

DAMAGE AND FRACTURE MECHANICS CONCEPTS IN CALCULATING CYCLICALLY LOADED PRESSURE EQUIPMENT

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Abstract: In the history of technology there occurred many accidents caused by the presence and propagation of cracks in a mechanical structure, especially cracks in weld seams or already existing in the material. The paper evaluates the damage caused by cracks, the superposition of effects by using fracture mechanics concepts (the stress intensity factor, the contour integral and the crack tip opening displacement), while in the case of superposition fracture modes I and II, the relation was checked against the experimental literature data.

Keywords: damage, crack propagation rate, effects superposition, critical stresses, fracture, principle of critical energy.

1. INTRODUCTION

The pressure equipment is used mainly in the chemical and petrochemical industry as well as in classical and nuclear power engineering. The appropriate design of such equipment operating under given conditions as well as the analysis of the equipment after a certain period of operation should be made on the basis of adequate computing relations so as to avoid the kind of damage.

The life assessment after a certain period of operation allowing for fully safe and continued equipment operation requires the calculation relations of the materials strength, by taking into account the previous material damage. In order to address these problems, the paper examines first the calculation of pressure equipment according to current regulations considered most popular by professionals (British Standard 2009 ASME Code, Section VIII - Division 3 and standard European EN 13445-3). Further below we have analyzed the experimental and theoretical research results to be used in solving the specific issues raised.

With mechanical structures, especially in the case of welds, there may occur brittle fractures that extend and are transmitted to the welded parts [1], causing the kind of damage that leads to the complete destruction of the structure. It is known that the weld may feature flaws, including cracks that produce high levels of residual stress.

When a part breaks down, the part is split into two or several pieces under the action of the applied loads [2]. This may occur slowly or quickly and the separation may be: brittle (with high speed crack propagation) and ductile (with slow crack propagation). The emergence and, in particular, the crack growth result in lower material strength until the load capacity loss occurs, followed in most cases by fracture.

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Papers [1-6] overview the basic concepts of fracture mechanics, the importance and role of structure cracks, the calculation methods for determining stress intensity, the conditions underpinning the propagation and inhibition of crack growth and the application of the critical energy principle to fracture mechanics.

In the European and U.S. standards [7-10] there have been set rules and methodologies for the calculation of pressure vessels using the concepts of fracture mechanics. The crack propagation laws that occur during the operation of cyclically loaded mechanical structures are numerous and they have been summarized in [11]. There have been published several papers [12-23] on the study of the crack propagation phenomenon in materials used in the construction of various mechanical structures, as well as computational models used to calculate their damage.

In the paper [24], one can find a standardized method for calculating the growth rate of fatigue crack, while [25-63] analyze the influences of operating parameters, some mechanical properties of materials as well as the structural geometric parameters on the crack growth rate. Relations for fatigue calculation, both for linear behavior materials and especially for power dependent materials with nonlinear behavior have been deduced from [64-69], with consideration of the influence of both average and residual stresses [70]. The calculation of the pressure equipment damage and lifetime are analyzed in [71-73].

The paper examines: the calculation of pressure equipment by using fracture mechanics concepts present in some standards, the status of specific fracture mechanics calculations in the literature, some experimental determinations of the crack growth rate, experimental results on the superposition of effects, as well as the experimental data analysis regarding the correlation between the type of loading and the concept of damage in order to highlight possible design options involving the use of the concept of damage.

2. CURRENT STATUS OF PRESSURE EQUIPMENT CALCULATION IN DESIGN STANDARDS BASED ON THE USE OF FRACTURE MECHANICS CONCEPTS

Further down there has been presented the evaluation of materials used in the calculation of pressure vessels based on fracture mechanics concepts in British Standard [7], ASME Code, 2009, Section VIII - Division 3 [8] and the European standard EN 13445-3 [10].

a. The British Standard [7] provides a design methodology for pressure vessels operating at temperatures below 0°C, so as to avoid brittle fracture. For this purpose there shall be determined:

- The design reference temperature θ_R , which must be less than the minimum design temperature.

$$\theta_R \leq \theta_D + \theta_S + \theta_C + \theta_H \quad (1)$$

where, θ_D is the minimum design temperature; θ_S is the adjustment temperature depending on the membrane stress calculated; θ_C is the temperature adjustment depending on the construction category (may vary between 0°C or - 10°C); θ_H is the adjustment temperature taking into account the heat treatment after welding;

- The design reference thickness, e , depending on the layout of the pieces to be welded and the heat treatment after welding;

- The impact test temperature, depending on the design reference temperature and the design reference thickness in the existing standard graphics;

- The impact energy.

The minimum allowable impact energy value in the impact test depends on the minimum stress and the size of the specimen used.

b. In the Normative 2009 ASME Code, Section VIII - Division 3 [8], the calculation of the pressure equipment by using fracture mechanics is based on the assumption that the crack initiation is complete and cracks are present in the most stressed areas of the vessel. To this end:

- One determines the final allowable crack depth as a basis for calculating the number of design loading cycles, N_p .

For single block design vessels the final allowable crack depth is considered either maximum 25% of the section thickness under consideration, or 25 % of the critical crack depth. For vessels with two or more wall layers, the final allowable crack depth in the inner layer must be equal to its thickness. For all other layers, the final allowable crack depth should not exceed 25 % of the thickness of that layer, except for the outer layer wherein the final allowable crack depth should not exceed 25 % of the theoretical critical depth:

- One calculates the stress intensity factor, K_I and the crack growth rate in thickness, with the Paris - Erdogan relation [8] for the deepest point on the crack periphery:

$$\frac{da}{dN} = C [f(R_K)] (\Delta K)^m \text{ [m/cycle]} \quad (2)$$

where:

$$\Delta K = K_{I,\max}^* - K_{I,\min}^* \quad (3)$$

is the variation of the stress intensity factor, where $K_{I,\max}^*$ and $K_{I,\min}^*$ are the stress intensity, maximum and minimum, respectively, all trials being considered except for the residual stress, while R_K is calculated with the formula:

$$R_K = \frac{K_{I,\min}^* + K_{I,res}}{K_{I,\max}^* + K_{I,res}} \quad (4)$$

where $K_{I,res}$, is the intensity factor corresponding to the residual stress.

