ANALYTICAL MODEL FOR SPECIFIC STRATIFIED COMPOSITE STRUCTURES

DURBACĂ ION, DURBACĂ ADRIAN-COSTIN*

POLITEHNICA University of Bucharest, Splaiul Independenţei, 313, Bucureşti, Romania

Abstract: This paper promotes a model for calculating the stress intensity factor at the crack tip in a bi-material composite laminated for damage tolerance model obtained using a brittle material that geometry. The model is based on assumptions about the elastic properties are homogeneous throughout the composite material and using finite element simulations leading to moderate results different from those obtained using the homogeneous model. Also, the model relies on the assumption that the system behaves as a homogeneous anisotropic material and when the field of stress at the crack tip is created on account of the effect of applied loads on crack tip and thus gives results that are closer to those of finite element simulations.

Keywords: composite laminate, tension, rupture, fracture loads applied

1. INTRODUCTORY CONSIDERATIONS

Geometric model of bi-material composite laminated, made by rolling (see Figure 1), consists of alternating layers of biaxial-compressive residual strain with the possibility of failure could be demonstrated by crack propagation efficiency of the proposed structure [1-3] .

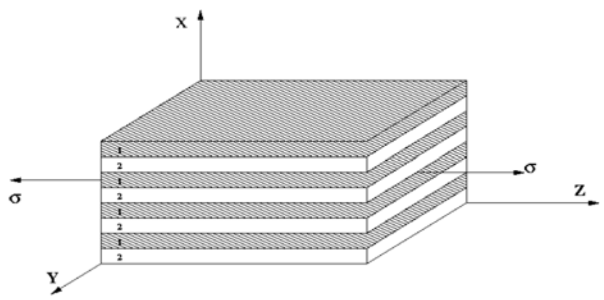

Fig. 1. Geometric model of bi-material composite laminated (2-3 is the plane of transverse isotropy).

This scheme, when applied to composite materials can be fragile, effectively induces changes in the force distribution of the material at deterministic probabilistic model, a certain amount of tension applied. Thus, the material develops a limit below which no yield strength, and over which it can be determined by the size of the largest defect structure.

* Corresponding author, email: ion.durbaca@yahoo.com

A stress intensity factor *K* depending on the inclusion of blood pressure effects of alternative waste-has been developed to describe the crack tip in layers retarding the compressive external stress applied, respectively:

$$K = \sigma_a \cdot \sqrt{\pi \cdot a} + \sigma_c \cdot \sqrt{\pi \cdot a}\left[\left(1 + \frac{t_1}{t_2}\right) \cdot \frac{2}{\pi} \cdot \sin^{-1}\left(\frac{t_2}{2a}\right) - 1\right] \tag{1}$$

where, σ_a is the applied stress; *a* is the half crack length; t_1 and t_2 is the thickness of the compressive and tensile layers; σ_c is the intensity biaxial tension-compression and residual layers is evaluated by the relationship [4]:

$$\sigma_c = \varepsilon \cdot E_1^{'}\left(1 + \frac{t_1 \cdot E_1^{'}}{t_2 \cdot E_2^{'}}\right)^{-1} \tag{2}$$

where, ε is the residual differential thermal strain; $E_1^{'} = E_1/(1 - \nu_1)$ is the modified Young's modulus of elasticity is as the first component of the composite material; ν_1 is Poisson's ratio corresponding to the first component of the composite material.

Equality (1) above shows that the intensity factor of crack tip voltage is proportional to crack length (being reduced to a large fraction there of), which takes place in the two-layer tablets, along each side traction layer. Thus, noting K_c critical stress intensity factor corresponding to the compressive layer material, $2a = t_2 + 2t_1$ and rearranging terms, one can obtain an expression for a threshold (limit) the stress below which a crack may not extend through the layers of compression as enough to cause a failure:

$$\sigma_{\lim} = \frac{K_c}{\sqrt{\pi \frac{t_2}{2}\left(1 + \frac{2t_1}{t_2}\right)}} + \sigma_c\left[1 - \left(1 + \frac{t_1}{t_2}\right) \cdot \frac{2}{\pi}\sin^{-1}\left(\frac{t_1}{t_2 + 2t_2}\right)\right] \tag{3}$$

