THEORETICAL ASPECTS REGARDING THE ANNULAR PLATE LOADING, SIMPLE LEANED ON THE BOTH EDGES, UNDER THE ACTION OF AN UNIFORM DISTRIBUTED PRESSURE ON A FACE

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Abstract. The paper, taking in discussing the loading of the annular plates given of a uniformly distributed pressure on one side, with simple leaning on the outlines, compares two versions of analysis. The appropriate expressions of the vertical displacement of the plate and of the radial and annular stresses for each study case are shown.

Keywords: annular plate, simple leaning, radial and annular stresses.

1. GENERALITIES

The industrial mechanical equipments, generally, or those working under pressure, particularly, contain a series of circular and annular plates, allowing the separation of the work environments or their transfer from one area to another. In such situations, to assess the state of deformations and stresses in the plate itself, but also in the adjacent structural elements, it must develop some appropriate methods of calculation. Depending on the specific conditions, the annular plates can be embedded on one or the both outlines, respectively that they may be resting on a outline or the both (Figure 1). This paper studies this last mode of leaning, in two versions, when the plate is subjected to the action of a pressure on one side (if the practice requires the presence of two differential pressures on the two sides of the plate, for calculating differential value of the pressure will be considered). The loading of the plate is considered in the elastic domain, as a possible influence of the temperature can be overwritten properly.
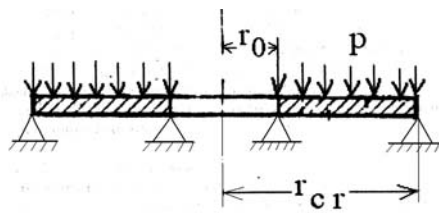

Fig. 1. Plate with the simple leaned outlines, under the action of uniform distributed pressure.

2. VERSION I

It is assumed that the inside leaning exist, simulating the presence of the second leaning (outside) by introducing of a uniform distributed force (Figure 2), to produce a vertical displacement of equal value, but opposite to that

created by the uniform distributed pressure. Equating the corresponding expressions for the vertical displacement of the plate, at the level of its outside outline, written as:

$$w^{(c)}_{p,\,x=\alpha} = w^{(a)}_{P^\bullet,\,x=\alpha}, \tag{1}$$

result (P^{*}_{1} represent the total force distributed on the outside outline of the plate, measured in N , Figure 2):

$$P^{\bullet}_{1} = 4\cdot\pi\cdot p\cdot r^{2}_{cr}\cdot\frac{C^{(c)}_{d,\,p,\,x=\alpha}}{C^{(a)}_{d,\,P^\bullet,\,x=\alpha}}, \tag{2}$$

where:

$$C^{(a)}_{d,\,P^\bullet,\,x=\alpha} = \frac{4\cdot(1+\nu_{p})\cdot\ln^{2}\alpha}{(1-\nu_{p})\cdot(\alpha^{2}-1)} - 2\cdot\ln\alpha + \left(\frac{3+\nu_{p}}{1+\nu_{p}} + \frac{2\cdot\alpha^{2}\cdot\ln\alpha}{\alpha^{2}-1}\right)\cdot\frac{\alpha^{2}-1}{\alpha^{2}}; \tag{3}$$
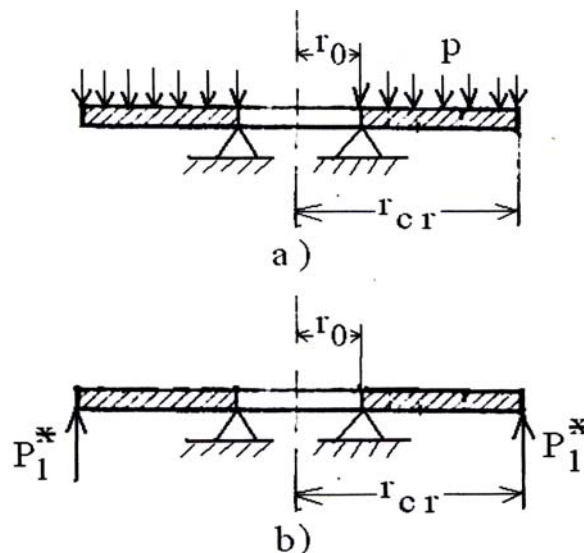
Fig. 2. Analysis scheme - version I.

$$C^{(c)}_{d,\,p,\,x=\alpha} = \frac{\alpha^{2}\cdot(1-\ln\alpha)-1}{2\cdot\alpha^{2}} + \frac{\alpha^{4}-1}{16\cdot\alpha^{4}} - $$

$$-\frac{\alpha^{2}-1}{2}\cdot\left[\frac{3+\nu_{p}}{1+\nu_{p}}\cdot\frac{1+\alpha^{2}}{4\cdot\alpha^{4}} + \frac{\ln\alpha}{\alpha^{2}-1} - \frac{1-\nu_{p}}{2\cdot(1+\nu_{p})\cdot\alpha^{2}}\right] - $$

$$-\frac{\ln\alpha}{1-\nu_{p}}\cdot\left[\frac{3+\nu_{p}}{4\cdot\alpha^{2}} - (1+\nu_{p})\cdot\frac{\ln\alpha}{\alpha^{2}-1}\right], \tag{4}$$

x and α having the significances given of the relations:

$$x = \frac{r}{r_{0}}; \; x = 1, \text{ for } r = r_{0}; \qquad x = \alpha, \text{ for } r = r_{cr}; \tag{5}$$

$$\alpha = r_{cr}/r_0,\tag{6}$$

2.1. Deformations expressions

The expression rotation on the inside leaning of the plate is derived as:

$$\varphi_{p,\,x=1}^{(A)} = \frac{p\cdot r_{cr}^{4}}{\mathfrak{R}_p\cdot r_0}\cdot\left[\frac{1}{4}\cdot C_{r,\,p,\,x=1}^{(c)} - \frac{C_{d,\,p,\,x=\alpha}^{(c)}}{C_{d,\,P^\bullet,\,x=\alpha}^{(a)}}\cdot C_{r,\,P^\bullet,\,x=1}^{(a)}\right],\tag{7}$$

as difference between the rotations produced by the uniform distributed pressure and that appropriate to the force $P_1^\bullet$, where:

$$C_{r,\,P^\bullet,\,x=1}^{(a)} = \frac{\left(1+\nu_p\right)\cdot\ln\alpha}{\left(1-\nu_p\right)\cdot\left(\alpha^2-1\right)} + \left(\frac{1}{1+\nu_p} + \frac{\alpha^2\cdot\ln\alpha}{\alpha^2-1}\right)\cdot\frac{1}{\alpha^2};\tag{8}$$

$$C_{r,\,p,\,x=1}^{(c)} = \frac{1}{2\cdot\alpha^2} + \frac{1}{4\cdot\alpha^4} - \frac{3+\nu_p}{4\cdot\left(1+\nu_p\right)}\cdot\frac{1+\alpha^2}{\alpha^4} - \frac{\ln\alpha}{\alpha^2-1} +$$

$$+ \frac{1-\nu_p}{2\cdot\left(1+\nu_p\right)\cdot\alpha^2} - \frac{1}{1-\nu_p}\cdot\left(\frac{3+\nu_p}{4\cdot\alpha^2} - \left(1+\nu_p\right)\cdot\frac{\ln\alpha}{\alpha^2-1}\right).\tag{9}$$

The angle of the neutral surface of the plate on the outside leaning can be determined by the formula,

$$\varphi_{p,\,x=\alpha}^{(A)} = \frac{p\cdot r_{cr}^{4}}{\mathfrak{R}_p\cdot r_0}\cdot\left[\frac{1}{4}\cdot C_{r,\,p,\,x=\alpha}^{(c)} - \frac{C_{d,\,p,\,x=\alpha}^{(c)}}{C_{d,\,P^\bullet,\,x=\alpha}^{(a)}}\cdot C_{r,\,P^\bullet,\,x=\alpha}^{(a)}\right],\tag{10}$$

deduced from the difference between the angles produced of the distributed force $P_1^\bullet$ and the uniform distributed pressure on the plate surface, where the following notations have used:

$$C_{r,\,P^\bullet,\,x=\alpha}^{(a)} = \frac{1}{\alpha}\cdot\left[\frac{\left(1+\nu_p\right)\cdot\ln\alpha}{\left(1-\nu_p\right)\cdot\left(\alpha^2-1\right)} - \ln\alpha + \frac{1}{1+\nu_p} + \frac{\alpha^2\cdot\ln\alpha}{\alpha^2-1}\right];\tag{11}$$

$$C_{r,\,p,\,x=\alpha}^{(c)} = \frac{1-2\cdot\ln\alpha}{2\cdot\alpha} + \frac{1}{4\cdot\alpha} - \frac{3+\nu_p}{4\cdot\left(1+\nu_p\right)}\cdot\frac{1+\alpha^2}{\alpha^3} - \frac{\alpha\cdot\ln\alpha}{\alpha^2-1} +$$

$$+ \frac{1-\nu_p}{2\cdot\left(1+\nu_p\right)\cdot\alpha} - \frac{1}{1-\nu_p}\cdot\left[\frac{3+\nu_p}{4\cdot\alpha^2} - \left(1+\nu_p\right)\cdot\frac{\ln\alpha}{\alpha^2-1}\right]\cdot\frac{1}{\alpha}.\tag{12}$$

The expression of the points displacement of the neutral surface of the plate is determined as a difference between the relationships specific to the uniform pressure action, respectively of the force $P_1^\bullet$, resulting:

$$w_{p,x}^{(A)} = \frac{p \cdot r_{cr}^4}{4 \cdot \Re_p} \cdot \left[C_{d,p,x}^{(c)} - \frac{1}{4} \cdot C_{d,P^\bullet,x}^{(a)} \cdot \frac{C_{d,p,x=\alpha}^{(c)}}{C_{d,P^\bullet,x=\alpha}^{(a)}} \right], \qquad (13)$$

with the notations:

$$C_{d,p,x}^{(c)} = \frac{x^2 \cdot (1 - \ln x) - 1}{2 \cdot \alpha^2} + \frac{x^4 - 1}{16 \cdot \alpha^4} -$$

$$- \frac{x^2 - 1}{2} \cdot \left(\frac{3 + \nu_p}{1 + \nu_p} \cdot \frac{1 + \alpha^2}{4 \cdot \alpha^4} + \frac{\ln \alpha}{\alpha^2 - 1} - \frac{1 - \nu_p}{2 \cdot (1 + \nu_p) \cdot \alpha^2} \right) -$$

$$- \frac{\ln x}{1 - \nu_p} \cdot \left[\frac{3 + \nu_p}{4 \cdot \alpha^2} - (1 + \nu_p) \cdot \frac{\ln \alpha}{\alpha^2 - 1} \right]; \qquad (14)$$

$$C_{d,P^\bullet,x}^{(a)} = \frac{4 \cdot (1 + \nu_p) \cdot \ln \alpha}{(1 - \nu_p) \cdot (\alpha^2 - 1)} \cdot \ln x - \frac{2 \cdot x^2 \cdot \ln x}{\alpha^2} - \left(\frac{3 + \nu_p}{1 + \nu_p} + \frac{2 \cdot \alpha^2 \cdot \ln \alpha}{\alpha^2 - 1} \right) \cdot \frac{1 - x^2}{\alpha^2}. \qquad (15)$$

2.2. Stresses expressions

To determine the expressions of the radial and annular stresses we proceed, as in the previous case, at the difference of the sizes characteristic to the distributed pressure action, respectively of the force $P_1^\bullet$, resulting in final:

► *for the radial stresses:*

$$\sigma_{r,p,x}^{(A)} = \frac{3 \cdot p \cdot r_{c,r}^2}{\delta_p^2} \cdot \left(\frac{1}{2} \cdot C_{r,\sigma,p}^{(c)} - 2 \cdot C_{r,\sigma,P^\bullet}^{(a)} \cdot \frac{C_{d,p,x=\alpha}^{(c)}}{C_{d,P^\bullet,x=\alpha}^{(a)}} \right), \qquad (16)$$

with the notations:

$$C_{r,\sigma,P^\bullet}^{(a)} = (1 + \nu_p) \cdot \left[\ln x - \frac{x^2 - 1}{\alpha^2 - 1} \cdot \left(\frac{\alpha}{x} \right)^2 \cdot \ln \alpha \right]; \qquad (17)$$

$$C_{r,\sigma,p}^{(c)} = \frac{3 + \nu_p}{4} \cdot (x^2 - 1) \cdot \frac{\alpha^2 - x^2}{\alpha^2 \cdot x^2} + (1 + \nu_p) \cdot \left(\ln x - \frac{\alpha^2 \cdot \ln \alpha}{\alpha^2 - 1} \cdot \frac{x^2 - 1}{x^2} \right); \qquad (18)$$

► *for the annular stresses:*

$$\sigma_{\theta,p,x}^{(A)} = \frac{3 \cdot p \cdot r_{c,r}^2}{\delta_p^2} \cdot \left(\frac{1}{2} \cdot C_{\theta,\sigma,p}^{(c)} - 2 \cdot C_{\theta,\sigma,P^\bullet}^{(a)} \cdot \frac{C_{d,p,x=\alpha}^{(c)}}{C_{d,P^\bullet,x=\alpha}^{(a)}} \right), \qquad (19)$$

with the notations::

$$C_{\theta,\sigma,P^\bullet}^{(a)} = \left(1 + \nu_p\right)\cdot\left[\ln x - \frac{x^2 + 1}{\alpha^2 - 1}\cdot\left(\frac{\alpha}{x}\right)^2\cdot\ln \alpha - \frac{1 - \nu_p}{1 + \nu_p}\right]; \quad (20)$$

$$C_{\theta,\sigma,p}^{(c)} = \frac{3 + \nu_p}{4}\cdot\left(1 + \frac{1}{\alpha^2} + \frac{1}{x^2}\right) + \left(1 + \nu_p\right)\cdot\ln x - \frac{1 + 3\cdot\nu_p}{4}\cdot\left(\frac{x}{\alpha}\right)^2 -$$

$$- \frac{\left(1 + \nu_p\right)\cdot\alpha^2\cdot\left(x^2 + 1\right)\cdot\ln \alpha}{\left(\alpha^2 - 1\right)\cdot x^2} - 1 + \nu_p. \quad (21)$$
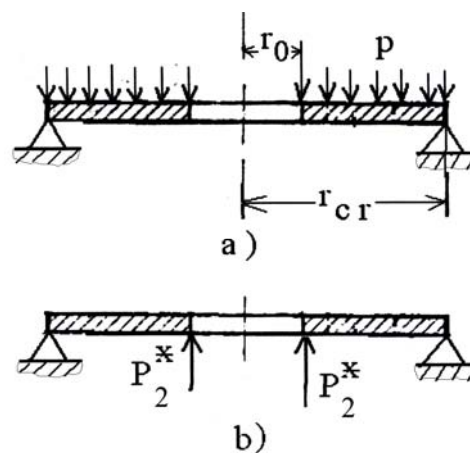

Fig. 3. Analysis scheme - the second version.

3. THE SECOND VERSION

This time it is envisaged the outside leaning as basis (Figure 3a), the force P_1^* being present on the inside outline of the plate (Figure 3b). To determine the appropriate relationship of the plate rotating on the outside leaning, we use the methodology set out in paper [1]. In this sense, we consider the plate from Figure 1, assuming the existence of a force P_1^* on the inside outline, having reverse acting to the pressure, materialized in the equality of the vertical displacements:

$$w_{p,x=1}^{(d)} = w_{P^\bullet,x=1}^{(b)}, \quad (22)$$

which leads to equality:

$$P_2^\bullet = \frac{1}{4}\cdot\pi\cdot p\cdot r_{cr}^2\cdot\frac{C_{d,p,x=1}^{(d)}}{C_{d,P^\bullet,x=\alpha}^{(a)}}, \quad (23)$$

where:

$$C_{d,p,x=1}^{(d)} = \frac{1}{\alpha^4} - 1 + \frac{8}{\alpha^2} - \frac{2\cdot\left(3 + \nu_p\right)\cdot\left(1 - \alpha^4\right)}{\left(1 + \nu_p\right)\cdot\alpha^4} +$$

$$+ \, 4 \cdot \frac{3 + \nu_p}{\left(1 - \nu_p\right) \cdot \alpha^2} \cdot \ln \alpha \, + \, 4 \cdot \frac{1 - \nu_p}{1 + \nu_p} \cdot \frac{1 - \alpha^2}{\alpha^4} \, +$$

$$+ \, 8 \cdot \frac{\ln \alpha - 1}{\alpha^2} \, + \, 8 \cdot \frac{\ln \alpha}{\left(\alpha^2 - 1\right) \cdot \alpha^2} \cdot \left(1 - \alpha^2 - 2 \cdot \frac{1 + \nu_p}{1 - \nu_p} \cdot \ln \alpha \right). \tag{24}$$

3.1. Deformations expressions

The rotation on the inside leaning has the form:

$$\varphi^{(B)}_{p, \, x = 1} \, = \, \frac{p \cdot r^4_{c, \, r}}{16 \cdot \Re_p \cdot r_0} \cdot \left[C^{(d)}_{r, \, p, \, x = 1} \, - \, \frac{C^{(d)}_{d, \, p, \, x = 1}}{C^{(a)}_{d, \, P^\bullet, \, x = \alpha}} \cdot C^{(a)}_{r, \, P^\bullet, \, x = 1} \right], \tag{25}$$

with the notation:

$$C^{(d)}_{r, \, p, \, x = 1} \, = \, \frac{1}{\alpha^2} \cdot \left[\frac{3}{\alpha^2} - \frac{3 + \nu_p}{1 + \nu_p} \cdot \frac{\alpha^2 + 1}{\alpha^2} - \frac{3 + \nu_p}{1 - \nu_p} \, + \right.$$

$$\left. + \, 2 \cdot \frac{1 - \nu_p}{\left(1 + \nu_p\right) \cdot \alpha^2} - \frac{8 \cdot \nu_p \cdot \ln \alpha}{\left(1 - \nu_p\right) \cdot \left(\alpha^2 - 1\right)} \right], \tag{26}$$

while for the outside leaning we set the expression:

$$\varphi^{(B)}_{p, \, x = \alpha} \, = \, \frac{p \cdot r^4_{c \, r}}{16 \cdot \Re_p \cdot r_0} \cdot \left[\begin{array}{c} C^{(d)}_{r, \, p, \, x = \alpha} \, - \\[2mm] - \, \dfrac{C^{(d)}_{d, \, p, \, x = 1}}{C^{(a)}_{d, \, P^\bullet, \, x = \alpha}} \cdot C^{(a)}_{r, \, P^\bullet, \, x = \alpha} \end{array} \right], \tag{27}$$

where the notation was introduced:

$$C^{(d)}_{r, \, p, \, x = \alpha} \, = \, \frac{1}{\alpha} \, +$$

$$+ \, \frac{2 \cdot \left(1 - 2 \cdot \ln \alpha\right)}{\alpha^3} - \frac{3 + \nu_p}{1 + \nu_p} \cdot \frac{\alpha^2 + 1}{\alpha^3} - \frac{3 + \nu_p}{\left(1 - \nu_p\right) \cdot \alpha^3} \, +$$

$$+ \, 2 \cdot \frac{1 - \nu_p}{\left(1 + \nu_p\right) \cdot \alpha^3} + \frac{4 \cdot \ln \alpha}{\alpha^2 - 1} \cdot \left(\alpha - \frac{1 + \nu_p}{1 - \nu_p} \cdot \frac{1}{\alpha}\right) \cdot \frac{1}{\alpha^2}. \tag{28}$$

