THEORETICAL ASPECTS ABOUT SIMULATING THE REACTION TO IMPACT OF APPLES ON RIGID PLANE SURFACES

CĂSĂNDROIU TUDOR[1], STAN GABRIEL CĂTĂLIN[1], VICTOR MOISE[1]

"Politehnica" University of Bucharest, Splaiul Independentei no. 313, sector 6, RO-060042 Bucuresti Romania

Abstract: The study presents a model of the fruits reaction to impact, expressed in a differential equation built on the hypothesis that the fruits are rheological Kelvin – Voight bodies with a linear viscoelastic behavior. The differential equation was integrated using the Runge-Kutta method using a Turbo-Pascal application conceived by the authors. The force-time and force-strain curves have been determined for the impact. These curves simulate the real ones, obtained during experiments and from science literature and they are similar. Knowing these curves helps obtaining the characteristics of the impact, necessary to evaluate the damage degree of the apples and/or the functional and design parameters of the mechanical systems.

Keywords: impact, viscoelastic, apples, mathematical model.

1. INTRODUCTION

During processing (sorting – packaging), the most frequent stress the fruits suffer is the impact between the fruits themselves and also between the fruits and the surfaces they have contact with. For agriculture products the impact is defined for contact velocity higher than 0,24 m/s, [1, 3], while for the activity process of mechanical systems the contact velocity is usually higher than 0,3 m/s. As a stress effect, the fruits get damaged. The highest losses of fruits during storing period, about 10 – 12 %, are due to mechanical damages (both internal and external) during manipulation, transportation etc. [1, 2]. Damages caused when tensions are higher than the elastic limit of the fruit's pulp, reaching the tensile limit, the tissue suffers concussions highlighted by the pulp's change of color in brown or cracks, [1, 3, 13].

Numerous experimental research were meant to evaluate correlation of the damage degree with the characteristics of the force – strain curve or force – time, during impact, in order to use the information obtained to appropriately choose the processing parameters of the mechanical systems and/or to evaluate the damage degree caused by a certain work regime, [3, 6, 9].

Experimental research based on the fruit's reaction to impact in order to appropriately correlate it with the solidity (consistence) of the structure (tissues) directly linked to the ripeness degree, were used in the fruits sorting process, [4, 5, 10]. Of high importance in this matter are the applications [4, 10] in which it is shown the indicator which best sorts the different degrees of ripeness correlated with the texture changes, either as the parameter defined by:

Corresponding author, email: gaby_cata_ro@yahoo.com

$$C = F_{\max} / t_c \;; \tag{1}$$

$$C^* = F_{\max} / t_c^2 \tag{2}$$

where F_{max} - the maximum force of impact, t_c– the time between the beginning of the impact until the F_{max} (time to peak) are reached. We specify the impact reaction can be used in quality assessment of different varieties of fruits related to their selection in order to improve the quality of the fruits varieties.

In many research studies, apples have been considered homogeneous and isotropic materials, [7,8], but recent research take into account the inhomogeneous and anisotropic characteristics in the apple's parenchymatous tissue, [4] in order to obtain more exact information.

All these highlights argues for further research, both theoretical and experimental.

The purpose of this study upon the fruits is to adapt an impact theoretical model (pattern) expressed in a differential equation built on the hypothesis that the fruits are rheological Kelvin – Voight bodies with a linear viscoelastic flow. The equation has been numerically integrated for a series of 8 coefficients of the apple's physical and mechanical known characteristics from which have been determined the impact force – time and the force - strain curves necessary to characterize the impact.

2. THEORETICAL CONSIDERATIONS

During the impact between viscoelastic bodies with rigid surfaces, an important role is played by the coefficient of restitution - the measure of the energy absorbed by the viscoelastic body. The absorbed energy is correlated with (affects) the mechanical damages suffered by apples.

The coefficient of restitution for an one-dimensional impact between two bodies in a translation motion, k, represents the relation (report, ratio) between the relative velocity of the two bodies involved (with opposite sign), at the end and at the beginning of the impact. Using figure 1a, the result is, [1]:

$$k = V_0 / V_i \tag{3}$$

This is a measure for the impact energy loss caused by internal sources: elastic waves, viscoplastic strains and frictions in the contact area, [11].

In order to consider irreversible the energy lost at the impact between a viscoelastic body and a rigid surface, the restitution coefficient has been numerically modeled taking the viscoelastic body as a Kelvin – Voight model (called an impact pair Figure1) who's damping (frictional) force is characterized by the relation:

$$F_d = \lambda_1 \delta^{\frac{3}{2}} \dot{\delta} \tag{4}$$

where

$$\lambda_1 = \frac{3}{2} \alpha k_c \tag{5}$$

The α parameter can be experimentally determined by measuring the restitution coefficient at the impact between a mobile fruit and a fixed rigid plane surface. For the Jonathan apples variety α = 0.42...0.69 [1, 4]. The law of the contact force becomes in this case (see Figure 1b), and δ is the strain (strain) in the contact area, k_c - contact rigidity, α is the coefficient from the equation:

$$k = 1 - \alpha V_i \tag{6}$$

The contact rigidity is given by the Hertz's Elastic Theory of Contact between the apple (seen as spherical body close to the contact point) and a rigid plane surface, [3, 9] where:

$$F_a = k_c \delta^{\frac{3}{2}} \tag{7}$$

From the Hertzian Contact Theory, [3, 9], the strain force F at the contact point between a sphere of d diameter and a rigid plane surface is:

$$F = 0.943 \cdot \frac{E}{1-\upsilon^2} \cdot d^{1/2} \delta^{\frac{3}{2}} \tag{8}$$

Comparing the equations (7) and (8), we get:

$$k_c = 0.943 \cdot \frac{E}{1-\upsilon^2} \cdot d^{1/2} \tag{9}$$

where E (Pa) – the Young modulus of elasticity for an apple, ν – Poisson's coefficient.

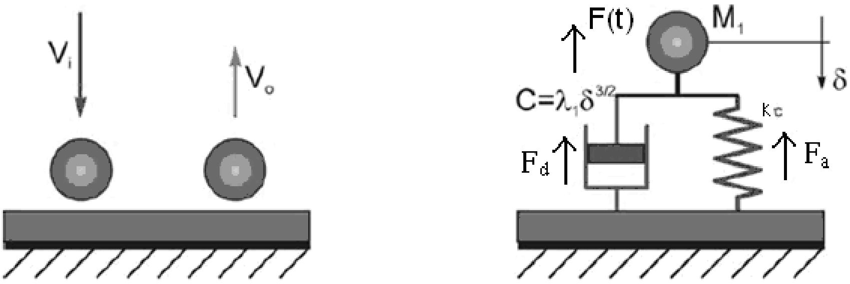

Fig.1. The model of the impact pair: a) model of the impact, b) equivalent viscoelastic model [11].

$$F(t) = F_d + F_a \tag{10}$$

Replacing the expressions of forces F_a şi F_d, we get the following equation:

$$F(t) = k_c \delta^{\frac{3}{2}} + \frac{3}{2} \alpha k_c \delta^{\frac{3}{2}} \dot{\delta} \tag{11}$$

From the motion equation for the central masses in an apple of mass M_1 during impact $M_1 \ddot{\delta} + F(t) = 0$, and taking into account the relation (11), we obtain:

$$M_1 \ddot{\delta} + \frac{3}{2} \alpha k_c \delta^{\frac{3}{2}} \dot{\delta} + k_c \delta^{\frac{3}{2}} = 0 \tag{12}$$

From this equation we can calculate the motion equation for the central masses in an apple in differential form, which expresses the strain δ of the apple during impact:

$$\ddot{\delta} + \frac{3}{2} \frac{\alpha k_c}{M_1} \delta^{\frac{3}{2}} \dot{\delta} + \frac{k_c}{M_1} \delta^{\frac{3}{2}} = 0 \qquad (13),$$

Integrating equation (13), results the variance of the strain during impact dependent of time, and using the equation (11), results the variation of the impact force dependent of time F(t) and the variation of the force depending on the strain, F(δ), representing the curves force - time or force – strain during impact. Using these curves we can appreciate the characteristics of the impact such as F_{max}, the duration of the impact, time to peak, the restitution coefficient.

The model can be adjusted to simulate the behavior (reaction) to impact for apples as viscoelastic materials. At lower impact velocity the contact surface behaves as an elastic hemispace.

This is the model created by Hunt and Crossley and it can be used to simulate the energy loss during impact between a rigid body and an elastic hemispace (the apple). For this reason the model can be adjusted to simulate the behavior during impact for apples, considering the fruits as viscoleastic materials, and at low impact velocity the contact surface behaves as an elastic hemispace.

Due to the natural variation of the parameters E, α, k_c, V_i, M_1 that change the values of the coefficients in the mathematical models for the fruits impact (linear viscoelastic bodies) expressed in the equations (13) and (11) for simulating the impact in 9 other different cases that generally describe the real situations.

In the equations (11) and (13) we have $A = \frac{3}{2}\alpha k_c$ so we can write them as**:**

$$F(t) = k_c \delta^{\frac{3}{2}} + A\delta^{\frac{3}{2}}\dot{\delta} \tag{14}$$

$$\ddot{\delta} + \frac{A}{M_1}\delta^{\frac{3}{2}}\dot{\delta} + \frac{k_c}{M_1}\delta^{\frac{3}{2}} = 0 \tag{15}$$

An updated physics model for the impact, elaborated by the authors, derived from the model in Figure 1b is based on the analysis of the force- strain curve in Figure 2.

Following the curve force – compression strain for a viscoelastic material (Figure 2) the OA segment represents the elastic behavior, AB is the segment where, along with the elastic behavior we can observe the viscous damping due to the viscous component having as result a viscoelastic behavior.

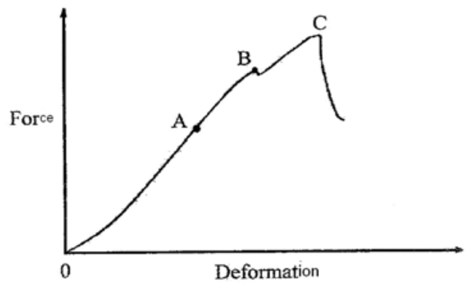

Fig. 2. The curve force - ideal strain for a viscoelastic material solicited to compression and constant velocity, [15].

On the AB segment of the curve force – strain can be observed multiplying damage upon cells spreading through the material, phenomenon assimilated to the plastic behavior observed at strains bigger than the ones corresponding to the point B bioyield point on the curve, (Figure 2).

Based on this concept and a suggestion in [16] we can make a supplement chart for the impact of viscoelastic bodies with rigid plane surface, from Figure 1b, shown in Figure 3 where: F_a – elastic segment of contact force; F_d – the absorbing viscous segment and F_p – dissipative segment given by the plastic strain.
In general form, the relation for the total force during impact will be:

$$F(t) = F_a(\delta) + F_d(\delta,\dot{\delta}) + F_p(\delta,\dot{\delta}) \tag{16}$$

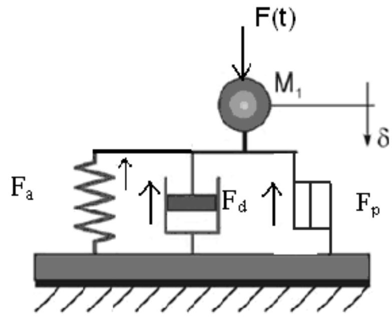

Fig. 3. Model for the pair impact – Kelvin equivalent model completed by the element with plastic behavior (Fp).

The contact force segment due to plasticity is present only at big strains followed by cells ruptures spreading through the product so that we can determine a critical value of this force F_{pc}

$$F_p(\delta,\dot{\delta}) = \begin{cases} 0, F_p < F_{pc} \\ F_p, F_p \geq F_{pc} \end{cases} \tag{17}$$

When the strains are minor, no ruptures are registered $F_p(\delta,\dot{\delta}) = 0$, equation (16) become:

$$F(t) = F_a(\delta) + F_d(\delta,\dot{\delta}) \tag{18}$$

Terms in eq. (18) can be expressed by the relations, [16,17]:

$$F_a(\delta) = K\delta^{3/2} \tag{19}$$

$$F_d(\delta,\dot{\delta}) = \beta\delta^{3/2}\dot{\delta} \tag{20}$$

where K – contact rigidity for impact, is the same K_c from eq. (9).

In equation (20) the coefficient β is dumping hysteresis factor which can be expressed in a new way in the equations [16, 17]:

$$\beta = \frac{3}{4}K\frac{1-k^2}{V_i} \tag{21}$$

where: k is the impact's restitution coefficient, given by the equation (6); V_i – the velocity at the beginning of the impact.

With the relations (19), (20), (21) replaced in (18) and the equation of the movement during impact, we get:

$$\ddot{\delta} + \frac{K}{M_1}\delta^{\frac{3}{2}} + \frac{3}{4}\frac{K}{M_1}\frac{1-k^2}{v_i}\delta^{\frac{3}{2}}\dot{\delta} = 0 \tag{22}$$

$$F(t) = K\delta^{\frac{3}{2}} + \frac{3}{4}K\frac{1-k^2}{v_i}\delta^{\frac{3}{2}}\dot{\delta} \tag{23}$$

If in the equation (21) we replace the restitution coefficient for impact k in equation (6), after calculating we get:

$$\beta = \frac{3}{4}K\alpha\left(2-\alpha^2 V_i\right) \tag{24}$$

Usually $V_i < 1$ m/s and considering that $\alpha < 1$, results that $\alpha^2 V_i << 1$, which permits neglecting the value of $\alpha^2 V_i$ în ec.(24), and we get:

$$\beta \cong \frac{3}{2}K\alpha = \frac{3}{2}k_c\alpha \tag{25}$$

Equation (25) is identical with (5), so $\lambda_1 \approx \beta$.

Following this hypothesis we can see that the differential equations of the force and strain during impact (15) and (17) are identical with (22) and (23).

3. NUMERICAL APPLICATION AND COMMENTS

For values from the real variation domain of the mechanical and geometrical characteristics of Jonathan apples $V_i = 0.65 \ldots 1.00$ m/s; $E = 2.5\cdot10^6 \ldots 6\cdot10^6$ Pa; $\alpha = 0.42 \ldots 0.69$ s/m, [1,3]. For these values we got the variation range for the coefficients $k_c = 0.794\cdot10^6 \ldots 1.672\cdot10^6$ N/m$^{3/2}$ şi $3/2\alpha k_c = 0.5\cdot10^6 \ldots 1.63\cdot10^6$ N/m$^{3/2}$. The values for all the 9 coefficients considered are given in the Table1.

Table 1. Values for the parameters V_i, E, α, k_c, k, F_{max}, δ_{max}, T, t_c, F_{max}/t_c, for 9 versions of the impact between apples on a rigid plane surface.

Nr. Crt	Nr. Cod	V_i (m/s)	E (Pa)·10^6	k	k_c (N/m$^{3/2}$)·10^6	F_{max}(N)		δ_{max}(mm)·10^{-5}		T(s)·10^{-4}		t_c(s)·10^{-4}		F_{max}/t_c(N/s)·10^4	
						Hunt Cross	Chen	Hunt Cross	Chen	H-C	Chen	H-C	Chen	H-C	Chen
1	2	1.00	3.2	0.5	0.892	67.89	67.96	164	170	59	58	19	21	3.573	3.236
2	3	0.8	3.5	0.52	0.975	53.59	53.89	133	137	60	59	21	22	2.551	2.449
3	4	0.65	4.8	0.55	1.337	47.35	47.63	100	103	55	54	18	21	2.630	2.268
4	3a	0.8	5.5	0.52	1.532	64.04	64.47	111	115	50	49	17	18	3.767	3.581
5	3b	0.8	3.5	0.56	0.975	53.83	53.87	135	138	59	59	21	21	2.563	2.565
6	3c	0.8	2.5	0.6	0.697	47.15	44.28	155	159	68	67	23	25	2.05	1.771
7	4a	0.65	6	0.58	1.672	52.15	52.00	92	95	50	49	17	19	3.067	2.736
8	4b	0.65	4.2	0.61	1.170	44.73	45.41	107	110	57	57	20	22	2.236	2.064
9	4c	0.65	2.85	0.64	0.794	38.87	38.99	126	129	67	66	24	26	1.619	1.499

Using the following apples characteristics d = 65 mm; E = $3\cdot10^6$Pa, ν = 0.37 [1, 4] and replacing the correspondent parameters in the equation (5) results k_c=$0.835\cdot10^6$ N/m$^{3/2}$. Using the data from equation (1), k-restitution coefficient= 0.577 and V_i- initial impact velocity= 1.0 m/s, and the equation (6)**,** we calculated that α = 0.423 s/m.

Considering the mass of the apple as being M_1 = 115g and the parameters k_c şi α, introduced in the equations (7), (9) it results:

$$F\left(t\right) = 0.835\cdot10^6\delta^{\frac{3}{2}} + 0.438\cdot10^6\delta^{\frac{3}{2}}\dot{\delta} \tag{26}$$

$$\ddot{\delta} + 3.809\cdot10^6\delta^{\frac{3}{2}}\dot{\delta} + 7.261\cdot10^6\delta^{\frac{3}{2}} = 0 \tag{27}$$

Integrating the differential equation (27) for the initial conditions t=0, δ(0)=0, and $\dot{\delta}(0) = V_i = 1m/s$, for a pace $\Delta t = 10^{-4} s$, using the Runge – Kutta method and a computer program created in Turbo-Pascal by the authors, [14], the following parameters have been calculated: the velocity variation $\dot{\delta}$, the strain δ, the impact force F and their values have been arranged in tables and graphically displayed in graphs (charts). With this values were drawn the graphs of strain variation – δ, velocity variation in time - $\dot{\delta}$, impact force variation depending on strain, impact force variation depending on time, respective Figure 4, Figure 5, Figure 6, Figure 7. It can be observed that the graphs are very similar to the ones obtained experimentally during the impact between apples on rigid plane surfaces, [12, 13].

Analyzing the fig.4 it results the impact time T=6,3 ms while δ(T)=0 and F(T)=0, when the contact with the impact surface is lost. This value is similar to the ones obtained during experiments [12] and it will be compared with other experimental parameters values. These charts are useful to evaluate the maximum force during impact (in this case $F_{\max} \cong 51N$), the stiffness coefficient during impact:

$$C = F_{\max} / t_c \tag{28}$$

where t_c is the time until F_{max} is reached ($C = 50.96 : 2.4 = 21.23$ N/s), necessary parameters for appreciating the degree of mechanical damage of the fruits and the ripeness degree.

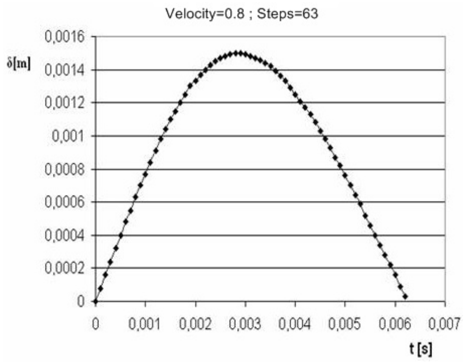

Fig 4. Strain variation in time.

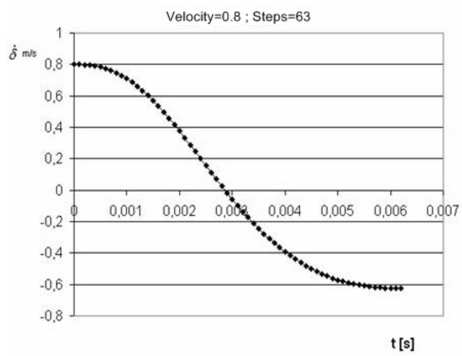

Fig. 5. Velocity variation in time.

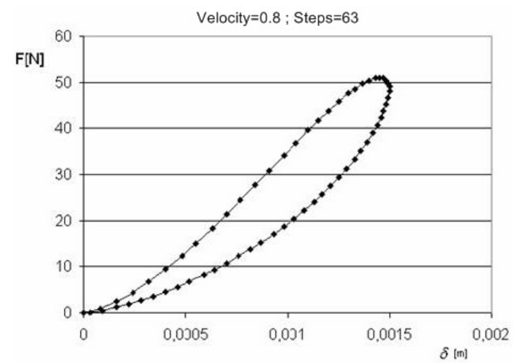

Fig. 6. Impact force variation depending on strain.

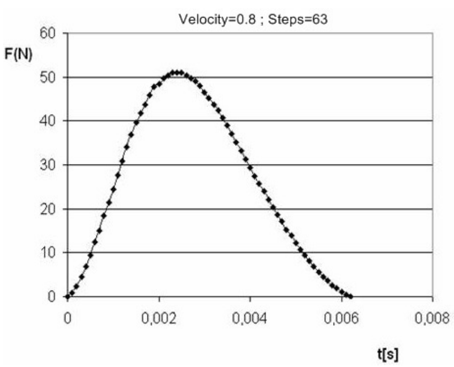

Fig. 7. Impact force variation depending on time.

Following the same methodology, other eight cases have been studied for values from their real variation domain of the mechanical and geometrical characteristics of the Jonathan apples variety specified in table 1.

For all the 8 studied cases, for specified values of differential coefficients equations (11) and (13), from the domains of values specified previously, the differential equations were integrating and the values obtains have been presents in graphs as in Figures 4, 5, 6, 7.

We observe for all the 8 studies cases, variation curves of the followed parameters (F(t), F(δ), δ(t), $\dot{\delta}$ (t)) have been similar to the ones in Figure 4 – Figure 7. This fact demonstrates that the equations (11) and (13) describe with accuracy the variation of the force F during impact depending on time or strain and also the local strain variation of the fruit δ at the impact point during impact, depending on time.

For the 9 versions in table 1 have been evaluated the coefficients for the equations (22) and (23) and, using the method previously described, have been integrated and represented in charts similar to the ones in Figure 4 – Figure 7, obtaining similar graphs. The values of the parameters F_{max}, δ_{max}, T, t_c şi C have been evaluated for the situations in the differential equations (11) and (13) and also (22), (23), proposed to simulate the response to impact of apples on rigid plane surface. These values are presented in Table 1.

It appears that the differences between the values of the parameters listed in the two situations is considered insignificant for all nine options presented, which proves that can be considered to simulate the impact response of apples on rigid plane surface, eq. (11), (13) or eq. (22), (23).

The obtained results are used during engineering activities related to the mechanical sorting of the fruits in order to predict the characteristics of the impact.

4. CONCLUSIONS

Starting from theoretical studies upon the impact between viscoelastic bodies on rigid plane surfaces (using the Kelvin – Voight model), applied on apples, a differential mathematical model has been elaborated which simulates the real behavior during impact. The solutions for these equations describe the force – time and the force – strain curves during impact that characterize the impact and are necessary to evaluate the degree of damage suffered by the apples and/ or the functional and constructive parameters of different mechanical systems included in the operations chain: transportation, sorting, packaging. This data is very useful as a starting point in choosing the appropriate functional parameters (design and exploitation) for the mechanical system used to process the apples. Based on the representation of the force – time and force – strain curves during the impact between fruits and the rigid plane surface, the maximum impact force can be determined and also the stiffness coefficient during impact. The same data can be used as a source of useful information to create software packs meant to evaluate in real time the impact's characteristic parameters, necessary in order to make the best decisions in certain circumstances. The method can be implemented in a sorting system using the principle of the differences between the stages of ripeness indicated by the impact between the free falling fruit and a rigid plane surface containing a force transmitter.

REFERENCES

[1] Căsăndroiu, T., Opriţa, N., Dumitrescu, L., Vintilă, M., Cercetării experimentale cu privire la evaluarea rezistenţei mecanice la ciocnire a merelor, Ştiinţe şi tehnologii alimentare, vol. II, nr. 5, 1994.

[2] Căsăndroiu, T., Voicu, Gh., Toma, M.L., Identification of adequate physical characteristic for impact demage susceptibility of an apple population, Proceedings of OPROTEH – 2005, Bacău.

[3] Mohsenin, N.N., Physical proprieties of plant and animal materials. Vol. 1, Gordon and Breach, Science publishers, Inc, N.Y., 1970.

[4] Căsăndroiu, T., Aspecte privind utilizarea ciocnirii ca metodă de evaluare a rigidităţii texturii fructelor 1. Cosideraţii teoretice. 2. Aplicaţie cu date experimentale, Construcţia de maşini, 11-12, 2003.

[5] Căsăndroiu, T., Vintilă, M., Evaluarea mărimilor ce caracterizează nivelul şocului suportat de mere, la ciocnirea cu suprafeţe plane rigide, fixe, Construcţia de maşini, 4, 2003.

[6] Hamann, D.D., Analysis of stress during impact of fruit considered to be viscoelastic, Transactions of the ASAE, 13, 1970, p. 893–900.

[7] Gao, Q., Pitt, R.E., Mechanics of parenchyma tissue based on cell orientation and microstructure, Transaction of the ASAE, 34(1), 1991, p.0232-0238.

[8] Abbott, J.A., Lu, R., Anisotropic mechanical proprieties of apples, Transaction of the ASAE, 39(4), 1996, p.1451-1459.

[9] Horsfield, B.C., Fridley, R.B., Claypool, L.L., Application of theory of elasticity to the design of fruit harvesting and handling equipment for minimum bruising, Transaction of the ASAE, 15(4), 1972, p.746-750.

[10] Lichtensteiger, M.J., Holmes, R.G, Hamdy, M.Y., Blaisdell, J.L., Impact parameters of spherical viscoelastic objects and tomatoes, Transaction of the ASAE, 31(2), 1988, p.595-602.
[11] Dogaru, F., Rezistenţa la impact a materialelor compozite, PhD Thesis, Universitatea Transilvania Braşov, 2005.
[12] Vintilă, M., Stadiul actual al cercetării privind analiza sistemului dinamic fruct-suprafaţă de contact şi prezentarea principalelor instalaţii de sortare-ambalare a fructelor, PhD report, 1995, UPB.
[13] Chen, P., Tang, S., Chen, S., Instrument for testing the response of fruits to impact, Proc ASABE Winter Meeting, Chicago, USA. Paper Number 85-3537, 1985.
[14] Moise, V., Maican, E., Moise, Şt.I., Metode numerice, Ed. Bren, Bucureşti, 2003.
[15] Dănilă, D.M., Contribuţii privind elaborarea metodologiei şi aparaturii pentru studiul comportamentului tuberculilor de cartof la solicitări mecanice de impact, PhD thesis, Universitatea Transilvania Braşov, 2008.
[16] Faik, S., Witteman, H., Modeling of Impact Dynamics: A Literature Survey, International ADAMS User Conference, 2000.
[17] Chen, H., De Baerdemaeker, J., Optimization of impact parameters for reliable excitation of apples during firmness monitoring, Journal of Agricultural Engineering Research, Vol.61, no.4, 1995, p.275-282.