The surface crack growth is calculated with the formula:

$$\frac{dl}{dN} = 2C [f(R_K)] (\Delta K)^m \quad (5)$$

Here ΔK is calculated based on the crack type, shape and its position (on the corner inner or outer surface around a hole, etc.) and the crack plane (circumferential or meridional).

If, $K_{I,\max}^* + K_{I,res} \leq 0$ then da/dN can be considered equal to 0.

The values of constants C and m are taken according to the vessel wall material.

- One calculates the number of stress cycles, N_p , which is a function of the loading order (sequence). The number of cycles can be calculated by numerically integrating the crack growth rate given by equation (2), by considering the value of K_I as constant over a span, Δa , in the crack growth, which is small compared with the crack depth. The calculation shall be repeated by using increasingly smaller intervals until we get an insignificant change in N_p ;

- One determines the maximum design pressure at room temperature.

c. The standard EN 13445-3 [10] makes use of the fracture mechanics method in order to determine the conditions for avoiding brittle fracture in C and CMn steels and low alloy steels featuring lower minimum stress of 460N/ mm². The impact test temperature is determined according to the stress value characteristic of the parent material, the impact energy KV and the reference thickness from the graphs that show the dependence of the design reference temperature, the impact test temperature and the reference thickness;

d. From the analysis of the current calculation methods taken from the design standards for pressure equipment, one has found that there is no calculation method allowing the superposition of the effects of loadings upon a mechanical structure.

For example, with the ASME Normative, calculation is quite difficult because for each type of crack there exists another factor to be determined, and the effect of the residual stress is evaluated separately. At the same time, the number of stress cycles, N_p , cannot be obtained from a single relation, but is repeated for increasingly smaller crack depth intervals, down to an insignificant change of N_p .

With the European standard EN 13445, the fracture mechanics method is confined to a limited domain of application, only for C and CMn steels and low alloy steels featuring a minimum stress lower than 460N/mm^2 , while for wall thicknesses below 10 mm there are no specific relations, thus one applies the relation used for the reference thickness of 10 mm.

The British Standard provides a calculation methodology applicable only to vessels operating at a temperature below 0°C , while for determining the reference thickness there is no unique relation; the latter is determined by considering the layout of the parts to be welded and the heat treatment after welding.

3. THE STATE OF THE ART IN THE USE OF CALCULATIONS BASED ON FRACTURE MECHANICS CONCEPTS IN LITERATURE

3.1. Crack growth rate

With variable cyclic loadings one uses the concept of crack propagation rate, da/dN , where a is the characteristic crack length and N is the number of loading cycles.

The doublelogarithm diagram (Figure 1) shows the dependence of the crack propagation rate logarithm, da/dN and the stress intensity variation factor logarithm, $\Delta K_I = K_{I,\max} - K_{I,\min}$, given by,

$$\Delta K_I = Y\Delta\sigma\sqrt{\pi \cdot a} \quad (6)$$

where $\Delta\sigma = \sigma_{\max} - \sigma_{\min}$ is the normal stress variation, Y is a factor that depends on the crack position and shape.

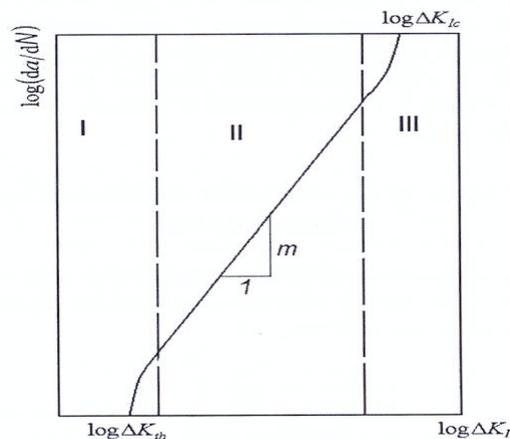


Fig. 1. Dependence of crack propagation rate on stress intensity factor variation.

In the diagram there are three domains: I - crack retarding domain, II - the domain of constant propagation rate, III - the domain of high propagation rate; ΔK_{III} is the threshold value of ΔK_I , while ΔK_{Ic} is the critical value of the stress intensity factor variation.

An overview of specialist literature yields over 120 relations [11] for calculating the crack propagation rate depending on the variation of the stress intensity factor ($\Delta K = \Delta K_I$), the maximum stress applied (σ_{\max}), fracture stress (σ_f), the yield strength (σ_y) and the cycle asymmetry coefficient (R) (Table 1).

Table 1. The main propagation laws of fatigue cracks [5, 11].

Propagation laws according to stress intensity factor variation $\Delta K = \Delta K_I$	Propagation laws based on the maximum stress applied $\sigma_{max} = \sigma$ and crack size	Propagation laws based on fracture toughness, $K_C = K_{I,C}$ and the stress intensity factor variation $\Delta K = \Delta K_I$
$\frac{da}{dN} = C(\Delta K)^m$ (7)	$\frac{da}{dN} = C\sigma^m \cdot a^p$ (8)	$\frac{da}{dN} = \frac{C \cdot (\Delta K)^m}{(1-R)K_C - \Delta K}$ (9)
$\frac{da}{dN} = C(\Delta K - \Delta K_{th})^m$ (10)	$\frac{da}{dN} = C\sigma^3 a$ (11)	$\frac{da}{dN} = 10^{-4} \cdot \left(\frac{\Delta K}{K_C}\right)^m$ (12)
$\frac{da}{dN} = C \cdot (1 + \beta)^n \cdot (\Delta K)^m$ (13)	$\frac{da}{dN} = C\sigma\sqrt{a}$ (14)	$\frac{da}{dN} = \frac{C \cdot (1 + \beta)(\Delta K)^3}{K_C - (1 + \beta)\Delta K}$ (15)
$\frac{da}{dN} = \left(\frac{A}{B-R}\right) \cdot (\Delta K)^m$ (16)	$\frac{da}{dN} = a \cdot \left(\frac{\sigma}{0,1E}\right)^2 - \frac{\rho_F}{2}$ (17)	$\frac{da}{dN} = \frac{C \cdot (\Delta K)^m}{\sqrt{((1-R)K_C - \Delta K)^2}}$ (18)
$\frac{da}{dN} = C(R) \cdot (\Delta K)^m \cdot a^p$ (19)	$\frac{da}{dN} = C \cdot \sigma^m$ (20)	$\frac{da}{dN} = C \cdot \frac{(\Delta K)^m}{K_C}$ (21)
$\frac{da}{dN} = C \cdot (\Delta K)^{\left(\frac{1}{mKT}\right)\left(\frac{2n}{n+1}\right)}$ (22)	$\frac{da}{dN} = C\sigma a$ (23)	$\frac{da}{dN} = \frac{C \cdot (\Delta K)^{N_1}}{(1-R)K_C - \Delta K} \cdot N_2$ (24)
$\frac{da}{dN} = C \cdot (\Delta K)^3$ (25)	$\frac{da}{dN} = C\sigma^2 a$ (26)	$\frac{da}{dN} = \frac{C(\Delta K)^m}{E\sigma_e K_{I,C}}$ (27)
$\frac{da}{dN} = \left(\frac{\Delta K}{E}\right)^2$ (28)	$\frac{da}{dN} = C\sigma^m a$ (29)	$\frac{da}{dN} = \frac{C(\Delta K - \Delta K_{th})^m \cdot K_{max}}{K_C - K_{max}}$ (30)
$\frac{da}{dN} = \frac{C}{E} \cdot (\Delta K^2 - \Delta K_{th}^2)$ (31)	$\frac{da}{dN} = \frac{C\sigma^3 a^{3/2}}{(\sigma_e - \Delta\sigma)^2 \cdot a_0^{1/2}}$ (32)	$\frac{da}{dN} = C \left(\frac{\Delta K - \Delta K_{th}}{K_C - K_{max}}\right)^m$ (33)
$\frac{da}{dN} = C(\Delta K - \Delta K_{th})^2$ (34)	$\frac{da}{dN} = C \cdot \sigma\sqrt{a} \cdot (1-R)^m$ (35)	$\frac{da}{dN} = \frac{C(\Delta K)^m}{(1-R)^r}$ (36)
$\frac{da}{dN} = C \cdot \left(\frac{\Delta K}{E}\right)^{3,5}$ (37)		

One should mention the following relations (recommendable for practical use) [16]:

- The Paris-Erdogan law (7), the most commonly used;
- Donahue’s Law (10), recommended for domain II;
- Forman’s law (9), originally proposed for domain III, further extended to domain II;
- Walker’s Law (36), for domain II but applicable to limited areas from domains I and III.

Some propagation laws were obtained from theoretical models while others were deduced experimentally.

3.2. Experimental determination of the crack growth rate for some steels

PKSingh [23] conducted tests on seamless pipes, SA333 Gr.6 carbon steel with the chemical composition given in Table 2 and the room temperature tensile properties listed in Table 3. We used 4.5 mm diameter specimens according to ASTM E 606 [24] obtained by cutting the same material.

Table 2. Chemical composition [%] of pipe material [23].

<i>C</i>	<i>Mn</i>	<i>Si</i>	<i>P</i>	<i>S</i>	<i>Al</i>	<i>Cr</i>	<i>Ni</i>	<i>V</i>	<i>N</i>
0.14	0.9	0.25	0.016	0.018	< 0.1	0.08	0.05	< 0.01	0.01

Table 3. Tensile properties of pipe material at room temperature [23].

Yield strength σ_y [MPa]	Tensile fracture stress σ_u [MPa]	Elongation of fracture (%)	Reduction in area when breaking (%)
302	450	36.7	72.96

Stress-strain curve used by Singh [23], is given by the relation:

$$\frac{\Delta\varepsilon}{2} = \frac{100}{E} \times \frac{\Delta\sigma}{2} \left(\frac{\Delta\sigma}{2k} \right)^{1/n} \quad (38)$$

where $k = 354.27$ MPa and $n = 0.1523$. The constants were obtained by adjusting the points resulting from the tests.

For a low number of fatigue loading cycles we used the Basquin-Coffin - Manson relation,

$$\frac{\Delta\varepsilon}{2} = \frac{\sigma_f}{E \cdot (2N_i)^b} + \varepsilon_f (2N_i)^c \quad (39)$$

where $\sigma_f = 586.06$ MPa; $b = -0.0757$; $\varepsilon_f = 24.06\%$; $c = -0.4814$; $E = 203$ GPa.

In relations (38) and (39) $\Delta\varepsilon$ is given in%, $\Delta\sigma$ in MPa, N_i is the number of load cycles resulting in specimen failure, E is the modulus of elasticity in MPa.

The crack growth rate curve was obtained by using three specimens from the same material as the pipe, for the asymmetry coefficient $R = 0.1$, $R = 0.3$, $R = 0.5$. The material constants were evaluated by adapting the experimental points to the law of Paris (7), where the constant values are given in Table 4 while da/dN is in m/cycle and ΔK in MPa \sqrt{m} .

Table 4. The values of constants C and m for various values of R .

<i>R</i>	<i>C</i>	<i>m</i>
0.1	3.807×10^{-12}	3.03445
0.3	4.061×10^{-12}	3.11734
0.5	4.079×10^{-12}	2.90897

As well known, the asymmetry coefficient has a negligible effect on the crack growth rate curve according to Paris's law while constants C and m do not cause significant variations in the asymmetry coefficient. The average values of C and m , in this case, are 3.982×10^{-12} and 3.0, respectively.

Experiments on pipe specimens with an outer diameter of 219 mm and 15.1 mm in thickness [23] with notches made on the outer cylindrical surface (Figure 2), the specimens undergoing pure bending, resulted in the following:

- The crack increased when the defect was over 0.1 mm in size;
- The crack initiation was observed across the wall thickness, from the maximum depth of the notch;

- The number of load cycles required for the crack to grow from 0.1mm depends on the maximum applied stress,
- The asymmetry factor and the initial notch size;
- The analysis of the crack growth rate was based on Paris's law (7).