The vertical displacement of the points of the median area of the plate, between the leaning outlines can be calculated by the expression:

$$w^{(B)}_{p,x} = \frac{p \cdot r^4_{c,r}}{64 \cdot \Re_p} \cdot \left[C^{(d)}_{d,p,x} - \frac{C^{(d)}_{d,p,x=1}}{C^{(a)}_{d,P^\bullet,x=\alpha}} \cdot \left(C^{(a)}_{d,P^\bullet,x=\alpha} - C^{(a)}_{d,P^\bullet,x} \right) \right],$$ (29)

with the notation:

$$C^{(d)}_{d,p,x} = \left(\frac{x}{\alpha}\right)^4 - 1 + \frac{8 \cdot x^2 \cdot (1 - \ln x)}{\alpha^4} - \frac{2 \cdot (3 + \nu_p)}{1 + \nu_p} \cdot \frac{(x^2 - \alpha^2) \cdot (\alpha^2 + 1)}{\alpha^4} +$$

$$+ 4 \cdot \frac{3 + \nu_p}{(1 - \nu_p) \cdot \alpha^2} \cdot \ln \frac{\alpha}{x} + 4 \cdot \frac{1 - \nu_p}{1 + \nu_p} \cdot \frac{x^2 - \alpha^2}{\alpha^4} + 8 \cdot \frac{\ln \alpha - 1}{\alpha^2} +$$

$$+ 8 \cdot \frac{\ln \alpha}{(\alpha^2 - 1) \cdot \alpha^2} \cdot \left(x^2 - \alpha^2 + 2 \cdot \frac{1 + \nu_p}{1 - \nu_p} \cdot \ln \frac{x}{\alpha} \right).$$ (30)

3.2. Stresses expressions

The expressions of the radial and annular stresses are derived, in this case, too, by the difference between those characteristic to the stresses corresponding to the actions given of the uniform distributed pressure and of the force $P^\bullet_2$, resulting:

► *for the radial stresses:*

$$\sigma^{(B)}_{r,p,x} = \frac{3 \cdot p \cdot r^2_{c,r}}{2 \cdot \delta^2_p} \cdot \left[C^{(d)}_{r,\sigma,p} - \frac{1}{4} \cdot \frac{C^{(d)}_{d,p,x=1}}{C^{(a)}_{d,P^\bullet,x=\alpha}} \cdot C^{(a)}_{r,\sigma,P^\bullet} \right],$$ (31)

with the notation:

$$C^{(d)}_{r,\sigma,p} = \frac{1 + \nu_p}{\alpha^2} \cdot \ln x - \frac{3 + \nu_p}{4} \cdot \frac{(x^2 - 1) \cdot (x^2 - \alpha^2)}{\alpha^2 \cdot x^2} -$$

$$- \frac{(1 + \nu_p) \cdot \ln \alpha}{\alpha^2 - 1} \cdot \frac{x^2 - 1}{x^2};$$ (32)

► *for the annular stresses:*

$$\sigma^{(B)}_{\theta,p,x} = \frac{3 \cdot p \cdot r^2_{c,r}}{2 \cdot \delta^2_p} \cdot \left[C^{(d)}_{\theta,\sigma,p} - \frac{1}{4} \cdot \frac{C^{(d)}_{d,p,x=1}}{C^{(a)}_{d,P^\bullet,x=\alpha}} \cdot C^{(a)}_{\theta,\sigma,P^\bullet} \right],$$ (33)

with the notation:

$$C^{(d)}_{\theta,\sigma,p} = \frac{1 + \nu_p}{\alpha^2} \cdot \ln x + \frac{3 + \nu_p}{4} \cdot \left(1 + \frac{1}{\alpha^2} + \frac{1}{x^2} \right) -$$

$$- \frac{(1 + \nu_p) \cdot \ln \alpha}{\alpha^2 - 1} \cdot \frac{x^2 + 1}{x^2} - \frac{1 + 3 \cdot \nu_p}{4 \cdot \alpha^2} \cdot x^2 - \frac{1 - \nu_p}{\alpha^2}.$$ (34)

4. CONCLUSIONS

In the preceding, a theoretical study was done referring to an annular plate under the action of a uniform distributed pressure (or a differential pressure), with simple leaning on the inside and outside outline.

Two versions of analysis are considered, eliminating one or another of the outlines and introducing distributed forces to produce similar vertical displacements as values, but opposite. In this way, the idea proposed by the work [2] is materialized. The aim is to deduce the expressions of the vertical displacements, of the rotation and of the radial and annular stresses developed in the current sections of the analyzed plate. The ulterior examples can highlight any aspects specific to each case.

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