THE EQUIVALENT STRESS CONCEPT IN MULTIAXIAL FATIGUE

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Abstract: Some equivalent stress methods, applicable for life prediction in case of multiaxial loading, are presented in this paper, such as empirical equivalent stresses, frequency and "signed von Mises" stress. To apply the latter method, a calculation program that allows the transformation of a fluctuating load cycle into a fully reversed cycle is developed. The authors extend the equivalent stress concept towards critical plane models. This is illustrated by a calculated equivalent shear stress based on the Yokobori criterion. Both the „signed von Mises" and Yokobori approaches have the advantage of allowing multiaxial fatigue calculations for limited durability, as opposed to classical methods which are applicable only for unlimited durability.

Keywords: equivalent stress, multiaxial fatigue, nonproportionality, signed von Mises, Yokobori, hodograph

1. INTRODUCTION

It is known that a large number of components and equipment in machine construction are subjected to multiaxial stresses variable in time. Life prediction under the conditions of triaxial variable loading remains an unresolved issue, having no unanimously accepted solutions up until present-day [1][2][3].

Multiaxial loadings can be classified as proportional (in-phase) and nonproportional (out-of-phase). During proportional loading the principal stress directions remain fixed in time and the principal stress ratio remains constant even though the loading directions rotate. As opposed to this, nonproportional loading is characterized by rotating principal directions and variable principal stress ratio.

Numerous theoretical and experimental studies have been carried out in order to highlight some characteristics of multiaxial fatigue durability in case of proportional (in-phase) and nonproportional (out-of-phase) loading. One of the most frequently used methods, especially when machine shaft calculation is considered, involves replacing the variable multiaxial loading with an equivalent uniaxial loading, or with an equivalent scalar damage parameter. The above can be justified, on one hand by the level of complexity and high costs of multiaxial testing, and on the other hand by the large amount of available experimental data regarding material behavior under variable uniaxial loading.

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Initially, researchers simply extended the existing static yield criteria in order to define equivalent stresses that described stress states characterized by even more general forms of the stress tensor. Thus, the maximum normal stress criterion (Rankine), the maximum shear stress criterion (Tresca) and the octahedral stress criterion (von Mises) were developed.

A number of equivalent stress methods, applicable for life prediction in case of variable multiaxial loading, are presented in this paper. The analyzed models are based on empirical equivalent stresses, frequency and "signed von Mises" equivalent stress. To apply the "signed von Mises" method, a calculation program that allows the transformation of a fluctuating load cycle into a symmetric cycle has been developed. The authors propose an extension of the equivalent stress concept towards the critical plane models, where the damage parameter is a normal or shear stress. This is illustrated by a calculated equivalent shear stress based on the Yokobori criterion [4], using the stress hodographs proposed by Skibicki [5].

2. DEFINING THE EQUIVALENT STRESS

2.1. Models based on empirical equivalent stresses

Given its simplicity, the definition of equivalent stress based on the yield criterion is the mostly utilized. Langer [6] proposed an equivalent stress based on Tresca's theory. When out-of-phase fully reversed sinusoidal cycles of bending and torsion are considered, the equivalent stress amplitude (SALT) can be computed:

$$SALT = \frac{\sigma_a}{\sqrt{2}}\left\{1 + K^2 + \left[1 + 2K^2\cos(2\Phi) + K^4\right]^{1/2}\right\}^{1/2} \tag{1}$$

where σ_a – bending stress amplitude
τ_a – torsion stress amplitude ($K = 2\tau_a / \sigma_a$)
Φ – phase angle between bending and torsion.

