GENERATION OF 3D SHAPES WITH SUPERELLIPSOIDS, SUPERTOROIDS, SUPER CYLINDERS AND SUPER CONES

ŢĂLU D.L. ŞTEFAN[*]

Technical University of Cluj-Napoca, Faculty of Mechanics, Department of Descriptive Geometry and Eng. Graphics, 103-105 B-dul Muncii Street, 400641, Cluj-Napoca, Romania

Abstract: The purpose of this paper is to present a CAD study for generating of 3D shapes with superellipsoids, supertoroids, super cylinders and super cones based on computational geometry. To obtain the relevant geometric information concerning the shape and profile for different 3D objects the Madsie Freestyle 1.5.3 application was used. Results from this study are applied in geometric constructions and computer aided design used in engineering and sculpture design.

Keywords: engineering design, sculpture design, superellipsoid, supertoroid, super cylinder, super cone, implicit surface, CAD

1. INTRODUCTION

The computer aided design and mathematical definition of 3D shapes is important for graphic, art and product designers, engineers, interior designers, and architects [1 - 3]. Mathematical visualization tools can act as interpretation support tools. Shape description is a crucial step in many computer graphics applications where usage of a mathematical description provides high geometric representativity [4 - 7].

Interactive technological tools for geometric modeling and graphical rendering have tremendous potential to address a broad range of objectives to explore and analyse possible new artistic shapes in virtual form [8, 9].

2. SUPERELLIPSOID, SUPERTOROID, SUPER CYLINDER AND SUPER CONE

2.1. Superquadrics

Superquadrics, as an extension of the basic quadric surfaces, constitute a class of surfaces which possess a natural parametric and implicit description that were introduced by Barr A. H. in 1981 [10]. The superquadrics are: the superellipsoid, the superhyperboloid of one and two sheets, and the supertoroid. A superquadric surface is defined as a spherical product of two parametric 2D curves, resulting in a parametric shape in 3D space.

In last decade, superquadrics have been extensively applied for their simple and flexible shape description and efficient computer graphical representation [11 - 14].

A superellipsoid is the spherical product of a pair of two superellipses being defined by the implicit equation [12]:

[*] Corresponding author, email: stefan_ta@yahoo.com
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$$\left(\left(\frac{x}{a_1}\right)^{2/\varepsilon_2}+\left(\frac{y}{a_2}\right)^{2/\varepsilon_2}\right)^{\varepsilon_2/\varepsilon_1}+\left(\frac{z}{a_3}\right)^{2/\varepsilon_1}=1. \qquad (1)$$

A supertoroid is the spherical product of a superellipse and another superellipse with $center_x > a_g$ that is defined by the implicit equation [12]:

$$\left(\left(\left(\frac{x}{a_1}\right)^{2/\varepsilon_2}+\left(\frac{y}{a_2}\right)^{2/\varepsilon_2}\right)^{\varepsilon_2/\varepsilon_1}-a_4\right)^{2/\varepsilon_1}+\left(\frac{z}{a_3}\right)^{2/\varepsilon_1}-1=0. \qquad (2)$$

The two exponents are squareness parameters; they are used to pinch, round, and square off portions of the solid shapes, to soften the sharpness of square, and to produce edges and fillets of any arbitrary degree of roundness [10].

This form provides an information on the position of a 3D point related to the superquadric surface, that is important for interior/exterior determination [10, 13].

We have an inside-outside function F (x, y, z):

- $F(x, y, z) = 1$ when the point lies on the superquadric surface;
- $F(x, y, z) < 1$ when the point is inside the superquadric surface;
- $F(x, y, z) > 1$ when the point is outside the superquadric surface.

The existence of the inside-outside functions means that superquadrics can be manipulated by means of solid boolean operations, such as union, intersection, and subtraction [10].

Superquadrics are an easy class of objects to use because they have well defined normal and tangent vectors. Normal vectors are used in intensity calculations during rendering. Both the normal and tangent vectors are used to calculate the curvature of the surface [14].

Superquadrics can be made to fit an even larger range of volumes by adding parameters to describe global deformations such as tapering, bending, twisting and boolean constraints. The main disadvantage of superquadrics is that they are intrinsically symmetric and thus the representation of many objects is difficult (small domain). The addition of the deformation and the tapering constraints also introduces problems with the efficiency and stability of the representation [15].

The superquadric models can't expand easily on complex shapes as easily as on smooth regular objects which results generally in over-segmentation. A problem with the use of superquadrics as a modeling primitive is that they are often not well defined by a single view.

2.2. Super cylinder and super cone

Super cylinder and super cone are defined according the known geometrical considerations and mathematical formula from [16].

3. GRAPHICAL REPRESENTATIONS

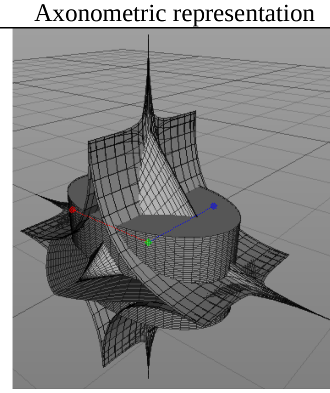
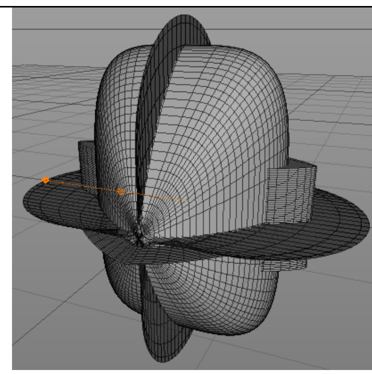
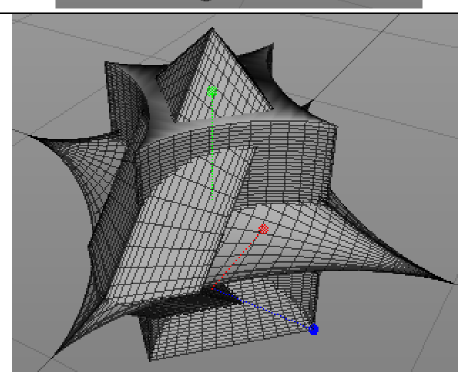
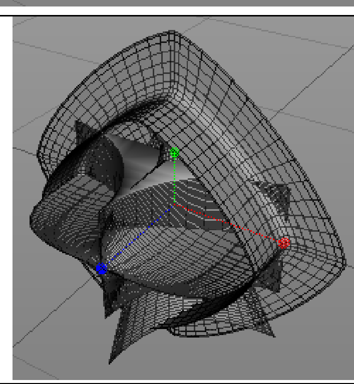
The Madsie Freestyle 1.5.3 application was used to generate the 3D objects with superellipsoids, supertoroids, super cylinders, super cones [16] and the graphical representations are given in Table 1.

The parameters for superellipsoid are:

- Radius X (r_1) - the radius of the ellipsoid along the X-axis;
- Radius Y (r_2) - the radius of the ellipsoid along the Y-axis;
- Radius Z (r_3) - the radius of the ellipsoid along the Z-axis;
- Stacks (n_1) - the number of segments along the Z-axis;

- Slices (n_2) - the number of radial segments around the ellipsoid;
- Stack Exponent (e_1) - the shape of the ellipsoid;
- Slice Exponent (e_2) - the shape of the ellipsoid.

Table 1. Graphical representations of 3D complex objects.