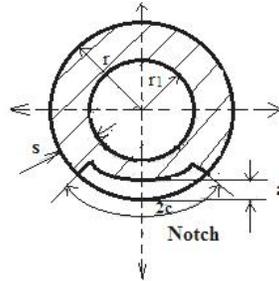


Fig. 2. Cylindrical specimen with rectangular notch on the outer surface [23].

The dependence of the crack growth rate, da/dN , and the characteristic crack size, a , across the pipe wall thickness is shown in Figure 3 for several values of the asymmetry coefficient, R .

According to the logarithm relation (7), we obtain:

$$y = b + m \cdot x \quad (40)$$

where we wrote:

$$y = \log \frac{da}{dN}; \quad x = \log \Delta K; \quad b = \log C$$

For different values of the asymmetry coefficient and cylindrical specimen wall thickness there were obtained the values of constants C and m [23], listed in Table 5.

Table 5. Constants and equations of crack growth rate curves (Figure 3) according to wall thickness, s and asymmetry coefficient, R .

R	s mm	C	m	Equation of curve
0.1	15.58	3.425×10^{-9}	3.11095	$y = -8.534 + 3.11095 \cdot x$
0.3	15.38	5.286×10^{-10}	3.31972	$y = -7.687 + 3.31972 \cdot x$
0.1	15.38	4.061×10^{-9}	3.11734	$y = -7.137 + 3.11734 \cdot x$
0.5	15.12	3.325×10^{-9}	3.23678	$y = -8.821 + 3.23678 \cdot x$
0.5	15.17	2.795×10^{-8}	2.50078	$y = -8.362 + 2.50078 \cdot x$
0.1	15.13	1.71×10^{-8}	2.80217	$y = -9.7 + 2.80217 \cdot x$

The six curves in Figure 3 were obtained after loading six specimens (tubes) with different pipe wall thickness, under different conditions (maximum load levels, number of load cycles) at several values of the asymmetry coefficient, R , thus also obtaining different values for the same value of R for constants C and m .

Work [25] based on recent studies [26, 27], whose object was the study of thick-walled containers tracks down the dependence of the crack growth rate, da/dN (microinch/cycle; 1 inch = 25.4 mm) depending on the stress intensity factor variation, ΔK for A533Grade B welding steel and the thermal influence zone factor considered. In these studies, both temperature variations and specimen size were considered. The following relations have been used:

- The stress intensity factor for a surface semielliptical defect, normally oriented to the circumferential stress:

$$K_I^2 = \frac{21a\pi\sigma^2}{Q} \tag{41}$$

where Q is a parameter that depends on the shape of the defect.

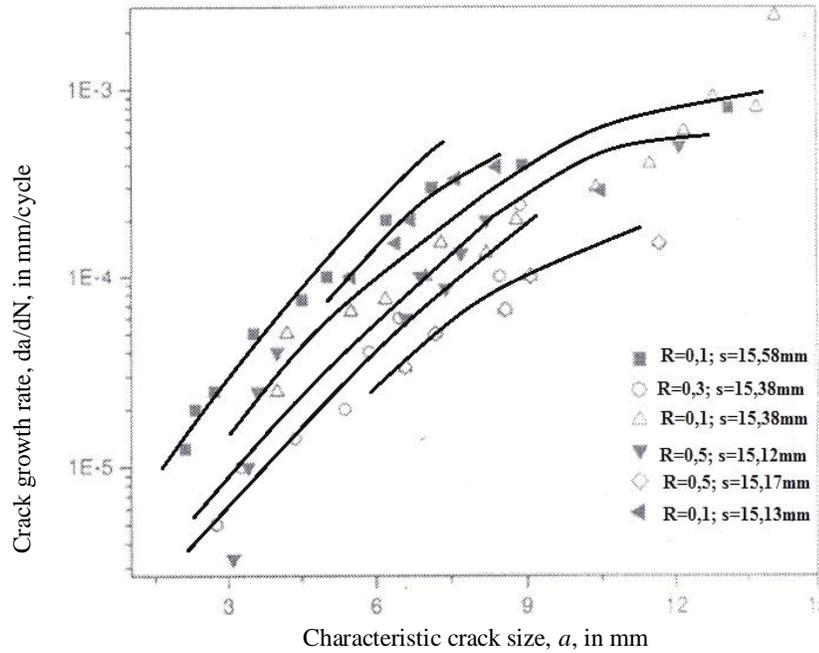


Fig. 3. Crack growth rate curves (s-cylindrical specimen wall thickness).

The critical size of the fracture crack with the formula:

$$a_{cr} = \frac{K_{Ic}^2 Q}{21\pi\sigma^2} \tag{42}$$

Number of load cycles down to failure, N, where $K_I = \sigma \sqrt{MPa}$ [25, 28-30]:

$$N = \frac{2}{(m-2)C_0 M^{m/2} \Delta\sigma^m} \cdot \left(\frac{1}{a_i^{(m-2)/2}} - \frac{1}{a_{cr}^{(m-2)/2}} \right), \text{ for } m \neq 2; \tag{43}$$

$$N = \frac{1}{C_0 M \Delta\sigma^2} \ln \frac{a_{cr}}{a_i}, \text{ for } m = 2, \tag{44}$$

where, n is the dependence slope of log (da/dN) on log (ΔK) (Figure 1), a_{cr} is the critical crack size, a_i is the initial crack size, C_0 is an empirical constant, $\Delta\sigma$ is the cyclic load variation, M is a parameter dependent on the crack geometry.

In studies about the influence of the initial crack size, a_0 , upon the number of load cycles down to failure, N, in the hypothetical case of a pressure vessel, it was found that with the depth of the defect over 12.7 mm and a length of over 127 mm, the lifetime corresponds to $N = 40\ 000$ cycles for $\Delta\sigma = 275.8$ MPa and $N = 130\ 000$ cycles for $\Delta\sigma = 184.096$ MPa [25].

In [31] there was experimentally determined the crack growth rate for a heat-resistant steel 13CrMo4-5, widely used in the manufacture of pressure vessels. The metal plate specimens used were subjected to mechanical variables of different frequencies, featuring cycles with the asymmetry coefficient $R \in [-1; 1]$, and maximum loading force $F_{\max} \leq 50$ kN [31].