Considering equation (1) when $\Phi = 0$, it yields:

$$SALT = \sqrt{\sigma_a^2 + 4\tau_a^2} \tag{2}$$

Modifying equation (1) with respect to the von Mises theory, the following form is obtained (ASME, 1978):

$$SEQA = \frac{\sigma_a}{\sqrt{2}}\left\{1 + \frac{3}{4}K^2 + \left[1 + \frac{3}{2}K^2\cos(2\Phi) + \frac{9}{10}K^4\right]^{1/2}\right\}^{1/2} \tag{3}$$

It can be seen that for proportional loadings ($\Phi = 0$) the von Mises stress is obtained:

$$SALT = \sqrt{\sigma_a^2 + 3\tau_a^2} \tag{4}$$

Gough et al. [7][8] proposed a fatigue criterion applicable for ductile materials, which represents the equation of an ellipse:

$$\frac{\sigma_a^2}{f_{-1}^2} + \frac{\tau_a^2}{\tau_{-1}^2} = 1 \tag{5}$$

Yielding:

$$\sqrt{\sigma_a^2 + \frac{f_{-1}^2}{\tau_{-1}^2}\tau_a^2} = f_{-1} \tag{6}$$

where: f_{-1} – fully reversed bending fatigue limit
τ_{-1} – fully reversed torsion fatigue limit

The left member of equation (6) can be considered as the equivalent stress amplitude. Considering a material having the characteristics $\tau_{-1} / f_{-1} = 0.5$, equation (6) becomes the Tresca criterion (equation (2)). For a material with $\tau_{-1} / f_{-1} = 1/\sqrt{3}$, equation (6) becomes the von Mises criterion (equation (4)).

Lee [9], based on Gough's formula, proposes the following form for an empirical equivalent stress:

$$SLEE = \sigma_a \left[1 + \left(\frac{f_{-1}}{2\tau_{-1}}K\right)^{2(1+\beta\sin\Phi)}\right]^{\frac{1}{2(1+\beta\sin\Phi)}} \tag{7}$$

where β is a material constant necessary for correlating the results with experimental data.

Analyzing the empirical equivalent stress formulations, it can be seen that they are relatively easy to compute for a broad variety of engineering applications. However, the Tresca and von Mises criteria generate non-conservative results [10], while Gough's formula can only be applied for proportional loading. The equation proposed by Lee introduces a material constant, thus narrowing the criterion's field of applicability.

2.2. The frequency based von Mises model

Pitoiset [11] and Preumont [12] define the equivalent stress as a variable stationary Gaussian process with non-zero mean stress, also known as Power Spectral Density, *PSD*. Plane stress will be considered, having the components:

$$\sigma = \begin{Bmatrix} \sigma_x(t) \\ \sigma_y(t) \\ \tau_{xy}(t) \end{Bmatrix} \Rightarrow \sigma^T = \begin{bmatrix} \sigma_x(t) & \sigma_y(t) & \tau_{xy}(t) \end{bmatrix} \tag{8}$$

Taking into account the following matrix:

$$Q = \begin{bmatrix} 1 & -1/2 & 0 \\ -1/2 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix} \tag{9}$$

the square of the equivalent stress can be computed in the von Mises hypothesis:

$$\sigma_{eq,vM}^2(t) = \sigma^T Q \sigma = Trace\{Q[\sigma\sigma^T]\} \tag{10}$$

After taking the expectation:

$$E[\sigma_{eq,vM}^2(t)] = Trace\{QE[\sigma\sigma^T]\} \tag{11}$$

where $E[\sigma\sigma^T]$ is the covariance matrix of the stress vector, related to the *PSD* matrix of the stress vector by

$$E[\sigma\sigma^T] = \int_{-\infty}^{+\infty} G_\sigma(\omega) d\omega \tag{12}$$

From these equations, we can define the *PSD* $G_{vM}(\omega)$ of the equivalent von Mises stress as a frequency decomposition of its mean square value where $G_\sigma(\omega)$ is the *PSD* matrix of the stress vector:

$$E[\sigma^2_{eq,vM}(t)] = \int_{-\infty}^{+\infty} G_{vM}(\omega) d\omega = \int_{-\infty}^{+\infty} Trace\{QG_\sigma(\omega)\} d\omega \tag{13}$$

$$G_{vM}(\omega) = Trace\{QG_\sigma(\omega)\} \tag{14}$$

This equation defines the von Mises stress as a scalar random process whose *PSD* is obtained from the *PSD* matrix of the stress components according to the von Mises quadratic combination rule.

