

ASSESSMENT OF AKURE ROAD NETWORK: A SHORTEST PATH APPROACH

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Abstract: In this paper, the road network of Akure (a city in south western Nigeria) was studied based on its shortest path properties. Three sections, each measuring 1 square mile, were sampled from the entire road network and developed into graphs using both a primal approach and graph theory in which the intersections and streets were represented as nodes and edges respectively. These graphs were assessed for the edge lengths and shortest paths between nodes from which their edge length distributions, average path lengths and efficiencies were also obtained. It was concluded that the city of Akure is self-organized with small world properties since the network case studies showed small path lengths.

Keywords: shortest path, primal approach, graph theory, edge length distribution, efficiency

1. INTRODUCTION

Transportation is a science that is concerned with the efficient movement of people and goods and which is undertaken to accomplish objectives that require transfer from one location to another. Nowadays, such transfer is largely facilitated by networks comprising of interconnected facilities and services that function together as a system.

A road network can be viewed as a typical example of a system; an assemblage of components and subsystems that work together to perform the same function. It follows that transportation networks inherently have a node and link structure, where the links represent linear features providing for movement, such as highways and rail lines, and the nodes represent intersections. Thus, the principal data content of a node is its name or number and location. Links usually have characteristics such as length, directionality, number of travel lanes and functional class.

Essentially, road networks consist of a lot of roads that are interwoven together [1]. A road network is a topological graph for expressing road information, and is composed of nodes and edges; the nodes indicate characteristic points such as intersections and terminal points while the edges point out the connectivity among nodes. It is generated through the connection procedure of different pairs of parallel line segments sequentially since a continuous road is looked upon as a sequence of pairs of parallel line segments [2].

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Considerable amount of research has been carried out to analyze the vulnerability of road networks and most of the methods were based on analyzing the topological structure of the network using graph theory [3]. Graph theory deals with problems that have a graph (or network) structure and is a field of mathematical ideas about graphs [4]. A graph is an abstract representation comprising of a number of points connected by lines where each point is usually called vertex and the lines are called edges.

Since the early sixties, much of the research in the area of urban network studies has tried to link the allocation of land uses to population growth through lines of transportation or sought to predict traffic flows given several topological and geometric characteristics of traffic channels or eventually investigate the exchanges of goods between settlements in the geographic space even in historical eras. Most, if not all these approaches have been based on a quite simple, intuitive representation of networks which in short turns intersections (or settlements) into nodes and roads (or lines of relationship) into edges [5].

This study assesses the properties of an urban city, namely Akure, located in Nigeria through network analysis based on its shortest paths. Akure, as a developing city, consists of several networks of roads which were developed in the early 70's. However, the increase in population associated with the growth of Akure into a city has caused a larger number of vehicles to continue to ply these roads that have been constructed over thirty years ago without any improvements causing the traffic to exceed the capacity of those roads.

Also, movement at the intersections becomes more conflicting due to increased number of vehicles and the arrangement of the legs of the intersection since a new road is more likely to connect to an important intersection. Hence, an evaluation of the road network in Akure becomes imperative so as to identify the locations where possible increase in the number of vehicles using the road (links) and intersections may occur and whose deactivation may breakdown the whole network leading to reduction in the efficiency of the network. This will allow attention to be focused where necessary so as to ensure improved road network performance.

2. BACKGROUND LITERATURE

Historically, the study of networks has mainly been the domain of a branch of discrete mathematics known as graph theory. Since its birth in 1736, when the Swiss mathematician Leonhard Euler published the solution to the Königsberg bridge problem (consisting in finding a round trip that traversed each of the bridges of the Prussian city of Königsberg exactly once), graph theory has witnessed many exciting developments and has provided answers to a series of practical questions in several areas of study including transportation engineering [6].

Transportation networks such as road, railway and airline networks are an integral part of the infrastructure in many countries. Therefore, their study is considered as an important field of research based on graph theory. Graphs are among the most ubiquitous models of both natural and human-made structures [7]. In the natural and social sciences, they model relations among species, societies and companies. In computer science, they represent networks of communication, data organization, computational devices as well as the flow of information.

In statistical physics, graphs can represent local connections between interacting parts of a system as well as the dynamics of a physical process on such systems. In a graph representing a road network, intersections and streets can be modeled by nodes and edges respectively. Two nodes have an edge in between if there is a street connecting the two corresponding intersections. Different streets of a road network have different lengths and are modeled by assigning to each edge a number, called the edge weight which represents a cost that reflects real-world values such as distance, travel time, transmission time and latency.

For a computer network, routers and the connecting network cables are mapped to nodes and edges, respectively. In a social network, the connections are not physical and individuals can be modeled by nodes; two nodes are connected by an edge whenever the corresponding individuals are friends. In other social networks, an edge may also indicate a private or professional relationship other than friendship.

2.1. Complex networks

Communication networks are nowadays the subject of intense research as modern society increasingly depends on them [8]. Many natural, technological and social systems find a suitable representation as networks made of a large number of highly interconnected units [9]. Typical examples include neural networks,

information/communication networks, electric power grids and transportation systems ranging from airports to street networks.

Most of the communication/transportation systems of the real world can be represented as complex networks in which the nodes are the elementary components of the system and the edges connect pairs of nodes that mutually interact by exchanging information [10]. Typical examples include the internet in which the nodes are the routers and the edges (or arcs) are the cables connecting couples of routers; the electrical power grid in which the nodes are the substations (generators or distribution substations) and the edges are the transmission lines; and a city road system in which the nodes are the crossings and the edges are the roads.