No.	Values of parameters	Axonometric representation
1	Superellipsoid: $r_1 = 2$, $r_2 = 2.5$, $r_3 = 2$, $n_1 = n_2 = 64$, $e_1 = 2$, $e_2 = 2.5$ Supertoroid: $r_4 = 2.5$, $r_5 = 1.5$, $n_3 = n_4 = 64$, $e_3 = 4$, $e_4 = 6.5$ Super cylinder: $r_6 = 2$, $r_7 = 1.3$, $h_1 = 1$, $n_5 = 64$, $n_6 = 64$, $e_5 = 1$ Capped - enabled. Super cone: $r_8 = 2.5$, $r_9 = 2.5$, $h_2 = 0$, $n_7 = 64$, $n_8 = 64$, $e_6 = 1.5$ Capped - enabled.	
2	Superellipsoid: $r_1 = 2$, $r_2 = 3$, $r_3 = 2$, $n_1 = n_2 = 64$, $e_1 = 0.5$, $e_2 = 1.5$ Supertoroid: $r_4 = 1.5$, $r_5 = 2$, $n_3 = n_4 = 64$, $e_3 = 8$, $e_4 = 1$ Super cylinder: $r_6 = 2.5$, $r_7 = 1$, $h_1 = 1$, $n_5 = 64$, $n_6 = 64$, $e_5 = 1.5$ Capped - enabled. Super cone: $r_8 = 2.5$, $r_9 = 2.5$, $h_2 = 0.2$, $n_7 = 64$, $n_8 = 64$, $e_6 = 1.45$ Capped - enabled.	
3	Superellipsoid: $r_1 = 2$, $r_2 = 1.5$, $r_3 = 2$, $n_1 = n_2 = 64$, $e_1 = 3.5$, $e_2 = 3$ Supertoroid: $r_4 = 1$, $r_5 = 0.5$, $n_3 = n_4 = 64$, $e_3 = 2$, $e_4 = 2$ Super cylinder: $r_6 = 1$, $r_7 = 1$, $h_1 = 1$, $n_5 = 64$, $n_6 = 64$, $e_5 = 3$ Capped - enabled. Super cone: $r_8 = 1$, $r_9 = 1$, $h_2 = 1$, $n_7 = 64$, $n_8 = 64$, $e_6 = 2$ Capped - enabled.	
4	Superellipsoid: $r_1 = 1$, $r_2 = 0.5$, $r_3 = 1$, $n_1 = n_2 = 64$, $e_1 = 4.5$, $e_2 = 0.5$ Supertoroid: $r_4 = 1$, $r_5 = 0.5$, $n_3 = n_4 = 64$, $e_3 = 1.5$, $e_4 = 8$ Super cylinder: $r_6 = 1$, $r_7 = 1$, $h_1 = 0.5$, $n_5 = 64$, $n_6 = 64$, $e_5 = 3.5$ Capped - enabled. Super cone: $r_8 = 1$, $r_9 = 1$, $h_2 = 1$, $n_7 = 64$, $n_8 = 64$, $e_6 = 4$ Capped - enabled.	

The parameters for supertoroid are:

- Major Radius (r_4) - the radius of the circle that the tube tracks;
- Minor Radius (r_5) - the radius of the tube;
- Major Segments (n_3) - the number of segments along the tube;
- Minor Segments (n_4) - the number of radial segments around the tube;
- Major Exponent (e_3) - the shape of the circle that the tube tracks;
- Minor Exponent (e_4) - the shape of the tube cross section.

The parameters for super cylinder are:

- Radius X (r_6) - the radius of the cylinder along the X-axis;
- Radius Z (r_7) - the radius of the cylinder along the Z-axis;
- Height (h_1) - the height of the cylinder;
- Stacks (n_5) - the number of segments along the height;
- Slices (n_6) - the number of radial segments around the cylinder;
- Exponent (e_5) - the shape of the cylinder;
- Capped - if enabled will put a polygon at the top and bottom of the cylinder.

The parameters for super cone are:

- Radius X (r_8) - the radius of the cone along the X-axis;
- Radius Z (r_9) - the radius of the cone along the Z-axis;
- Height (h_2) - the height of the cone;
- Stacks (n_7) - the number of segments along the height;
- Slices (n_8) - the number of radial segments around the cone;
- Exponent (e_6) - the shape of the cone;
- Capped - if enabled will put a polygon at the bottom of the cone.

Because there are some numerical issues in computation with both very small or very large values of the exponents, in this study, for safety, they are chosen in the range of 0.01 to about 8.

4. CONCLUSIONS

This paper explores a computational method for generation of complex 3D shapes with superellipsoids, supertoroids, super cylinders and super cones. Several examples are provided to demonstrate the performance of the developed models. These shapes can be used to model various shapes as well as shapes inbetween.

The Madsie Freestyle 1.5.3 application helps in obtaining conclusions referring to shape of complex 3D objects, but also facilitate the creation of new analogies in shape design. New generated shapes allow a new perspective to sculptural research, architecture and design.

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