Under these conditions the von Mises stress is considered to be a countable variable stress, taking into account the effects of multiaxiality. The methodology proposed for biaxial loading can be extended for triaxial stress state, starting from the definitions of the stress vector and matrix *Q*.

2.3. Models based on the sign of the von Mises equivalent stress as a function of time
Based on the definition of the von Mises equivalent stress, given by the equation:

$$\sigma_{eq}(t) = \frac{1}{\sqrt{2}} \sqrt{[\sigma_1(t) - \sigma_2(t)]^2 + [\sigma_2(t) - \sigma_3(t)]^2 + [\sigma_3(t) - \sigma_1(t)]^2} \tag{15}$$

a generally non-Gaussian uniaxial equivalent process is generated, having non-zero mean stress. Furthermore, the generated cycle is of repeated tension, containing non-coherent frequency components.

The above is illustrated for a case of plane stress, composed of tension and torsion, $\sigma_x(t)$ and $\tau_{xy}(t)$, presumed to be sinusoidal and having the same frequency, but out-of-phase by Φ, as follows:

$$\sigma_x(t) = \sigma_m + \sigma_a \cdot \sin \omega t \;\; [MPa] \tag{16}$$

$$\tau_{xy}(t) = \tau_m + \tau_a \cdot \sin(\omega t + \Phi) \;\; [MPa] \tag{17}$$

where σ_a = 120 *MPa*, σ_m = 0 *MPa*, τ_a = 60 *MPa*, τ_m = 30 *MPa* and $\Phi = \pi/2$ *rad*.
The two stress components mentioned above, generate the following path loading presented in Fig.1:

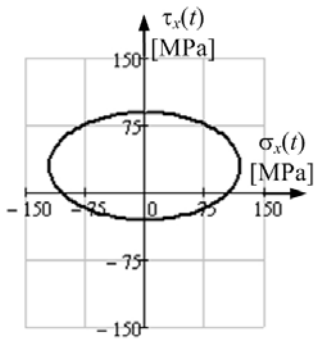

Fig. 1. Path loading generated by the considered stress components, having a coefficient of nonproportionality *A* = 0.75.

In Fig.2 – Fig.4 are shown the variations in time of the two stress components, and of the von Mises equivalent stress $\sigma_{eq,vM}(t)$ respectively.

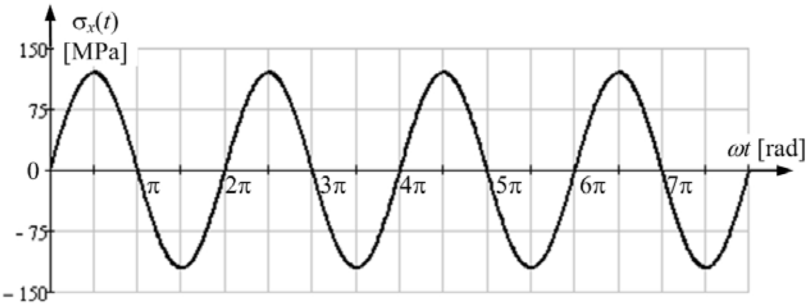

Fig. 2. Variation in time over 4 complete cycles of the considered normal stress.

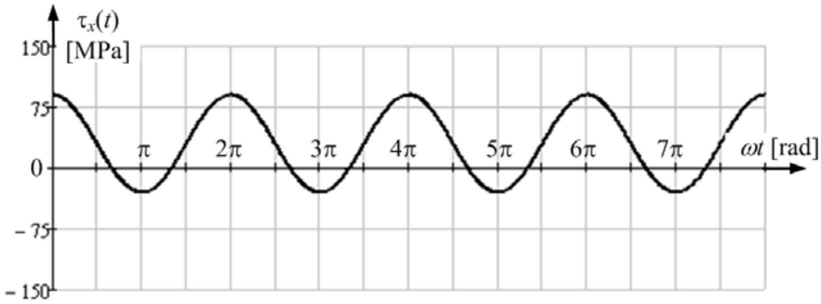

Fig. 3. Variation in time over 4 complete cycles of the considered shear stress.

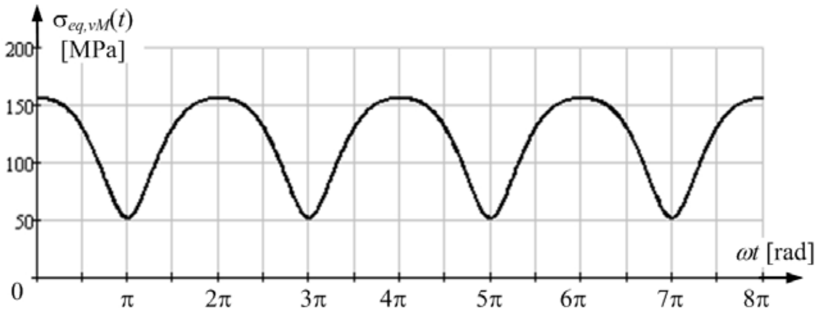

Fig. 4. Variation in time over 4 complete cycles of the von Mises equivalent stress $\sigma_{eq,vM}(t)$.

It can be clearly seen, that the $\sigma_{eq,vM}(t)$ equivalent stress does not correctly reflect the real load spectrum and cannot be considered as a substitute for the given stress components, since it does not retain the negative stress values. In order to eliminate this ambiguity of the von Mises equivalent stress, Bishop [13] proposed a correction. According to Bishop, the equivalent stress should be a "signed von Mises" stress, the sign being generally given by the first principal stress, as its general form is shown in equation (18):

$$\sigma_{eq,vMs}(t) = \sigma_{eq,vM}(t) \cdot sign(\sigma_1(t)) \tag{18}$$

The computation of the "signed von Mises" equivalent stress can run into complications, since in order to plot $\sigma_{eq,vMs}(t)$ it is also necessary to plot the variation in time of the two principal stresses $\sigma_1(t)$ and $\sigma_2(t)$. The authors developed a calculation program, which allows the plotting of $\sigma_{eq,vMs}(t)$ taking into account the original stress components given in equations (16) and (17). The flowchart of the calculation program is presented in Fig.5.

Based on the steps presented in Fig.5, the "signed von Mises" stress has been computed over 4 complete cycles ($\omega t \in 0 \ldots 4\times 2\pi$), resulting in the form presented in Fig.6, considering the specific load history in equations (16) and (17). Fig.6 shows that the equivalent stress cycle is fully reversed, having the extreme values $\sigma_{eq,vMs}^{max} = 155.9\ MPa$ and $\sigma_{eq,vMs}^{min} = -155.9\ MPa$, presenting certain discontinuities when the principal stress ratio changes. Obviously, this is a conservative approach with respect to engineering applications, however, it is extremely difficult to apply in case of random load spectra.

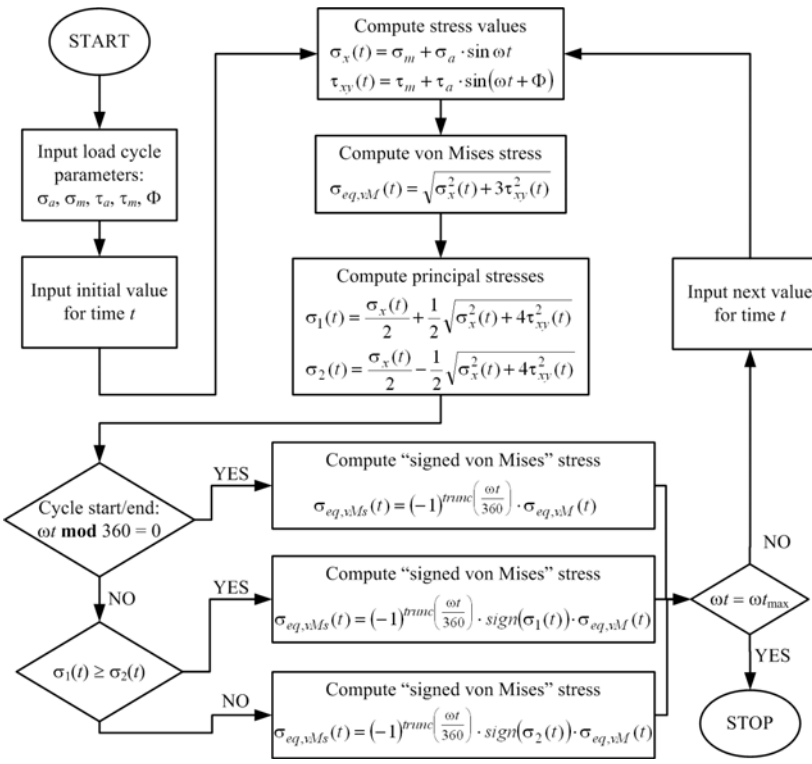

Fig. 5. Calculation program flowchart for transforming a pulsating equivalent stress cycle into a fully reversed equivalent cycle.

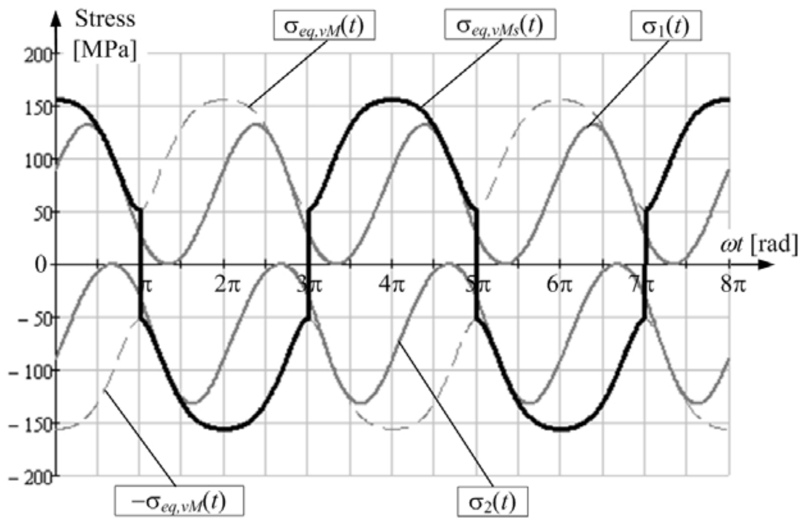

Fig. 6. Variation in time over 4 complete cycles of the "signed von Mises" equivalent stress $\sigma_{eq,vMs}(t)$, the von Mises equivalent stress $\sigma_{eq,vM}(t)$ and the principal stresses $\sigma_1(t)$ and $\sigma_2(t)$.

Another approach regarding the von Mises equivalent stress has been proposed by Braccesi et al. [14][15]. This method consists of building a new coherent stress tensor as a function of frequency. The obtained von Mises stress $\sigma^*_{eq,vM}$ is then corrected by subtracting a mean value in order to obtain a final cyclic stress with zero-mean value.

$$\sigma^{**}_{eq,vM}(t) = \sigma^{*}_{eq,vM}(t) - mean\{\sigma^{*}_{eq,vM}(t)\} \quad (19)$$

According to the above, the new equivalent stress defining the process is the following:

$$\sigma_{eq}(t) = \sigma^{**}_{eq,vM}(t) \frac{\max\limits_{t \in T}\{\sigma_{eq,vM}\}}{\max\limits_{t \in T}\{\sigma^{**}_{eq,vM}\}} \tag{20}$$

Braccesi [14] shows that the proposed method eliminates some of the ambiguities of the classical von Mises approach.

3. MODELS BASED ON MULTIAXIAL FATIGUE CRITERIA

The above presented considerations support the claim that the approach of multiaxial fatigue using equivalent stress concepts runs into a series of problems, especially when the variable character of the external load is taken into account. Furthermore, such methods generally extend the classical theories of Rankine, Tresca or especially von Mises. According to these methods, the planes where failure occurs are the ones on which principal stresses, maximum shear stresses or octahedral shear stresses act respectively.

The loading's variable character makes that in many cases the failure plane does not coincide with the planes predicted above, thus the study of multiaxial fatigue cannot be made based on static yield criteria. These criteria, albeit all the corrections, cannot take into account the succession of stress states that define a multiaxial load spectrum.

As a result of the variable character of the load spectrum and because of the fatigue process's inception particularities, fatigue criteria are much more complex than classical static yield criteria. A fatigue criterion has to take into account the components of a multiaxial fatigue cycle $[\sigma_{ij}(t)]_T$, defined over a time period T (amplitude, mean value, maximum value, alternating components), and the fatigue limits as well (infinite or finite durability for a number of N cycles) for fully reversed bending f_{-1}, fully reversed tension σ_{-1}, repeated tension σ_0, fully reversed torsion τ_{-1}. The general expression of such a criterion is given in equation (21):

$$X + \alpha \cdot Y \leq \beta \tag{21}$$

where X and Y are normal and shear stresses in certain planes considered to be critical, while α and β are values determined as functions of the material's fatigue characteristics.

The left member of equation (21) can be considered as an equivalent stress, which besides some components of the stress tensor, in comparison to the von Mises approach, also contains certain material characteristics with respect to variable loading conditions.

A calculation program was developed in order to compute these equivalent stresses accepting the Yokobori criterion [4]. According to Yokobori, equation (21) becomes:

$$\max_{t \in T}[\tau_n(t)] + \alpha \cdot \sigma_{n,\max} = \beta \tag{22}$$

where $\alpha = 2\tau_{-1} / \sigma_{-1} - 1$ and $\beta = \tau_{-1}$. The authors have chosen the following material characteristics: $\sigma_{-1} = 660$ *MPa* and $\tau_{-1} = 410$ *MPa* [16]. The left member of equation (21) can be considered as an equivalent stress $\tau_{eq,TY}(t)$, which leads to failure when it reaches its critical value, in other words equaling τ_{-1}. Obviously, $\tau_{eq,TY}(t)$ can be used to compute finite fatigue life as well, given the *S-N* curve in fully reversed torsion is known.

The calculation program developed by the authors allowed the extended computation of the equivalent stress $\tau_{eq,TY}(t)$, as a function of time and as a function of critical plane position θ as well. The equivalent stress has been plotted in polar and in Cartesian coordinates too.

In Fig.7-Fig.9 the stress hodographs proposed by Skibicki [5] are presented, applied for the Yokobori equivalent stress at three chosen moments of the load cycle $\omega t = 0$, $\omega t = 0.77\pi$ *rad* and $\omega t = 1.5\pi$ *rad* respectively.

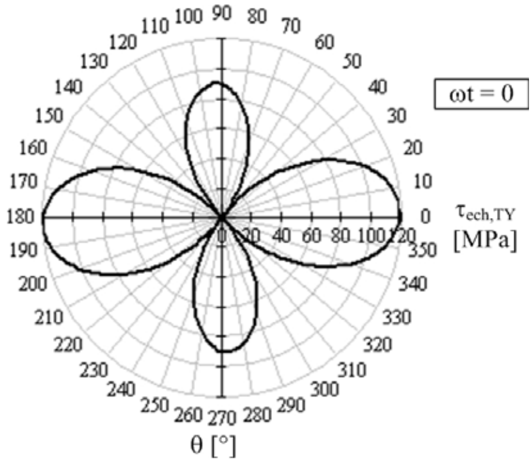

Fig. 7. Hodograph of Yokobori stress on planes inclined by the angle θ at the chosen moment $\omega t = 0$ *rad.*

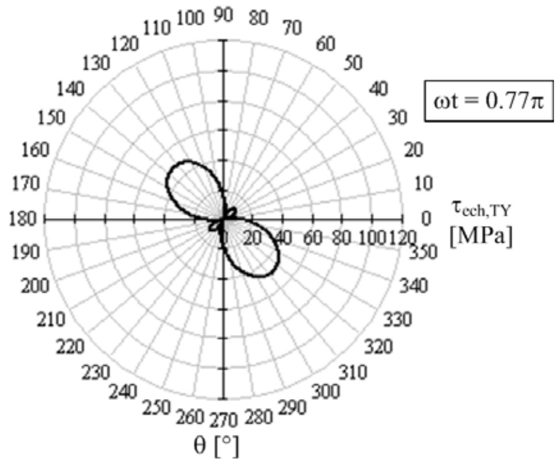

Fig. 8. Hodograph of Yokobori stress on planes inclined by the angle θ at the chosen moment $\omega t = 0.77\pi$ *rad.*

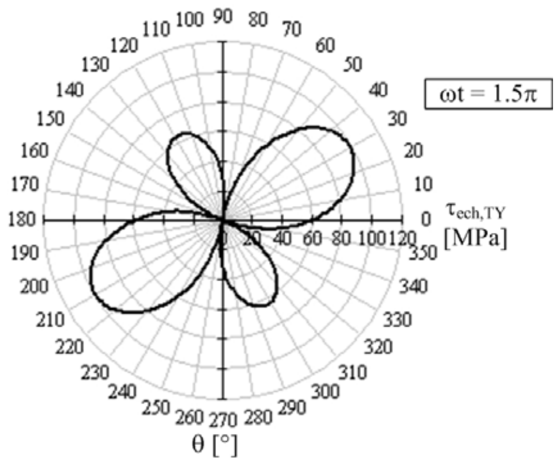

Fig. 9. Hodograph of Yokobori stress on planes inclined by the angle θ at the chosen moment $\omega t = 1.5\pi$ *rad.*

By analyzing these hodographs, the position of the planes on which the Yokobori equivalent stress reaches its extreme value can be easily determined. The calculation program developed by the authors allows the determination of these angles using the either graphical or analytic methods.

Thus, for the given load spectrum, the Yokobori equivalent stress's maximum value is $\tau_{eq,TY}^{\max} = 119.79\,MPa$, which is reached at the moment of the load cycle $\omega t = 1.94\pi$ *rad*, on a plane inclined by the angle $\theta = 5.64°$ or $185.64°$. The stress hodograph corresponding to these values is presented in Fig.10.

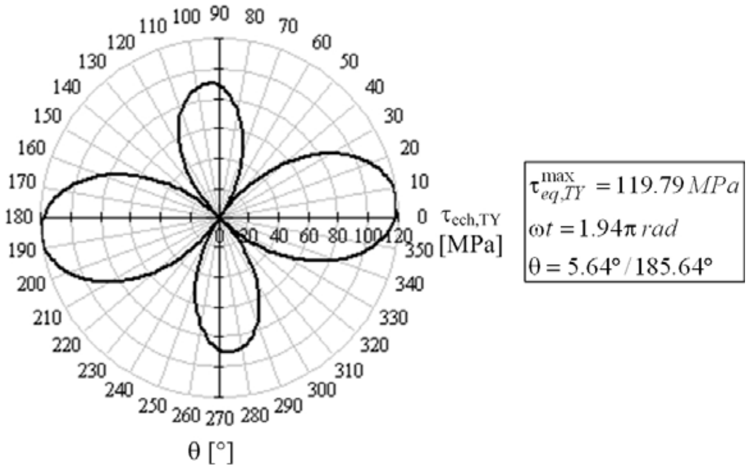

Fig. 10. Hodograph of Yokobori stress on planes inclined by the angle θ at the moment $\omega t = 1.94\pi$ *rad* when the maximum value is reached.