Systems in nature and in technology are made of a large number of highly interconnected dynamical units [6]. The first approach to capture the global properties of such systems is to model them as graphs whose nodes represent the dynamical units (for instance, the neurons in the brain or the individuals in a social system) and the links stand for the interactions between the units. This may be regarded as a very strong approximation, since it means translating the interaction between two dynamical units (which usually depends on time, space and many other details) into a simple binary number indicating whether there exists a link between the two corresponding nodes or not. Nevertheless, in many cases of practical interest, such an approximation provides a simple but still very informative representation of the entire system.

2.2. Characterization of real world networks

The three properties of real world networks considered most important are small shortest path lengths, high clustering and low cost [11]. The first of these three properties, namely shortest path distances, is investigated in this study for assessing the properties of Akure Metropolis.

In most real-world networks, it is possible to reach a node from another one, going through a number of edges that is small when compared to the total number of existing nodes in the system. However, in order to measure the typical separation between two generic nodes in a graph denoted by the symbol ‘ G ’, a characteristic path length parameter denoted by the symbol ‘ L ’ was introduced which is given in equation 1.

$$L(G) = \frac{1}{N(N-1)} \sum_{\substack{i,j \in G \\ i \neq j}} d_{ij} \quad (1)$$

where ‘ N ’ is the total number of nodes in ‘ G ’ and ‘ d_{ij} ’ is the shortest path length between nodes ‘ i ’ and ‘ j ’ (i.e. the minimum number of edges covered in order to reach ‘ j ’ from ‘ i ’).

In a similar manner, the efficiency (E) of a network can be obtained from measures of the shortest path lengths on the basis that the efficiency between node ‘ i ’ and ‘ j ’ denoted by the symbol ‘ ϵ_{ij} ’, is assumed to be inversely proportional to the shortest path length, i.e. $\epsilon_{ij} = 1/d_{ij}$. When there is no path linking ‘ i ’ and ‘ j ’, it is assumed that $d_{ij} = +\infty$ and therefore $\epsilon_{ij} = 0$. Consequently, the global efficiency of a graph ‘ G ’ (connected or non-connected) denoted by the symbol $E_{glob}(G)$ is defined as the average of ϵ_{ij} and given by equation 2.

$$E_{glob}(G) = \frac{1}{N(N-1)} \sum_{\substack{i,j \in G \\ i \neq j}} \epsilon_{ij} = \frac{1}{N(N-1)} \sum_{\substack{i,j \in G \\ i \neq j}} \frac{1}{d_{ij}} \quad (2)$$

In the case of topological (unweighted) graphs, $E_{glob}(G)$ assumes growing values of efficiency from 0 to 1 and therefore no normalization for comparisons of different networks is required. An extension of this assumption to weighted graphs (e.g. metric systems) is also possible.

2.3. Classes of networks

Networks can be roughly classified into three major groups namely regular networks, random networks and small-world networks.

2.3.1. Regular networks

These are the most common types of networks whose nodes have the same degree. They have common basic cycles such as square or cubic lattices [12].

The first model to be presented is the regular lattice as shown in figure 1 which is not different from a regular ring. Research has shown that this model is a bad representation of reality because of its excessive regularity [11]. In fact, in a network with N nodes and K edges ($k = 2K/N$ edges per node), we have to make the condition of $K \ll N(N-1)/2$ in order to assure low cost.

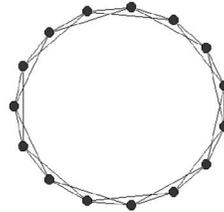


Fig. 1. A typical regular network (with node degree, $k = 4$).

2.3.2. Random networks

The study of random networks has been motivated by the observation that real networks often appeared to be random [12]. A random graph has ' N ' labeled/ nodes connected by ' n ' edges, which are chosen randomly from the $N(N-1)/2$ possible edges. They defined important properties of random networks such as the distribution of node degree (which is the number of edges every node is connected to) as those that follow a binominal distribution. They further described the probability of the presence of sub-graphs and defined the commonly used clustering coefficient which indicates the probability that two node neighbors are connected as well as the average path length.

Random networks are the most investigated and were introduced by Erdos and Renyi (ER), who were the first to study the statistical aspects of random graphs by probabilistic methods [11]. Hence, they are often referred to as Erdős-Rényi (ER) random graphs [13]. In a random network (or graph), non-metric distance is defined along the edges and it can be constructed from an initial condition with N nodes and no connections by adding K edges (in average k per node), randomly selecting the couples of nodes to be connected. A typical example of random graph is as shown in Figure 2.

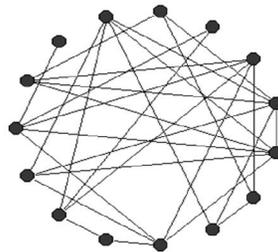


Fig. 2. Erdős-Rényi model.

2.3.3. Small world networks

A network is called a small world if it is analogous to a phenomenon known as six degree separation which was first proposed by Hungarian writer, F. Karinthy in 1929 and tested experimentally by Milgram in 1967 [13]. In existing models, a small characteristic path length and low clustering or large characteristic path length and high clustering was observed.

Watts and Strogatz (WS) recently introduced a new model that is often referred to as small-world network in an effort to describe the transition from a locally ordered system to a random network. The topological properties of the network generated by this model have been the subject of much attention lately [14].

In order to interpolate between regular and random graphs, D. J. