

THEORETICAL STUDIES CONCERNING ELASTIC BARS DEFORMATION FROM GRAPES HARVESTING MACHINE

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Abstract: This paper presents a theoretical model for calculating the elastic bars subjected to bending plane, with application to gear the shaking Braud grape harvesters. Based on this model we can determine the deformation of the bars with the elliptic integrals.

Keywords: elastic bars, deformed bars, elliptic integrals

1. INTRODUCTION

Thin bars or slick bars are straight bars or curved ones having the cross-section dimensions very small, in comparison with its length and radius of curvature.

It is presumed that under the action of loads, the elastic line of a thin bar (deformed shape of the axis) remaining plane, it can take different shapes than in initial state. Because of small thickness of the bar, its bending deformations remain small, and its tensions don't exceed the proportionality limit [1].

In the theory of thin bars, deflections can be very small and no quantitative limit is imposed regarding their values [2]. In these circumstances, relations from linear statics can no longer be applied. In particular, the bars equilibrium conditions must be written taking into account deflections, which means that the principle of initial dimensions non-variations is no longer valid, the same goes to the superposition principle [3].

The relatively simple character of problems from main class results from the fact that the differential equation of elastic line of a bar can be integrated with the help of elliptic integrals [4].

For this study, shaking equipment with elastic bars, model BRAUD, has been used. The working parts are made out of high flexible nylon, the two layers or bars being tied up kinematical and displaced with 180° . The working stage is presented in Figure 1.

One end of the bar is fixed on to the oscillating support plate (which regarding to the rotational angle determines the amplitude of deflection), the other end being articulated at a balance-wheel.

Mounted with an initial curvature, the elastic bars assure a convergent entrance, a long active area and one divergent exit [5].

The work aims to establish the mathematical equations describing the deformation bar at work, and finally its speed and acceleration of relative to the axis line of the vine.

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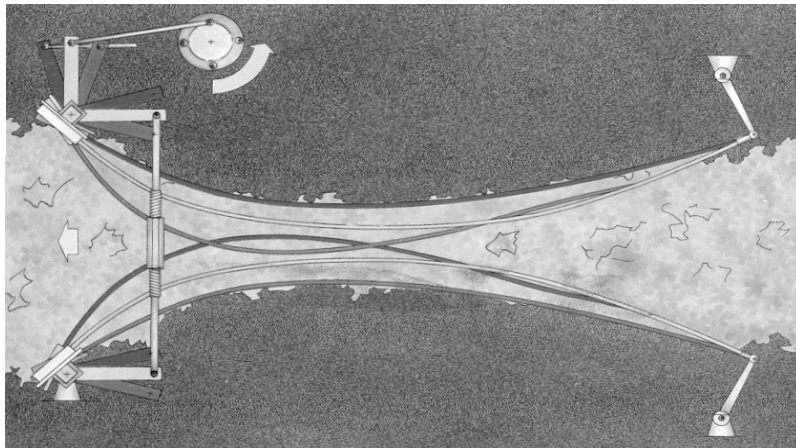


Fig. 1. The working stage of the equipment.

2. PLANE BENDING DIFFERENTIAL EQUATION OF ELASTIC BAR

Let us consider that the elastic bar from Figure 2 has initially straight shape, with length l and radius of curvature $R = \infty$. The end A of the bar describes an arc with the radius $OA = a$, having center in O, center chosen as origin for a fix system of axes XOY. The end B of the bar describes an arc with the radius $O_1B = b$, and center O_1 . With respect to the system XOY, the coordinates of point O_1 are given by:

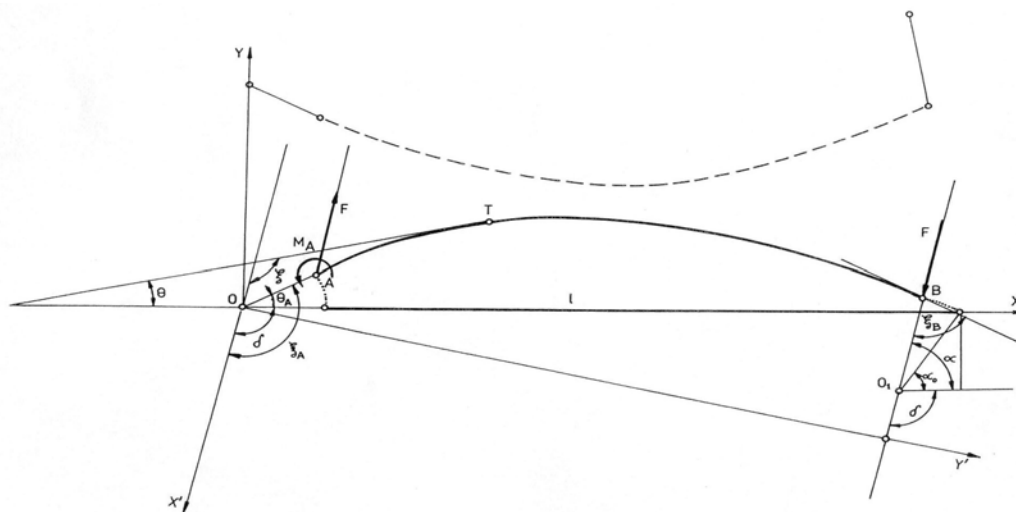


Fig. 2. Notations used for the elastic bar.

$$O_1 : \begin{cases} x = a + l - b \cos \alpha_0 \\ y = -b \sin \alpha_0 \end{cases} \quad (1)$$

The bar changes its shape as a result of rotation with angle θ_A of a rigid piece in which end A is fixed. Because of this, the angle in point A made by the tangent to the deformed bar with the axis OX is equal to θ_A . It is considered that the deformed bar has the same length as the initial bar.

The reactions at the ends of the deformed bar are: bending moment from fixed end M_A and F forces at the two ends of the bar. Taking into account that the element O_1B is jointed at both ends, it results that force F has

direction O_1B . From the equilibrium condition, the forces from the ends of the bar must be equal, parallel and opposite in sign.

An $X'OY'$ system of coordinates is fixed, with axis OX' parallel with force F and rotated clockwise with angle δ with respect to OX . In general, the angle δ doesn't remain constant during bending process because direction of force F varies, in such a way that system $X'OY'$ is mobile.

It is considered a current point T on the deformed bar. The tangent in point T forms both angle θ and angle ξ , with positive directions of axes OX and OX' . Between these angles, a relation exists:

$$\theta = \xi - \delta \quad (2)$$

Arc s is considered as independent variable for determination of deformed line shape, measured between points O and T , considering that in case of bending, the length of this arc remains constant. This hypothesis can be accepted because system deflections produced by bending are much bigger than the ones which appear in case of compression or tension of arc contour. Between coordinates of point T from two systems, relations exist:

$$\begin{cases} x = x' \cos \delta + y' \sin \delta \\ y = y' \cos \delta - x' \sin \delta \end{cases} \quad (3)$$

From equilibrium condition of elastic bar it can be written:

$$M_A = F(y'_B - y'_A) \quad (4)$$

In current point T , the bending moment is given by:

$$M = F(y'_B - y') = M_A - F(y' - y'_A) \quad (5)$$

It has been considered the positive moment if, under its action, the bar bends itself even more. The advantage of the mobile axes system is the fact that the arm of force F can be written as difference between coordinates O and T .

