

## THEORETICAL ANALYSIS OF DYNAMIC RESPONSE OF A BRIDGE SECTION LOADING BY THE IMPULSIVE FORCE

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Summary: The work in seismic design of bridges is needed to develop physical models and mathematical theory, based on which will be assessed and quantified the dynamic response of bridge structures subjected to stress arising from road traffic or seismic activity.

The need for this modeling is imposed three requirements: to ensure the structural integrity of the bridge elements for subjected impulsive, proper choice of dynamic insulation elements placed as a interface between the superstructure and infrastructure, as well as analysis of effects on joints in these situations demand. This paper proposes a physical model of a general nature, enabling customization depending on the specific constructive bridge, or the way it is solicited. Customize this model was developed for this study based on the existing viaduct Transylvania highway located between km 29 +602.75 and km 29 +801.25.

Keywords: bridge, vibration, dynamic

### 1. INTRODUCTION

Bridges and viaducts are vital structures railway and traffic networks, for which there are embedded in their structure, systems for isolation and impulsive damping actions resulting from road traffic and the seismic activities. Due to the intense and varied demands, which the dynamic isolation systems are subjected to, it degrades over time and thus changing the response to the stresses it is subjected. It is the moment when these systems must be replaced with new ones. Determining precisely when it is needed to replace these systems it is very important, because the abnormal system operations can cause unexpected movement when they are stresses by impulsive actions. Determining precisely when it is needed to replace these dynamic insulation systems, it is done by monitoring in time the kinematic and energy parameters of vibration in response to the same type of requests.

In other words, at commissioning dynamic isolation systems there will be performed a series of experimental measurements to quantify the kinematics and energy parameters of the deck vibrations of a section of bridge.

At certain periods of time, these types of experimental measurements will be repeated for a comparative analysis of the targeted parameters. In this way, highlighting the differences that arise between the parameters values at different time, data provide information about normal function of dynamic isolation systems. This essay will highlight the theoretical point of view of parameters, to be monitored regularly by experimental measurements.

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## 2. DEVELOP PHYSICAL AND MATHEMATICAL MODEL

To reflect the parameters of "control" it is considered a physical model with six degrees of freedom that can be custom built depending on the specific model which is being prepared.

This model, presented in Figure 1, consider a section of bridge deck as a rigid solid support through some tri-orthogonal viscoelastic type support, Figure 2.

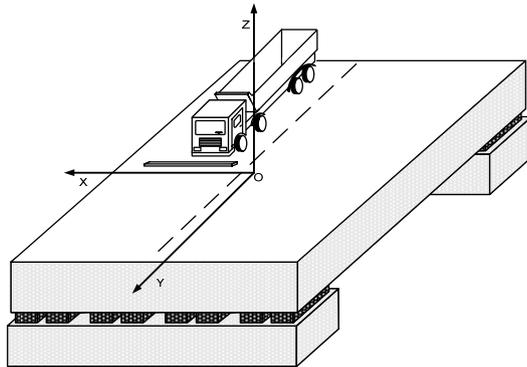


Fig. 1. A sketch section of the bridge, a passing truck over an obstacle.

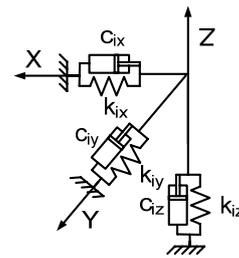


Fig. 2. Tri-orthogonal viscoelastic type support.

For this purpose a section of bridge deck can be considered a solid rigid with tri-orthogonal viscoelastic type links. The matrix equation system characterizing oscillatory motion can be written as follows [1]:

$$\underline{\underline{I}}\underline{\underline{\ddot{q}}} + \underline{\underline{C}}\underline{\underline{\dot{q}}} + \underline{\underline{K}}\underline{\underline{q}} = \underline{\underline{f}} \quad (1)$$

were:  $\underline{\underline{q}}$  is the vector of generalized coordinates;  $\underline{\underline{\dot{q}}}$  is the vector of generalized velocities;  $\underline{\underline{\ddot{q}}}$  is the vector of generalized accelerations;  $\underline{\underline{f}}$  is the vector of generalized forces;  $\underline{\underline{I}}$  is the matrix of inertia,  $\underline{\underline{C}}$  is the damping matrix;  $\underline{\underline{K}}$  is the rigidity matrix.

Rigid movements based on the generalized coordinates are defined as follows:

- X - lateral forced vibration (sliding);
- Y - forced longitudinal vibration (forwarding);
- Z - vertical forced vibration (lifting);
- $\varphi_x$  - forced vibration in pitch (pitching)
- $\varphi_y$  - forced vibration roll (rolling);
- $\varphi_z$  - forced gyration vibration.

Main elastic supports of elastic axes are parallel to the axes of bearing reference. In this case, the movements represented by the coordinates variation corresponding to the six degrees of freedom, uncouple follows:

- coupled translational motion along the X axis and rotation around the Y axis ( $X, \varphi_y$ );
- coupled translational motion along the Y axis and rotation around the X axis ( $Y, \varphi_x$ );
- translational movement along the Z axis independent of the other ways;
- rotation around Z axis ( $\varphi_z$ ) independent of the other ways.

In this case, the system of differential equations becomes:

a) Couple mode ( $X, \varphi_y$ ):

$$\begin{cases} m\ddot{X} + \dot{X} \sum_1^n c_{ix} + \dot{\phi}_y \sum_1^n z_i c_{ix} + X \sum_1^n k_{ix} + \phi_y \sum_1^n z_i k_{ix} = 0 \\ J_y \ddot{\phi}_y + \dot{X} \sum_1^n z_i c_{ix} + \dot{\phi}_y \sum_1^n (c_{iz} x_i^2 + c_{ix} z_i^2) + X \sum_1^n z_i k_{ix} + \phi_y \sum_1^n (k_z x_i^2 + k_x z_i^2) = e_x F_z \end{cases} \quad (2)$$

where  $n$  is the number of bearings,  $n = 16$ .

b) Couple mode ( $Y, \phi_x$ ):

$$\begin{cases} m\ddot{Y} + \dot{Y} \sum_1^{16} c_{iy} - \dot{\phi}_x \sum_1^{16} c_{iy} z_i + Y \sum_1^{16} k_{iy} - \phi_x \sum_1^{16} k_{iy} z_i = F_y \\ J_x \ddot{\phi}_x - \dot{Y} \sum_1^{16} z_i c_{iy} + \dot{\phi}_x \sum_1^{16} (c_{iy} z_i^2 + c_{iz} y_i^2) - Y \sum_1^{16} z_i k_{iy} + \phi_x \sum_1^{16} (k_{iy} z_i^2 + k_{iz} y_i^2) = -e_y F_z \end{cases} \quad (3)$$

c) Shift on OZ axe:

$$m\ddot{Z} + \dot{Z} \sum_1^{16} c_{iz} + Z \sum_1^{16} k_{iz} = -F_z \quad (4)$$

d) Rotation around OZ axe:

$$J_z \ddot{\phi}_z + \dot{\phi}_z \sum_1^{16} (c_{ix} y_i^2 + 2c_{iy} x_i^2) + \phi_z \sum_1^{16} (k_{ix} y_i^2 + 2k_{iy} x_i^2) = e_x \cdot F_y \quad (5)$$

were:  $m$  is the weight of the deck,  $k_{ix}$ ,  $k_{iy}$ ,  $k_{iz}$  are the dynamic stiffness isolation systems on three system directions considered reference;  $k_z = 650 \times 10^6$  N/m;  $c_{ix}$ ,  $c_{iy}$ ,  $c_{iz}$  are the damping coefficients of dynamic systems contained in the three system directions considered reference;  $c_z = 1.5 \times 10^6$  Ns/m;  $F_y$  is the force application on horizontal direction of the bridge deck;  $F_z$  is the force application on horizontal direction of the bridge deck,  $e_x$  is the distance on OX direction between the center of the mass of the bridge section and point of impact  $e_x = 2$  m,  $e_y$  is the distance on OY direction between the center of mass of the bridge section and point of impact,  $e_y = -2$  m,  $e_z$  is the distance on OZ direction between the center of mass of the bridge section and point of impact;  $e_z = -1.4$  m.

#### 4. DEFINING THE PARAMETERS OF DYNAMIC REQUESTS

To establish the system excitation is considered a truck weighing 41 tons, Figure 3, passing over an obstacle with a height of 40 mm at a speed of 20 km / h. This application corresponds to the dynamic measurements performed on the highway "Transilvania" the viaduct located between km 29 +602.75 and km 29 +801.25. The simulation characteristic dates are summarized in the table next:

On each axle passing over the considered barrier are two forces solicitation directions OZ and OY, acting on the bridge deck:  $F_{1z} = 4.6793 \times 10^5$  N;  $F_{2z} = 4.6157 \times 10^5$  N;  $F_{3z} = 8.2699 \times 10^5$  N;  $F_{4z} = 8.2699 \times 10^5$  N;  $F_{1y} = -1.4751 \times 10^5$  N;  $F_{2y} = -1.4551 \times 10^5$  N;  $F_{3y} = -2.6071 \times 10^5$  N;  $F_{4y} = -2.6071 \times 10^5$  N.

Considering the shape's excitation pulse resulted at the passing of the wheel over the obstacle of rectangular shape and impact duration of 0.03s, in Figure 4 is represented the excitation functions shape both vertically and horizontally (Figure 5) [2]. For the case considered as a section of the viaduct, have the following characteristics, [3]:  $m = 992000$  kg,  $J_x = 120.533 \times 10^6$  Kg·m<sup>2</sup>,  $J_y = 15.133 \times 10^6$  Kg·m<sup>2</sup>,  $J_z = 134.091 \times 10^6$  Kg·m<sup>2</sup>.

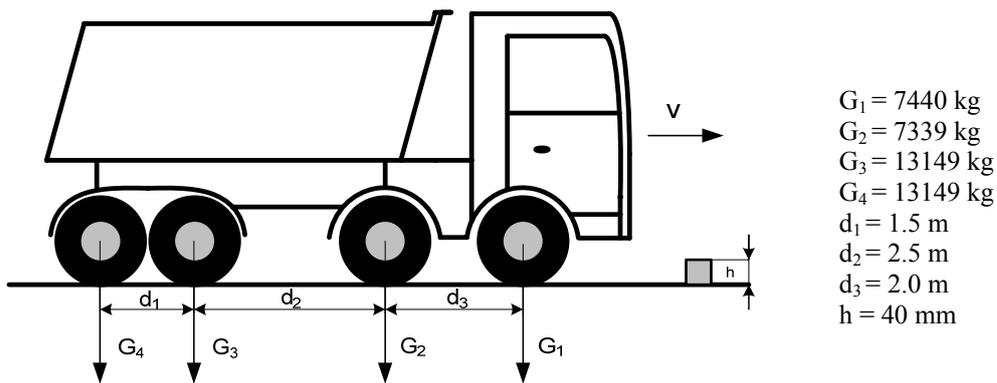


Fig. 3. The truck used for dynamic testing.

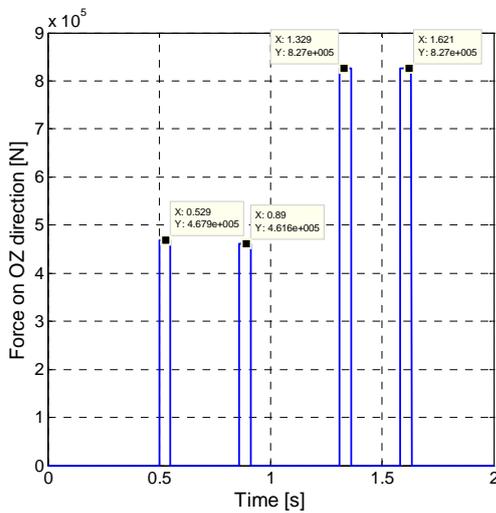


Fig. 4. The loads on the vertical direction.

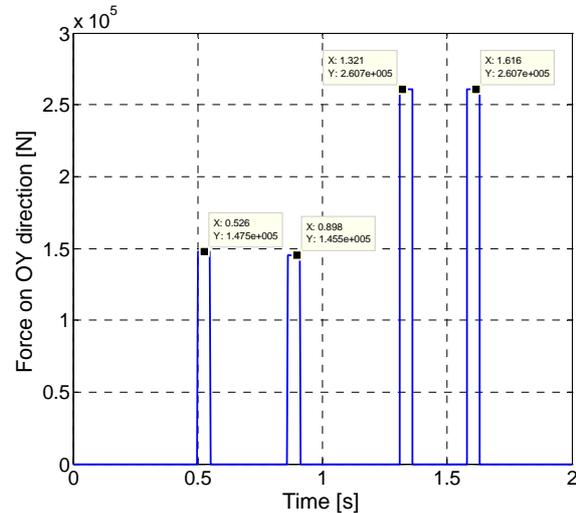


Fig. 5. The loads on the horizontal direction.

### 5. ANALYSIS OF DYNAMIC RESPONSE PARAMETERS OF BENDING DECK

The equation characterizing this motion is described by the following relation:

$$m\ddot{Z} + \dot{Z} \sum_1^{16} c_{iz} + Z \sum_1^{16} k_{iz} = -F_z \quad (6)$$

Solving these second order differential equations was performed using MATLAB software package, targeting developments following parameters:

- Moving direction OZ: representation in time and frequency, Figures 6 - 7;
- Acceleration in the direction OZ: representation in time and frequency, Figures 8 - 9;
- The energy dissipated by damping, Figure 10;
- Representation in phase plan to characterize the stability of motion, Figure 11.

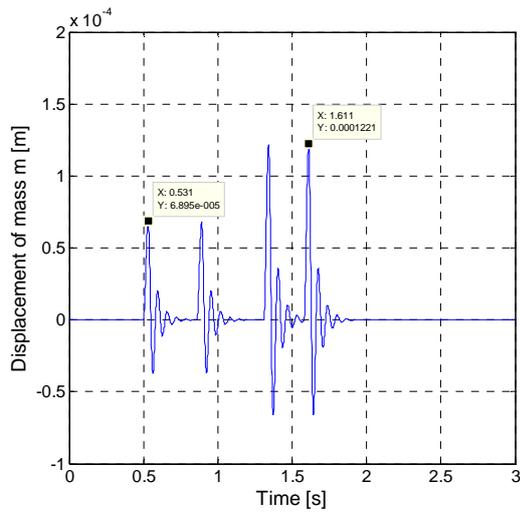


Fig. 6. Displacement on mass m.

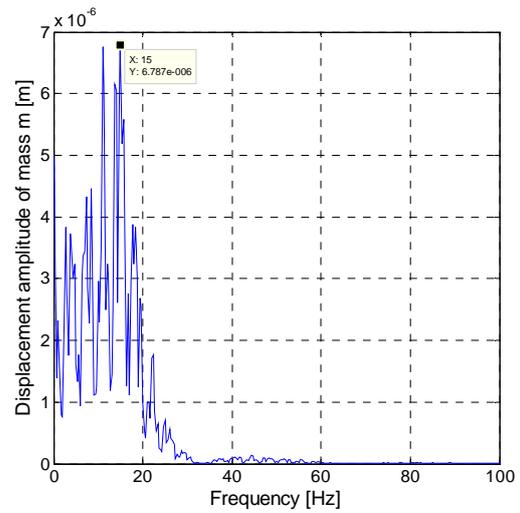


Fig. 7. Spectral representation of displacement.

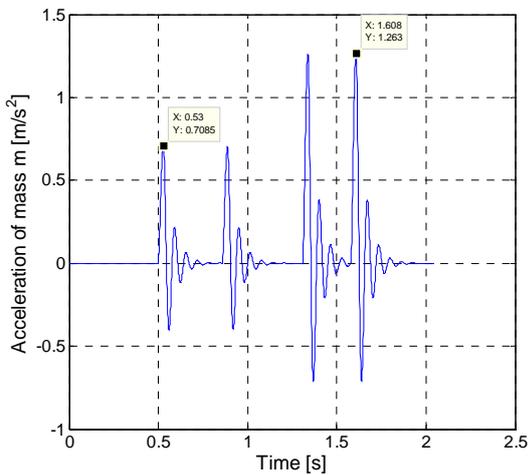


Fig. 8. Acceleration of mass m.

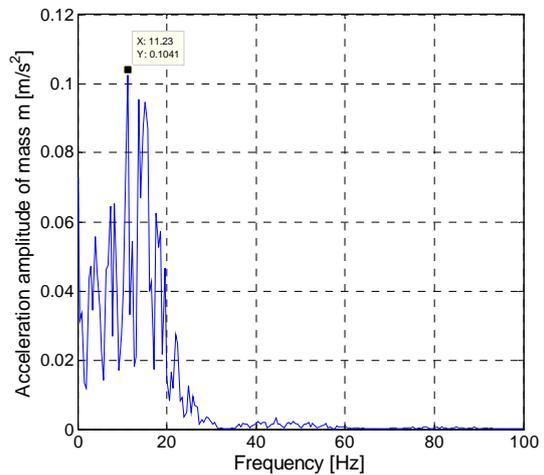


Fig. 9. Spectral representation of acceleration.

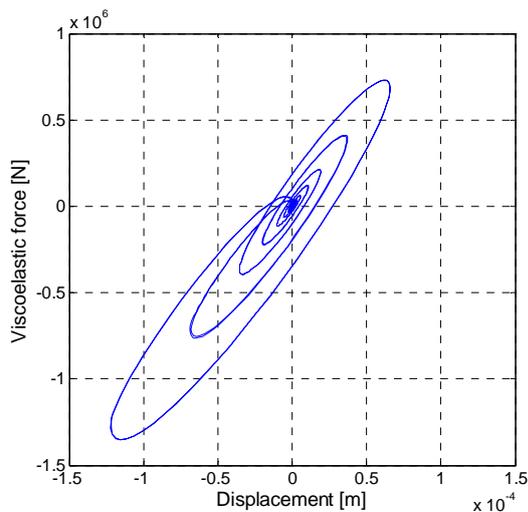


Fig. 10. Hysteresis loop,  $W=264$  J.

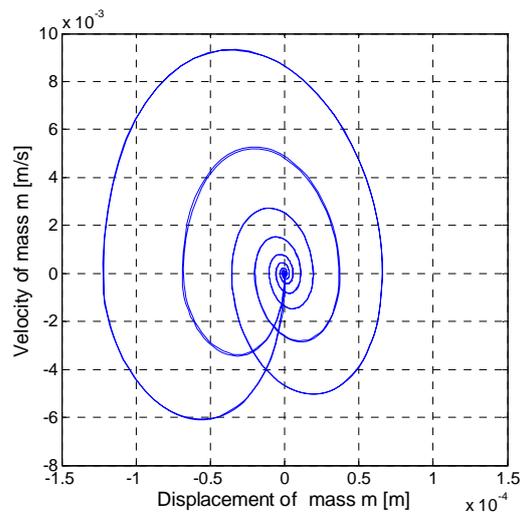


Fig. 11. Phase plane representation.

From these representations can conclude the following aspects:

- Displacement on OZ direction has the maximum value of 0.00012 m and the spectral representations indicate the dominant component in the range 11-15 Hz, Figures 6 - 7;
- Acceleration of oscillatory motion in the OZ direction has maximum value at  $1.26 \text{ m/s}^2$ , and the spectral band of the dominant components is situated in the range 11-15 Hz, Figures 8 - 9;
- The energy dissipated over the considered period is 264 J, since the movement values are reduced on the OZ direction. Figure 10;
- Representation in the phase plane shows that the movement is stable, Figure 11.