In order to determine the stress intensity factor variation, ΔK , one used the relation recommended by standard ASTM E 647-05 [31]:

$$\Delta K = \Delta F g_e(a, W) = \frac{\Delta F}{B\sqrt{W}} \cdot \frac{2 + \alpha}{(1 - \alpha)^{3/2}} \cdot (0.866 + 4.64\alpha - 13.32\alpha^2 + 14.72\alpha^3 - 5.60\alpha^4) \quad (45)$$

where a - crack length, W, B - characteristic specimen dimensions, $\alpha = a / W$.

There were applied variable loads with sinusoidal cycles with frequencies $\nu = 5$ Hz and 20 Hz. The asymmetry coefficient featured: $R = 0.1$, $R = 0.3$ and $R = 0.5$.

The results, interpreted according to the Paris-Erdogan law (7), are given in Table 6.

Table 6. Summary of experimental results [31].

Frequency of load cycles, ν Hz	Asymmetry coefficient, R	The results obtained with Paris law, with da/dN , in mm/cycle and ΔK , in $MPa\sqrt{m}$	
		C	m
20	0.1	$8.219 \cdot 10^{-8}$	2.248
20	0.5	$1.226 \cdot 10^{-8}$	2.920
5	0.1	$2.057 \cdot 10^{-8}$	2.667
5	0.5	$5.464 \cdot 10^{-8}$	2.306
20	0.5	$4.891 \cdot 10^{-8}$	2.406
20	0.1	$1.313 \cdot 10^{-8}$	2.167

The experimental results for steel 13CrMo4-5 showed that:

- For all the conditions of specimen sampling and testing, the growth rates of fatigue cracks were lower than those obtained by calculation according to the standards in force;
- A great influence on the crack growth rate is exerted by the asymmetry coefficient, R , and the frequency of load cycles, ν .

One can draw the following practical recommendations [31] for steels with yield limit $R_{p0.2} \leq 600$ MPa when using the Paris-Erdogan law (with da/dN in mm/cycle and ΔK_I in $MPa\sqrt{m}$):

- For structures of ferritic, austenitic and duplex steels operating in the open air or non-aggressive environments, at temperatures below 100°C , the Paris-Erdogan law is used with $m = 3$ and $C = 1.65 \cdot 10^{-8}$;
- For ferritic or duplex steel structures in marine environment at temperatures below 20°C , $C = 7.27 \cdot 10^{-8}$ and $m = 3$;
- For steel structures operated at temperatures below 600°C , $m = 3$ and $C = 1.65 \cdot 10^{-8} (E_{RT}/E_{ET})^3$,

where E_{RT} – steel modulus of elasticity at ambient temperature; E_{ET} - steel modulus of elasticity at high operating temperature;

- The threshold stress intensity factor, ΔK_{th} is dependent on the environment surrounding the structure operation and the asymmetry coefficient of the variable loading cycles, R . With steel structures of any type that operate in the open air or in non-aggressive environments, but also with ferritic steels with cathodic protection operated in the marine environment, one can use the values $\Delta K_{th} = 2.0$ $MPa\sqrt{m}$, for $R \geq 0.5$; $\Delta K_{th} = 5.5-6.8R$, for $0 \leq R < 0.5$ and $\Delta K_{th} = 5.5$ $MPa\sqrt{m}$, for $R < 0$, and for any type of steel without cathodic protection, used in the marine environment, one will consider $\Delta K_{th} = 0$.

4. DAMAGE CAUSED BY CRACKS

Material damage can generally be defined as a reduction of the material capacity from a certain point of view, such as, for example, the reduction in mechanical strength, in deformation capacity etc. [5, 71]. In fact, any material ages over time, so it deteriorates, the process being irreversible. In the case of engineering, when a material undergoes stresses that are lower than the material yield limit, it is accepted that the damage is reversible.

The presence of cracks may lead to a significant deterioration of mechanical structures.

The damage of structures through external actions, written as $D(t)$, where t is the loading duration, is a dimensionless variable whose values range between 0 and 1. For $D(t)=0$ the structure is not damaged, and for $D(t) = 1$ the structure is destroyed.

With cracked structures two extreme situations may be encountered [5]:

- Structures whose damage increases to $D(t) = 0.8 \dots 0.9$ within a time interval $t = 0.1t_r$, where t_r is the time down to failure, beyond which the damage grows very slowly down to failure (Figure 4, curve 1), is the case of structures allowed to operate after crack initiation;
- Structures wherein the damage increases slowly over time $t = (0.9-0.95)t_r$, beyond which the propagation occurs very rapidly (Figure 4, curve 2), in which case the cracked structure is not allowed to continue operation.

A rare case is one when the time variation of the damage is linear (Figure 4, curve 3).

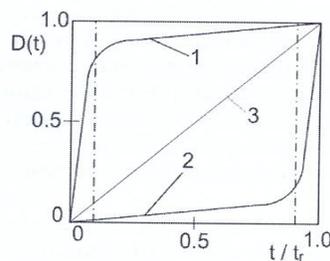


Fig. 4. Damage variation in time [5].

In literature there are many relations that account for structure damage caused by the action of a the single cyclic load [72], written as: D_c and calculated, on the basis of a reduction induced through cyclical loading, of a certain material characteristic; D , when it is a measure of the strain accumulation or durations of mechanical, thermal loading etc.

Out of the relations classified in [40, 72] we should mention:

- Shanley's relation [40, 41],

$$D = \sum \left(\frac{a_0}{a_c} \right), \quad (46)$$

where a_0 is the initial crack size and a_c is the critical crack size;

- The Ibrahim and Miller relation [40, 42], based on crack growth in two stages,

$$D = \frac{a_0}{a_f}, \quad (47)$$

where a_0 is the initial crack size; a_f is the final crack size;

- Jinescu's relation [5, 72],

$$D(a) = \left(\frac{a}{a_{cr}} \right)^{\frac{\alpha+1}{2}} \quad (48)$$

where, a is the current crack size, $\alpha = 1/k$ comes from the law of structural material behavior ($\sigma = M_\sigma \cdot \epsilon^k$, where σ is the applied normal stress, ϵ is the strain and M_σ and k are the material constants).

It is recommendable to use relation (48) that includes the material behavior through exponent α .

The influence of residual stresses on damage is seen in works [4, 5, 45].

The calculation of the total deterioration of the mechanical structures subjected to simultaneous or sequential loadings with two or more loads of the same kind, of different types, or of a different nature is achieved by the superposition of effects based on the principle of critical energy [4, 5] by algebraically adding the individual deteriorations.

5. SUPERPOSITION OF EFFECTS DUE TO MULTIAXIAL LOADING

An important issue is the superposition of several types of loadings of the same kind or of different kinds upon the mechanical structure and the procedure of calculating the total effect of these loadings on such structures [4].

5.1. Effect superposition of stresses corresponding to the three fracture modes

One considers the loading with normal stress, σ , according to the failure mode I and tangential stresses, τ_{II} and τ_{III} corresponding to the failure modes II and III.

In the case of materials with nonlinear behavior ($\sigma = M_\sigma \cdot \epsilon^k$), on the basis of the critical energy principle [4-6, 54-56], we obtained a general relationship,

$$P_T = \left(\frac{K_I}{K_{I,c}} \right)^{\alpha+1} \cdot \delta_\sigma + \left(\frac{K_{II}}{K_{II,c}} \right)^{\alpha_1+1} + \left(\frac{K_{III}}{K_{III,c}} \right)^{\alpha_1+1} \quad (49)$$

where P_T is the total participation of the specific energies induced by stresses σ , τ_{II} and τ_{III} ; $K_{I,c}$ - fracture toughness in fracture modes I, II and III, δ_σ is a dimensionless factor which imparts the stress direction: if stress σ opens the crack, $\delta_\sigma = 1$ and if the stress closes the crack, $\delta_\sigma = -1$; α and α_1 are exponents dependent on the material behavior and the loading speed [5]: $\alpha = 1/k$ and $\alpha_1 = 1/k_1$ (where k_1 is provided by the behavior law, $\tau = M_\tau \cdot \gamma^{k_1}$, where τ is the tangential stress, γ is the specific slip, M_τ and k_1 are constants).

When reaching the critical state the condition must be fulfilled [5, 6]:

$$\left(\frac{K_I}{K_{I,c}} \right)^{\alpha+1} \cdot \delta_\sigma + \left(\frac{K_{II}}{K_{II,c}} \right)^{\alpha_1+1} + \left(\frac{K_{III}}{K_{III,c}} \right)^{\alpha_1+1} = P_{cr} \quad (50)$$

where $P_{cr} \leq 1$ is the critical participation.

Another method of calculating the total result induced by the superposition of effects in fracture mechanics lies in using concepts δ (opening movement at crack tip) and J (full contour integral). One uses relationship involving concepts δ and J , mapped on [4-6, 56]:

$$\left(\frac{\delta_I}{\delta_{I,c}} \right)^{\frac{\alpha+1}{2}} + \left(\frac{\delta_{II}}{\delta_{II,c}} \right)^{\frac{\alpha_1+1}{2}} + \left(\frac{\delta_{III}}{\delta_{III,c}} \right)^{\frac{\alpha_1+1}{2}} = P_{cr} \quad (51)$$

where $\delta_I, \delta_{II}, \delta_{III}$ are the opening displacement values at the crack tip for fracture modes I, II, III, while $\delta_{I,c}, \delta_{II,c}, \delta_{III,c}$ are the corresponding critical values;

$$\left(\frac{J_I}{J_{I,c}}\right)^{\frac{\alpha+1}{2}} + \left(\frac{J_{II}}{J_{II,c}}\right)^{\frac{\alpha+1}{2}} + \left(\frac{J_{III}}{J_{III,c}}\right)^{\frac{\alpha+1}{2}} = P_{cr} \tag{52}$$

where J_I, J_{II}, J_{III} are the contour integrals for the corresponding fracture modes while $J_{I,c}, J_{II,c}, J_{III,c}$ are the critical values of the contour integral for the corresponding fracture modes.

5.2. Analysis of experimental results

In [1] there are introduced the dependencies, derived experimentally, between the stress intensity factor for fracture mode I, K_I , and the stress intensity factor for fracture mode II, K_{II} , considered for a mechanical structure loaded according to the two fracture mechanisms (Figure 5). The material used is an aluminum alloy.

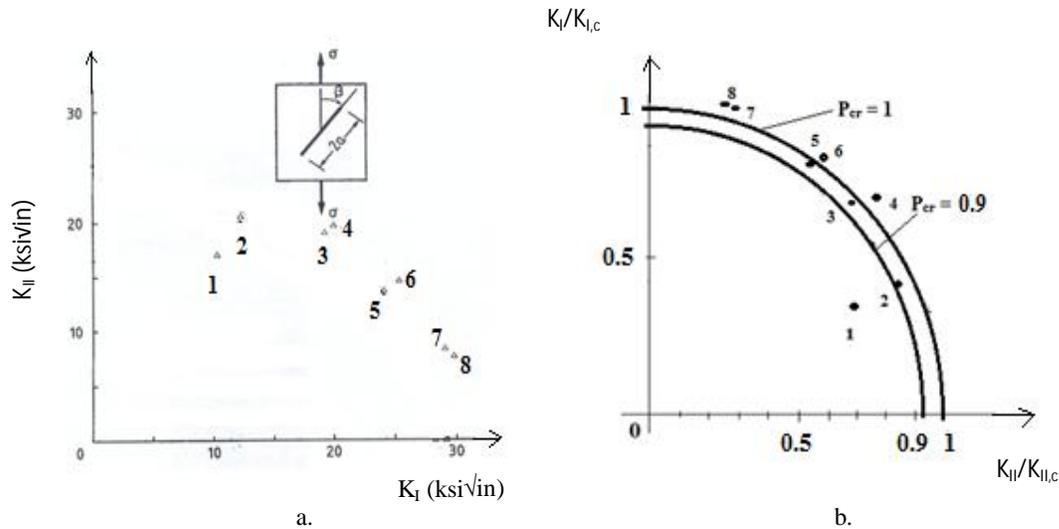


Fig. 5. Dependence between the stress intensity factors, K_I and K_{II} :
 a – experimental values [1]; b – the curve corresponds to relation (53),

a particular case of the general relation (50). The points 1 - 8 are the experimental ones for Figure 5, a [1].