At point T , deflected bar has a curvature $1/\rho$ (ρ is the radius of curvature), the relation being known:

$$\frac{1}{\rho} = \frac{d\theta}{ds} \quad (\text{in } XOY) \quad (6)$$

$$\frac{1}{\rho} = \frac{d\xi}{ds} \quad (\text{in } X'OY) \quad (7)$$

In relation $\frac{1}{\rho} - \frac{1}{R} = \frac{M}{EI}$, known from strength of materials, for $R = \infty$ and because the curvature of the elastic

line in the considered point is given by $\frac{d\theta}{ds}$ or by $\frac{d\xi}{ds}$ can be written:

$$M = EI \frac{d\xi}{ds} \quad (8)$$

and for system $X'OY'$:

$$\frac{d\xi}{ds} = \frac{F}{EI} (y'_B - y') \quad (9)$$

Relation (9.) is being derived with respect to s arch and is obtained:

$$\frac{d^2\xi}{ds^2} = -\frac{F}{EI} \frac{dy'}{ds} \quad (10)$$

From the element ds, it could be written $\frac{dx'}{ds} = \cos \xi$ and $\frac{dy'}{ds} = \sin \xi$ replacing in relation (10) it is obtained:

$$\frac{d^2\xi}{ds^2} = -\frac{F}{EI} \sin \xi \quad (11)$$

Relation (11) represents the differential equation of the elastic line of the deformed bar in the mobile system of axes X'OY'.

It is more convenient to use instead of forces, adimensional parameters, tied to F force. The adimensional relation is written $\beta = l \sqrt{\frac{F}{EI}}$; with this, differential equation (11) becomes:

$$l^2 \frac{d^2\xi}{ds^2} = -\beta^2 \sin \xi \quad (12)$$

In this equation, the unknown function is the tilting angle of the tangent in point T of the deformed bar with respect to OX' axis, ξ .

2.1. First integral and the quality study of equilibrium forms

The differential equation can be easily integrated. For this, it can be written as:

$$l^2 \frac{d}{ds} \left(\frac{d\xi}{ds} \right) = -\beta^2 2 \sin \frac{\xi}{2} \cos \frac{\xi}{2} \quad (13)$$

Both members of the equation are multiplied with $\frac{d\xi}{ds}$, simplifications are made and each member is integrated.

The left side variable is $\frac{d\xi}{ds}$ and the right side variable is ξ . The result is written as:

$$l \left(\frac{d\xi}{ds} \right)^2 = 4\beta^2 \left(C_1 - \sin^2 \frac{\xi}{2} \right) \quad (14)$$

With the obtained relation (14), a quality analysis can be made of the equilibrium forms of elastic bar. In this analysis, the main role is played by the integration constant C_1 . From relation (14) it results that C_1 must be positive. The following cases are apart: if C_1 is improper, the elastic line has no inflexion points and such a form is called form without inflexion; if C_1 is proper, then it must respect the condition: $\left(\sin \frac{\xi}{2} \right)_{\max} \leq C_1 \leq 1$.

For the case in which the constant $C_1=1$ it is obtained an equation which through integration allows the determination of parameter β .

Next, the analysis of the elastic line of deformed bar will be made depending on two behavior situations of the integration constant.

2.2. The second integral and deduction of basic formulas

a. Integration constant $C_1 < 1$.

Before we integrate the equation (14) we denote the constant $C_1 = K^2$ and introduce a new variable ψ , defined by

relation: $\sin \frac{\xi}{2} = K \sin \psi$, where $K \leq 1$. Both members are derived, $\cos \frac{\xi}{2}$ is expressed through $\sin^2 \frac{\xi}{2}$ and it results:

$$d\xi = 2K \frac{\cos \psi}{\sqrt{1 - K^2 \sin^2 \psi}} d\psi \quad (15)$$

For relation (14.) the shape is obtained:

$$\frac{\beta}{l} ds = \frac{d\xi}{2\sqrt{K^2 - \sin^2 \frac{\xi}{2}}} \quad (16)$$

Introducing (15) into (16) and after simplifications, integrating the equation between origin point and the current point of variable s , it is obtained:

$$\frac{\beta}{l} s = \int_{\psi_0}^{\psi} \frac{d\psi}{\sqrt{1 - K^2 \sin^2 \psi}} \quad (17)$$

in which ψ_0 is an elliptic variable in the origin point and ψ is an elliptic variable in the current point.

b. Integration constant $C_1 > 1$.