A useful representation to identify spectral participation of each excitation pulse is spectrogram of signal acceleration, presented in several forms in Figures 12 - 15. This kind of representation proves its usefulness especially in cases where the mechanical system load is made by multiple excitations. It is noted that for high frequency vibrations are responsible the excitations three and four from the train of pulses.

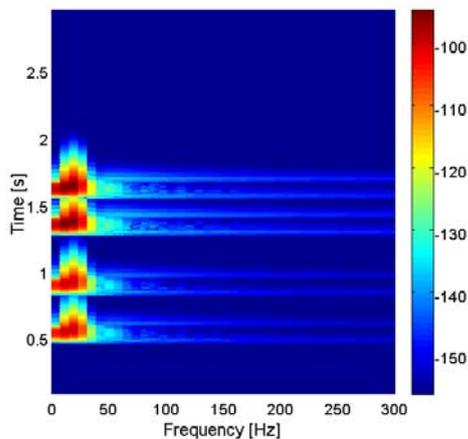


Fig. 12. 2D spectrogram of acceleration signal.

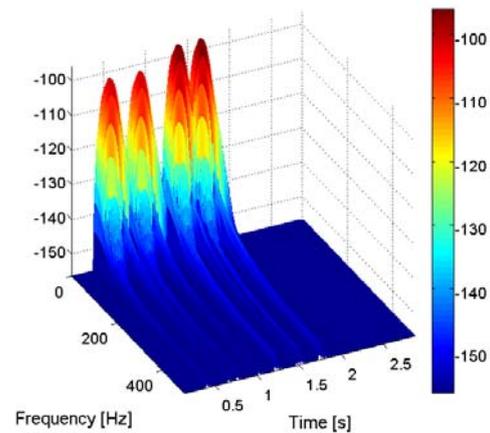


Fig. 13. 3D spectrogram of acceleration signal.

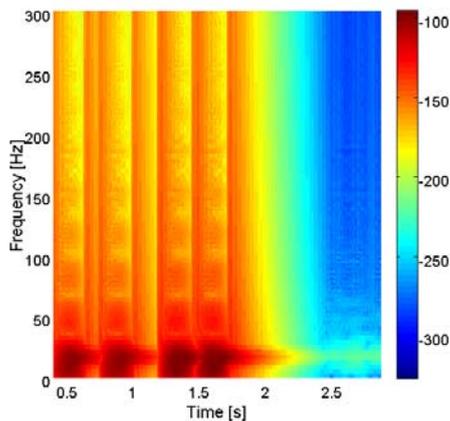


Fig. 14. 2D spectrogram of acceleration signal.

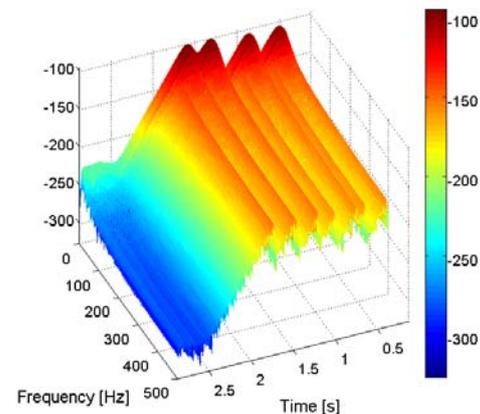


Fig. 15. 3D spectrogram of acceleration signal.

## 6. CONCLUSIONS

This study shows one way of characterizing the dynamic response of a bridge or viaduct section, if its loading by loads from road traffic. The utility of the dynamic study is demonstrated by the following aspects:

- characterization of the dynamics of these systems allows the proper choice of isolation systems against dynamic actions from road traffic;
- based on the comparative quantification of the dynamic parameters established by this study can be characterized operating conditions of devices, and therefore is establishing their level of degradation.

This study can be completed by the dynamics characterization of the considered section of bridge on the other degrees of freedom, obtaining in this way a more complete characterization of the considerate case.

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