of the wave and intend to use this for an efficient damping device are being explored by the authors and are the subject of a patent.
The analysis developed in this paper can be adapted to any situation of pressure wave propagation in a pipe: it can be applied for adiabatic moving fluids (V≠ 0), for barotropic processes, for ideal or viscous fluids. It requires only basic knowledge of fluid mechanics and skill in the use of the mathematical apparatus.

REFERENCES

1. Krasilnikov, V.A.: *Sound waves in air, water and solids*, Bucharest, Technical Publishing House, **1957**;
2. Ursoniu, C., Dumitrescu, C.: Noise pollution and its consequences, Timisoara, Facla, **1976**;
3. Maestrello, L., Parikh, P., Bayliss, A.: Instability and sound emission from a flow over a curved surface, *ASME – Journal of Vibration, Acoustics, Stress and Reliability in Design*, **1988**, **110** (4), 538-544;
4. Tabraiz, S., Ahmad, S., Shehzadi, I., Bilal, M.A.: Study of physio-psychological effects on traffic wardens due to traffic noise pollution; exposure-effect relation, *Journal of Environmental Health Science and Engineering*, **2015**, **13** (1), 13-30;
5. Münzel, T., Schmidt, F.P., Steven, S., Herzog, J., Daiber, A., Sørensen, M.: Environmental Noise and the Cardiovascular System, *Journal of the American College of Cardiology*, **2018**, **71** (6), 688-697;
6. Kerns, E., Masters, E.A., Them, C.L., Calvert, G.M.: Cardiovascular conditions, hearing difficulty, and occupational noise exposure within US industries and occupations, *American Journal of Industrial Medicine*, **2018**, **61** (6), 477-491;
7. Zannin, P.H.T., Bunn, F.: Noise annoyance through railway traffic – a case study, *Journal of Environmental Health Science and Engineering*, **2014**, **12** (1), 14;
8. Lynch, P.: The origins of computer weather prediction and climate modeling, *Journal of Computational Physics*, **2008**, **227** (7), 3431-3444;
9. Alford, R.M., Kelly, K.R., Booret, D.M.: Accuracy of Finite-Difference Modeling of the Acoustic Wave Equation, *Geophysics*, **1974**, **39** (6), 759-918;
10. Yee, K.: Numerical solution on initial boundary value problems involving Maxwell's equations in isotropic media, *IEEE Transactions on Antennas and Propagation*, **1966**, **14** (3), 302-307;
11. McFall, K.S., Mahan, J.R.: Artificial Neural Network Method for Solution of Boundary Value Problems With Exact Satisfaction of Arbitrary Boundary Condition, *IEEE Transactions on Neural Networks*, **2009**, **20** (8), 1221-1233;
12. Alterman, Z., Karal, F.C.Jr.: Propagation of elastic waves in layered media by finite-difference methods, *Bulletin of the Seismological Society of America*, **1968**, **58**, 367-398;
13. Fenton, J.D.: The numerical solution of steady water wave problems, *Computers and Geosciences*, **1988**, **14** (3), 357-368;
14. Antunes, A.J.M., Leal-Toledo, R.C.P., Teixeira da Silveira Filho, O., Magalhães Toledo, E.: Finite difference method for solving acoustic wave equation using locally adjustable time-steps, *Procedia Computer Science*, **2014**, **29**, 627-636;
15. Emmanuel, J., Okhuese, V.Al.: Numerical solution for weather forecasting using finite difference scheme, *Journal of Mathematics*, **2020**, **16** (3), 49-56;
16. Albert, B., Petrila, T., Mathematical models and numerical simulations for the blood flow in large vessels, *INCAS Buletin*, **2012**, **4** (4), 3-10;
17. Chakravarty, S., Mandal, P.K.: Two-dimensional blood flow through tapered arteries under stenotic conditions, *International Journal of Non-Linear Mechanics*, **2000**, **35**, 779-793;
18. Paprota, M.: On pressure forcing techniques for numerical modelling of unidirectional nonlinear water waves, *Applied Ocean Research*, **2019**, **90**, 101857;
19. Tveito, A., Winther, R.: *Introduction to Partial Differential Equations: A Computational Approach*, Springer, **1998**, 39-82;

20. Michelle, L., Fornberg, G.B., Reeger, J.A.: Stability ordinates of Adams predictor-corrector methods, *BIT Numerical Mathematics*, **2015**, **55** (3), 733-750;
21. Langtangen, H.P.: Finite *difference methods for wave motion*, **2016**, available at : http://hplgit.github.io/num-methods-for-PDEs/doc/pub/wave/pdf/wave-4print-A4-2up.pdf. Accessed November 2019 ;
22. Lundgren, L., Mattsson, K.: An efficient finite difference method for the shallow water equations, *Journal of Computational Physics*, **2020**, **422**, 109784;
23. Eriksson, S.: A dual consistent finite difference method with narrow stencil second derivative operators, *Journal of Scientific Computing*, **2018**, **75** (2), 906-940.