For the integration of equation (14), the constant C_1 is denoted with $1/K^2$ and a new variable is introduced ψ ,

defined by the relation: $\sin \frac{\xi}{2} = K \sin \psi$, where $K \leq 1$. A same mathematical procedure is made in the precedent case and it is obtained:

$$\frac{\beta}{l} s = K \int_{\psi_0}^{\psi} \frac{d\psi}{\sqrt{1 - K^2 \sin^2 \psi}} \quad (18)$$

Relations (17) and (18) establish the link between the amplitude ψ , initial and current one, and the force parameter β . In both cases, $K \leq 1$ and the elliptic integrals have real values.

2.3. Determination of the shape of elastic line in X'OY' and the bending moment

For obtaining the expressions which determine the shape of the elastic line, we start from relation dx' and dy' are expressed through the angle $\xi/2$. The expressions for the sinus function are drawn and for ds from relations (16).

a. Integration constant $C_1 < 1$.

Expressing dx' is obtained ordinate as:

$$\frac{dx'}{l} = \frac{2}{\beta} \sqrt{1 - K^2 \sin^2 \psi} d\psi - \frac{ds}{l} \quad (19)$$

In a similar way is preceded in the case of dy' :

$$\frac{dy'}{l} = \frac{2K}{\beta} \sin \psi d\psi \quad (20)$$

Integrating the relations (19) and (20) from the origin of arches s till a current point T , it is obtained:

$$\frac{x' - x_0}{l} = \frac{2}{\beta} \int_{\psi_0}^{\psi} \sqrt{1 - K^2 \sin^2 \psi} d\psi - \frac{s}{l} \quad (21)$$

$$\frac{y' - y_0}{l} = \frac{2K}{\beta} (\cos \psi_0 - \cos \psi) \quad (22)$$

Determination of bending moment is made starting from relation (8) taking into account relation (16), after doing the simplifications, the final form is obtained:

$$M = \frac{2K}{\beta} Fl \cos \psi \quad (23)$$

b. Integration constant $C_1 > 1$.

Proceeding in a similar way, relations are obtained:

$$\frac{x' - x_0}{l} = \frac{2}{\beta K} \int_{\psi_0}^{\psi} \sqrt{1 - K^2 \sin^2 \psi} d\psi - \left(\frac{2}{K^2} - 1 \right) \frac{s}{l} \quad (24)$$

$$\frac{y' - y_0}{l} = \frac{2}{\beta K} \left(\sqrt{1 - K^2 \sin^2 \psi_0} - \sqrt{1 - K^2 \sin^2 \psi} \right) \quad (25)$$

The bending moment will have the following expression:

$$M = \frac{2}{\beta K} Fl \sqrt{1 - K^2 \sin^2 \psi} \quad (26)$$

The form of the elastic line of the deformed bar is determined by equations (21), (22), (24) and (25). In these equations, besides the parameter β , the K modulus and initial amplitude ψ_0 gain implicit and explicit form, which are determined from boundaries conditions. The variables are arch s and current amplitude ψ , tied to s .

In general, determination of parameters ψ_0 and ψ is reduced to resolving a system of transcendental equations, in which the unknowns are under the sign of the elliptic integrals. These integrals are solved through trials, using table of elliptic integrals.

Because the system of axes $X'OY'$ is rotated with the angle δ with respect to XOY , relations (3) could be written, which introduced in the precedent relations will determine two systems of equations for the elastic line of a bar, for two cases.

On the basis of mathematical relations established, particularizations can be made of equations which define the elastic line of the bar, from which values of ψ_0 and parameter β are extracted, with these values elliptic integrals can be solved.

3. CONCLUSION

From the construction of acting mechanism of the shaking equipment it will be determined the law of variation for angle θ , depending on the constructive elements and time. With these, replacing it in mathematical relations presented earlier, it can be obtained for coordinates of current point T, functions which depend on time. Through deriving them we can determine the component of velocity and acceleration with respect to those two systems of axes of coordinates.

This method of calculus allows to determine the kinematic parameters of the elastic bars, depending on revolutions of acting mechanism or the frequency of oscillation (shaking).

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