In order to draw the curves mapping out the dependence of the $K_I/K_{I,c}$ ratio and the $K_{II}/K_{II,c}$ ratio, there were determined from Figure 5.a, the two values of fracture toughness for the two modes of fracture, $K_{I,c}=29\text{ksi}\sqrt{\text{in}}$, $K_{II,c}=25\text{ksi}\sqrt{\text{in}}$, respectively, and the corresponding values K_I and K_{II} for the eight points in the figure. Based on these values, in Figure 5.b, there was drawn the curve that yields the dependence of the $K_I/K_{I,c}$ ratio and the $K_{II}/K_{II,c}$ ratio.

The analysis of Figure 5.b shows that in the case of loading effect superposition according to fracture modes I and II of the aluminum alloy under study [1], the rupture occurs when there is fulfilled the condition imposed by relation,

$$\left(\frac{K_I}{K_{I,c}}\right)^2 + \left(\frac{K_{II}}{K_{II,c}}\right)^2 = P_{cr} \tag{53}$$

resulting from relation (50) for $\delta_\sigma = 1, k = k_I = 1, K_{III} = 0$, wherein, according to the experimental data (Figure 5.a), $P_{cr} = 0.92 \div 1.0$.

6. CALCULATION OF CRITICAL STRESSES BY CONSIDERING CRACK DAMAGE

6.1. Critical stresses for statically loaded cracked structures

In cracked structures one encounters the superposition of the effect caused by the crack with the effect of the stress away from the crack. On the basis of the critical energy principle [4, 57] the total participation, P_T , is equal to the sum of the participations of specific energies corresponding to stress, σ , and the stress, $\Delta\sigma$ [57]:

$$P_T = P(\sigma) + P(\Delta\sigma) \quad (54)$$

One considers the case of the nonlinear material behavior of mechanical structures, under normal stress load, σ . It has been shown that in this case [5, 57],

$$P(\Delta\sigma) \equiv D(a) = \left(\frac{a}{a_{cr}} \right)^{\frac{\alpha+1}{2}}$$

The critical state is reached when:

$$P_T = P_{cr} \quad (55)$$

With notations: σ_{cr} - critical normal stress of the material without cracks and $\sigma_{cr}(a)$ - critical normal-stress of the material with one crack, one obtained relation [57]:

$$\sigma_{cr}(a) = \sigma_{cr} \cdot [1 - D(a)]^{\frac{1}{\alpha+1}} \quad (56)$$

showing that the presence of crack reduces the critical stress.

6.2. Critical stresses for cyclically loaded structures with cracks

If a mechanical structure is cyclically loaded with normal stresses ranging between σ_{min} and σ_{max} , one calculates the average stress, σ_m and the stress amplitude, σ_a , with relations:

$$\left. \begin{aligned} \sigma_m &= 0,5(\sigma_{max} + \sigma_{min}) \\ \sigma_a &= 0,5(\sigma_{max} - \sigma_{min}) \end{aligned} \right\} \quad (57)$$

The total participation of the specific energy, P_T is [4]:

$$P_T(\sigma) = P(\sigma_a) + P(\sigma_m) + P(a) \quad (58)$$

out of which we obtain relation [5, 57]:

$$\left(\frac{\sigma_a}{\sigma_{a,cr}} \right)^{\alpha+1} + \left(\frac{\sigma_m}{\sigma_{m,cr}} \right)^{\alpha+1} \cdot \delta_{\sigma_m} = P_{cr} - D(a) \quad (59)$$

where $\sigma_{a,cr}$ is the fatigue strength of the structure with N loading cycles while $\sigma_{m,cr}$ may be the yield limit or failure resistance which depends on whether failure occurs at stresses $\sigma < \sigma_c$ or at $\sigma_c \leq \sigma < \sigma_r$; δ_{σ_m} depending on the sign of σ_m ($\delta_{\sigma_m} = 1$ for $\sigma_m > 0$ and $\delta_{\sigma_m} = -1$ for $\sigma_m < 0$).

The damage incurred by cyclic loading caused by the crack is calculated with (48).

The critical crack depth can be calculated based on the stress intensity factor for the most unfavorable loading. For example, for a cylindrical vessel under pressure one obtains for a_{cr} the relation [5]:

$$a_{cr} = \frac{1}{\pi} \cdot \left(\frac{K_{I,c}}{Y_1 \cdot \sigma} \right)^2 \quad (60)$$

The critical participation in this case, $P_{cr} = P_{cr}(a)$, is calculated with respect to the damage caused by the crack, with relation [4, 5, 44, 58]:

$$P_{cr}(a) = 1 - D(a) \quad (61)$$

Relation (48) is used for calculations worked out with respect to the critical state.

The stress amplitude upon reaching the critical state for structures with cracks after N loading cycles, according to works [5, 57], is:

$$\sigma_{-1,s}(a; N) = \sigma_{-1,s}(N) \cdot \left[P_{cr} - D(a) - \left(\frac{\sigma_m}{\sigma_{m,cr}} \right)^{\alpha+1} \cdot \delta_{\sigma_m} \right]^{\frac{1}{\alpha+1}} \quad (62)$$

where $\sigma_{-1,s}(N)$ is the strength of the crackless structure under alternating symmetrical loading with normal stresses; $\sigma_{-1,s}(a; N)$ is the strength of the cracked structure under alternating symmetrical loading with normal stresses.

Relation (62) for specimens under alternating symmetrical loads ($\sigma_m = 0$), where $P_{cr} = 1$, becomes:

$$\sigma_{-1}(a; N) = \sigma_{-1}(N) \cdot [1 - D(a)]^{\frac{1}{\alpha+1}} \quad (63)$$

which is similar to relation (56) used for static loading.

If a structure is loaded simultaneously with normal and tangential stresses, the effects of the two loads are superposed and the total effect is calculated according to the principle of critical energy [4, 44]. The calculation of the number of load cycles with normal stresses, by considering the mean stress, the residual stress and the deterioration is presented in paper [73].

If the verification or calculation of a mechanical structure with cracks is made with respect to the the allowable state, one resorts to relations that are similar to those for the critical state. One replaces the critical stresses with allowable stress and the deterioration related to the critical state with the deterioration related to the allowable state [4, 5].

7. THE ANALYSIS OF EXPERIMENTAL DATA OF CYCLIC LOADING AND THEIR CORRELATION WITH THE CONCEPT OF DETERIORATION

In order to illustrate the dependence of failure resistance on deterioration and vice-versa, one has presented the results obtained in [63] for a rectangular shaped tile.

Starting from this experiment, for the ceramic plate with one crack located at different sloping angles, α and with crack length $2a$ (Figure 6.a) under uniaxial compression loading, one could determine the values of the compressive strength variation, $\Delta\sigma_c$, at different values of crack length (Figure 6.b) and the amount of damage.

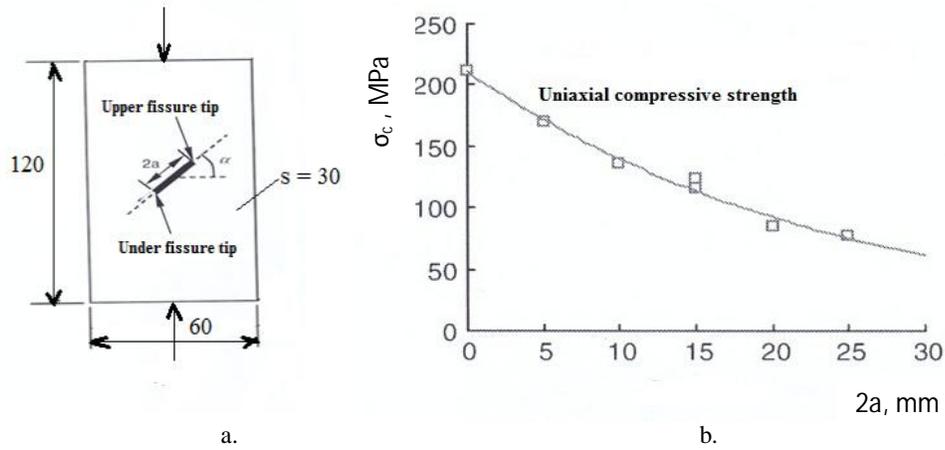


Fig. 6. a. – The crack geometric parameters;
 b. – uniaxial compressive strength variation depending on crack size [63].

The variation of the material compressive strength is

$$\Delta\sigma_c = \sigma_c(0) - \sigma_c(a) \tag{64}$$

where $\sigma_c(0)$ is the compressive strength of the material without cracks and $\sigma_c(a)$ is the compressive strength of the material with a crack. From relations (56) and (64) it follows

$$\Delta\sigma_c = \sigma_c(0) \cdot \left[1 - (1 - D(a))^{\frac{1}{\alpha+1}} \right] \tag{65}$$

Since the material (sandstone) has brittle behaviour, $k = 1$ and $\alpha = 2$, so that relation (65) becomes,

$$\Delta\sigma_c = \sigma_c(0) \cdot \left[1 - (1 - D(a))^{\frac{1}{2}} \right] \tag{66}$$

out of which we obtain the resulting expression for damage,

$$D(a) = 1 - \left[1 - \frac{\Delta\sigma_c}{\sigma_c(0)} \right]^2 \tag{67}$$

Table 7 presents the variation of compressive strength values obtained experimentally, $\Delta\sigma_{c,exp}$, and damage $D(a)$ calculated with equation (67) for the different values of crack length, $2a$.

Table 7. Variation of experimentally determined compressive strength and damage values calculated with equation (67).

$2a$ (mm)	5	10	15	20	25
$\Delta\sigma_{c,exp}$ (MPa)	30.76	65.38	88.46	115.38	126.92
$D(a)$	0.283	0.546	0.688	0.821	0.866

One considered $\sigma_c(0) = 200$ MPa.

Analyzing the data in Table 7 one can find that the material damage increases with the crack growth.

8. CONCLUSIONS

The paper analyzed the calculation relations for pressure vessels by resorting to fracture mechanics concepts (stress intensity factor) and the concept of damage (damage due to the presence of cracks). There were investigated the particularities of calculations set out in the American standard ASME Code, 2009, Section VIII - Division 3, European standard EN 13445-3 and the British Standard PD 5500: 2009. It was found that the calculation method set out in these standards have a limited scope and take no account of the damage caused by cracks.

Calculation according to the ASME Normative is quite difficult because for each type of crack one has to determine another stress intensity factor, the effect of residual stresses must be evaluated separately, while the determination of the number of loading cycles requires a rerun of the calculation for increasingly smaller crack depths.

With standard EN 13445-3, the fracture mechanics method has a scope of application limited to certain steels.

The British Standard provides a calculation methodology based on fracture mechanics applicable to vessels operating at negative temperatures, while there is no unique relationship for the determination of the reference thickness. The literature review revealed that there are over 120 relations for calculating the crack propagation rate, which is expressed either depending on the stress intensity factor variation, the crack size, the maximum stress applied, the fracture toughness, the asymmetry coefficient or the equipment wall thickness. It was found that the number of loading cycles depends on the value of the maximum loading stress, the asymmetry factor and the initial crack size.

We analyzed the damage resulting from cracks, highlighting the relations for calculating damage, where equation (48) features the highest level of generality. The superposition of effects was analyzed for the case of multiaxial loadings by evaluating the results of the effect superposition with the aid of such concepts as the stress intensity factor, the crack tip opening displacement and the crack contour integral. Their relation for the superposition of effects for fracture modes I and II was verified with literature experimental data, thus confirming the high degree of generality of relation (50). We analyzed the calculation of the critical stresses by considering the damage caused by the cracks under static and cyclic loadings. This resulted in relations (56), (62) and (63).

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