Coefficients of safety were computed as the ratio of the two presented equivalent stresses' ("signed von Mises" equivalent stress $\sigma_{eq,vMs}(t)$ and Yokobori equivalent stress $\tau_{eq,TY}(t)$) extreme values and the material's fully reversed torsion fatigue limit. Thus, for $\sigma_{eq,vMs}(t)$ the safety coefficient turned out to be $c_{vMs} = 2.63$, while in case of $\tau_{eq,TY}(t)$, the calculation yielded $c_{TY} = 3.42$. Based on these values, it can be stated that for a nonproportional load spectrum, the Yokobori equivalent stress model is more conservative than the "signed von Mises" model.

4. CONCLUSIONS

A number of equivalent stress methods, applicable for life prediction in case of variable multiaxial loading, are presented in this paper. The analyzed models are based on empirical equivalent stresses, frequency and "signed von Mises" stress. To apply the "signed von Mises" method, a calculation program that allows the transformation of a fluctuating load cycle into a fully reversed cycle has been developed.

According to the classical method, multiaxial fatigue calculation is based on the determination of distinct safety coefficients for tension and torsion loading, respectively. Based on these two values, a global safety coefficient is calculated, using the Tresca theory. The obtained safety coefficient is then compared with the prescribed value. The classical method is mostly applicable for infinite durability calculation. However, in many cases equipment and machinery is designed to operate safely for a limited period of time (in the domain of elastic strains). Limited durability requires the determination of the number of cycles to failure as accurately as possible.

Based on this necessity, replacing the cyclic load combinations with an equivalent uniaxial cyclic load history has been considered. Initially, this equivalent load history was based on tension or bending stress, but with the development of critical plane models, shear stresses have become more utilized. According to the von Mises theory (which is the most widely accepted equivalent stress theory), the equivalent stress history is composed of fluctuating tensile cycles. Fluctuating tensile cycles come with two major inconveniences with perspective to fatigue calculation: firstly, they do not reflect the real loading conditions, since they totally lack the compressive components, and secondly fatigue curves used for the determination of the number of cycles to failure are generally designed for fully reversed cycles. Hence the idea of transforming a complex cyclic load history into an equivalent one with fully reversed cycles (the "signed von Mises" approach discussed in this paper). Thus, the main advantage of this approach is that based on the amplitude or maximum value of an equivalent stress or load cycle, using the fatigue curves corresponding to the given material, the number of cycles to failure can be determined more accurately than by the classical method.

The authors propose an extension of the equivalent stress concept towards the critical plane models, where the damage parameter is a tensile or shear stress. This aspect is illustrated using the multiaxial fatigue model proposed by Yokobori, by calculating the maximum value in time of an equivalent shear stress, applying the modern method of stress hodographs. With respect to the classical method, this approach also has the advantage of a more accurate fatigue life determination using the torsion fatigue curve in this purpose.

The calculation program developed by the authors also allows the plotting of the Yokobori equivalent stress in polar and in Cartesian coordinates as well, considering a nonproportional load spectrum. The determination of the critical plane positions, where the equivalent stress reaches extreme values, is also possible.

Finally, the “signed von Mises” and Yokobori equivalent stress approaches are compared based on safety coefficients. It is found that the Yokobori approach is more conservative than the “signed von Mises” approach when a nonproportional load spectrum is considered.

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