As shown in equation (3) above, it can be used to predict the effectiveness of such a system to obtain predictable tensions in brittle materials. This translates into increased endurance limit according to the factor of intensity of compressive residual stress in alternating layers of composite material and that the reverse change in relation to the appropriate size scale thickness of fabric layers. Also, the effect of endurance limit has been validated in experimental studies into a composite alumina / mullite [3, 5], and silicon systems / silica layers with thickness much smaller [6]. This expression can also be used to optimize the voltage limit can be achieved in a given system by adjusting the material thickness ratio between layers achieved. The great contribution of some scientists *Hbaieb* and *McMeeking* [7] showed that the limit of optimum strength for materials with low hardness of about 0,3 $\varepsilon E_1^{'}$, can be achieved by maintaining the ratio between the thickness of layers made between 1 ÷ 2.8.

The approximate bi-material laminate composite with an elastic homogeneous system, according to relations (1) and (3) assess of the stress intensity factor at the crack tip and the tension resulting σ_{lim} limit. As such, all predictions resulting from the relations written above are limited in their accuracy when the modules of elasticity (Young) and Poisson's coefficient corresponding to the two laminated composite materials are significantly different. Hbaieb and McMeeking [8] conducted a finite element simulation of this geometrical model and have presented extensive results on the variation of threshold voltage in relation to the different ratios of coefficients specific modules and bi-component materials. Figure 2 shows a comparison of results from finite element model predictions reported in the corresponding analytical expression. Thus, the results begin to differ greatly depending on the ratio of modules E_1/E_2, that have different values of unity.

Results of finite element model provides a great insight into the phenomenon of prevention / limitation of cracking due to the presence of tension and compression at the same time, greater efficacy than the analytical model compared to the corresponding parameters of different materials, materials assuming homogeneity. The drawback is that the finite element analysis can not be easily applied to optimize the material and geometrical parameters to achieve the highest possible voltage limits in relation to how the analytical form of an expression may be accessible. For this reason, is being sought an alternative model to predict the intensity of pressure in the crack tip and its limit who can take into account the heterogeneity of bi-component material and an expression of efficiency in a way more real.

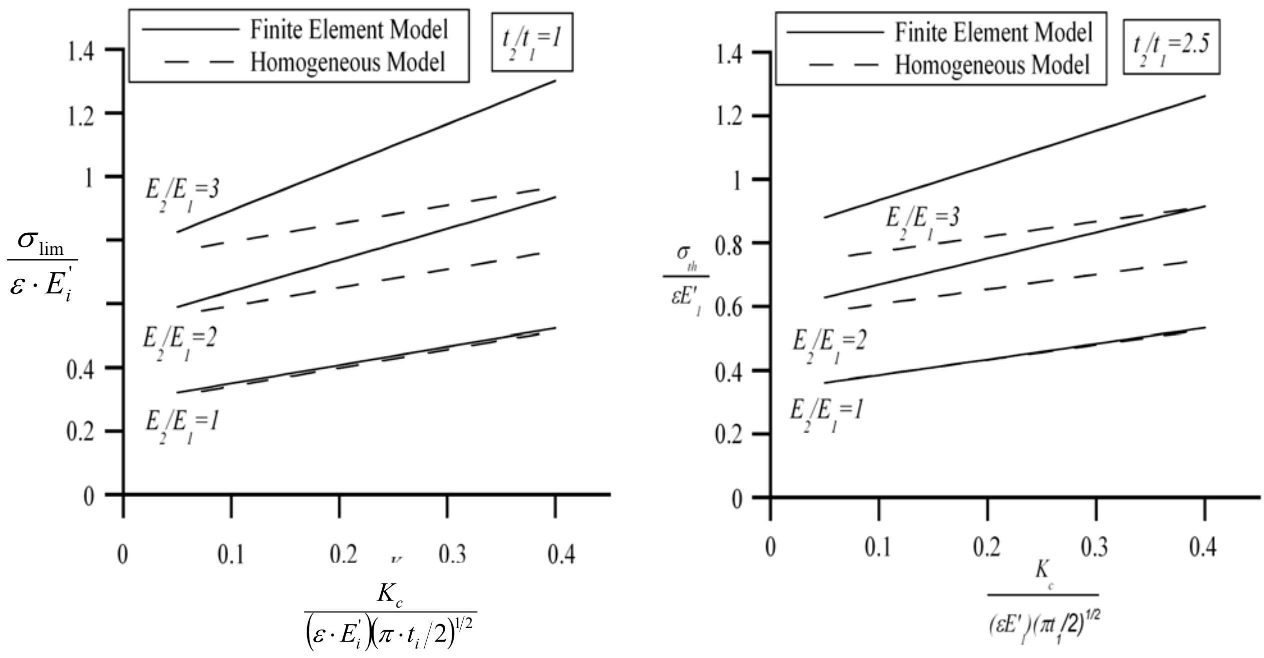

Fig. 2. Comparison of the results of analytic model and FEA for two different ratios of the thickness of layers of bi-component material.

2. DESCRIPTION OF ANALYTICAL MODEL FOR A BI-MATERIAL COMPOSITE LAMINATED ORTHOTROPIC

To arrive at an analytical expression for the stress intensity factor at the crack tip is considered a bi-laminate composite as an anisotropic material in a limited sense.

It is assumed that the tensions set up at the crack tip due to the imposed limit of composite material components in the composite are similar to those analyzed for the type of rupture would have identical geometry in an anisotropic material. In literature [9], using an expression for energy release rate at crack tip in an anisotropic material specific stress intensity factor K_i limit value corresponding enigmatic problem:

$$G_i = C\ K_i^2 \tag{4}$$

where, C is calculated according to the matrix elements for anisotropic materials. For the case of a bi-material laminate, is considered orthotropic composite (property materials have different properties on the direction / different planes), and the corresponding crack opening of a bursting Mode 1; C is assessed by the expression:

$$C = \sqrt{\frac{A_{11} \cdot A_{22}}{2}} \left[\sqrt{\frac{A_{22}}{A_{11}}} + \frac{2A_{12} + A_{66}}{2A_{11}} \right]^{\frac{1}{2}} \tag{5}$$

in which A_{ij} are the elements of the compliance matrix. The stress intensity factor for the isotropic boundary value problem is that evaluated by *Rao*, according to [3], which takes into account the effect of the residual stress at the crack tip, and which we will refer to as $K_1^{izotr.}$.

The energy release rate thus obtained can be associated, similar previous case, the stress intensity factor at the crack tip, which for an anisotropic material is expressed as:

$$K_1^{anizotr.} = \sqrt{\frac{E}{\left(1-\nu^2\right)}} \cdot G_1^{1/2} \tag{6}$$

and will take into account the state plane displacement (*SPD*). Modulus of elasticity (*E*) and Poisson's ratio (υ) in rel (6) are established for the area at the crack tip material components, which in this case is the compressive layer material. The relations (4) and (6), we obtain the report:

$$F = \frac{K_1^{anizotr.}}{K_1^{izotr.}} = \sqrt{\frac{EC}{\left(1-\nu^2\right)}} \tag{7}$$

2.1. Equations

To obtain the compliance matrix for a bi-laminate composite orthotropic (with transverse isotropy in the plane 2-3), the general elasticity problem is solved, so as shown in Figure 1, above. Apply tension along the axis of symmetry for isotropic and calculated displacements (deformations). The report stresses applied displacement corresponding matrix elements resulting elastic constants of material relating to specific individual components of the bi-laminate composite and their relative thickness. For an orthotropic material, there are only five independent elastic constants and the compliance matrix has the form:

$$\begin{bmatrix} A_{11} & A_{12} & A_{13} & 0 & 0 & 0 \\ A_{21} & A_{22} & A_{23} & 0 & 0 & 0 \\ A_{31} & A_{32} & A_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & A_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & A_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & A_{66} \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{12} & 0 & 0 & 0 \\ S_{12} & S_{22} & S_{23} & 0 & 0 & 0 \\ S_{12} & S_{23} & S_{22} & 0 & 0 & 0 \\ 0 & 0 & 0 & 2(S_{22}-S_{23}) & 0 & 0 \\ 0 & 0 & 0 & 0 & S_{66} & 0 \\ 0 & 0 & 0 & 0 & 0 & S_{66} \end{bmatrix} \tag{8}$$

where:

$$S_{11} = \frac{E_B^2 t_A t_B (1-\nu_A - 2\nu_A^2) + E_A E_B \left[t_B^2\left(1-\nu_A\right) + t_A^2\left(1-\nu_B\right) + 4 t_A t_B \nu_A \nu_B\right] + E_A^2 t_A t_B \left(1-\nu_B - 2\nu_B^2\right)}{E_A E_B \left(t_A + t_B\right)\left[E_B t_B \left(1-\nu_A\right) + E_A t_A \left(1-\nu_B\right)\right]} \tag{9}$$

$$S_{12} = \frac{t_A \nu_A \left(1-\nu_B\right) + t_B \nu_B \left(1-\nu_A\right)}{E_B t_B \left(1-\nu_A\right) + E_A t_A \left(1-\nu_B\right)} \tag{10}$$

$$S_{23} = \frac{\left(t_A + t_B\right)\left[E_B t_B \nu_B \left(1-\nu_A^2\right) + E_A t_A \nu_A \left(1-\nu_B^2\right)\right]}{E_B^2 t_B^2 \left(1-\nu_A^2\right) + 2 E_A E_B t_A t_B \left(1-\nu_A \nu_B\right) + E_A^2 t_A^2 \left(1-\nu_B^2\right)} \tag{11}$$

$$S_{22} = \frac{\left(t_A + t_B\right)\left[E_B t_B \left(1-\nu_A^2\right) + E_A t_A \left(1-\nu_B^2\right)\right]}{E_B^2 t_B^2 \left(1-\nu_A^2\right) + 2 E_A E_B t_A t_B \left(1-\nu_A \nu_B\right) + E_A^2 t_A^2 \left(1-\nu_B^2\right)} \tag{12}$$

$$S_{44} = 2\left(S_{22} - S_{23}\right) = \frac{2\left(t_A + t_B\right)\left(1-\nu_A\right)\left(1-\nu_B\right)}{E_B t_B \left(1+\nu_A\right) + E_A t_A \left(1+\nu_B\right)} \tag{13}$$

$$S_{66} = \frac{E_B t_A (1+\nu_A) + E_B t_B (1+\nu_B)}{E_A E_B (t_A + t_B)} \tag{14}$$

The factor F – expression (7), on the relationship between $K_1^{anizotr.}$ and $K_1^{izotr.}$ is calculated using relations (9) - (14). Although the above expressions have apparently complicated form, however, they offer new perspectives by using them. However, they have been checked by applying special conditions: $E_A = E_B$ and $\nu_A = \nu_B$.

The expression threshold stress for this model is:

$$\sigma_{\lim} = \sqrt{\frac{(1-\nu_1^2)}{E_1 C}} \frac{K_c}{\sqrt{\pi \frac{t_2}{2}\left(1+\frac{2t_1}{t_2}\right)}} + \sigma_c \left[1 - \left(1+\frac{t_1}{t_2}\right)\frac{2}{\pi}\sin^{-1}\left(\frac{t_1}{t_2 + 2t_2}\right)\right] \tag{15}$$

The expression (15) obtained above, made the prediction a higher value of the stress limit for brittle ceramic composites. The simulation results compared with forecasts made by FEA homogeneous model. It is noted that such analysis only affects the first term of the relationship obtained for the stress intensity factor (1). The first term that is independent of any residual tension in the system, reflecting, in turn, influence the critical stress intensity factor for layered composite material properly, and most importantly, the thickness of graded layers.

3. RESULTS

In Figure 3 below shows a comparison between the threshold stress values obtained by finite element simulation model of isotropic material and orthotropic model, described above, for different ratios of the respective modules of elasticity and thickness of specific layers of bi-component materials.

The FEA results differ more than the homogeneous model results by increased disparity between the modules of elasticity, and higher values of critical stress intensity factor of the film layers of specific materials. Somewhat different mode of deformation anisotropy reflects the mismatch effect on the change of resistance index limit critical stress intensity factor. How interpretation of the curves in Figure 3 is not influenced by changes to model homogeneous and leads to an underestimation of the threshold stress by emphasizing small breaks.

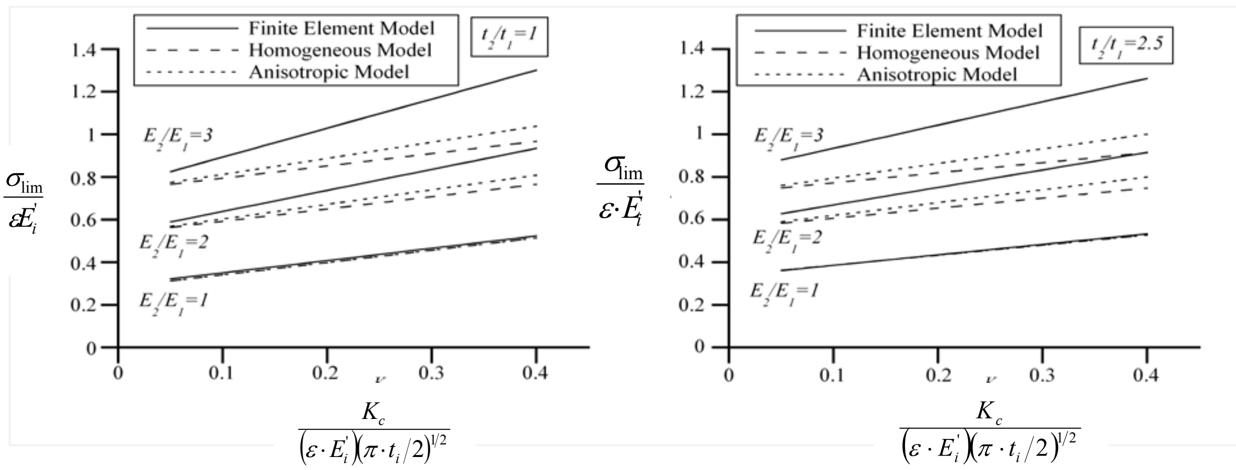

Fig. 3. Comparative results between the FEA and analytical modeling methods.

4. CONCLUSIONS

The modeling strategy presented in this paper represents an important step towards optimizing the parameters to achieve the highest possible level voltage corresponding to a geometric pattern made of a composite bi-material laminate in a given system [9]. There is a significant discrepancy between the results of the analytical model homogeneous and finite element analysis, where the elastic constants of two materials are different, and this becomes more significant impact on reducing the size strata values. In this area a number of works are carried out towards applying the proposed analytical models to improve the reliability of composite structures involving thick layers of material components with sub-micron levels. Optimizing the thickness of layers of materials within a composite laminate layer requires an improved analytical model, since the use of finite element simulations for the same purpose is quite difficult. However, the analytic model presented remains only an approximation of the optimal solution for a bi-material laminate composite. The challenge is accurately modeling the deformation at the crack tip, approaching the interface between a layer of compressive and tensile another, finally determined, the stress limit. Also to be developed and pursued research on issues relevant situations of failure in laminated composite materials.

REFERENCES

[1] Alămoreanu, E., Negruţ, C., Jiga, G., Calculul structurilor din materiale compozite, UPB, 1993.
[2] Gheorghiu, H., Hadăr, A., Constantin, N., Analiza structurilor din materiale izotrope si anizotrope, Editura Printech, Bucuresti, 1998.
[3] Rao, M. P., Sanchez-Herencia, A. J., Beltz, G. E., McMeeking, R. M. and Lange, F. F., Science 286, 1999, p. 102-105.
[4] Hillman, C., Suo, Z. G. and Lange, F. F., Journal of the American Ceramic Society 79, 1996, p. 2127-2133.
[5] Rao, M. P. and Lange, F. F., Journal of the American Ceramic Society 85, 2002, p. 1222-1228.
[6] Paranjpye, A., MacDonald, N. C. and Beltz, G. E., A microscale composite material for fracture resistant MEMS architecture, Unpublished, Work in Progress.
[7] McMeeking, R. M. and Hbaieb, K., Zeitschrift Fur Metallkunde 90, 1999, p. 1031-1036.
[8] Hbaieb, K. and McMeeking, R. M., Mechanics of Materials 34, 2002, p. 755-772.
[9] Paranjpye, A., Beltz, G.E. and MacDonald, M. C., An Analytical Model for the Effect of Elastic Modulus Mismatch on Laminate Threshold Strength, Mat. Res. Soc. Symp. Proc. Vol. 791, 2004 Materials Research Society.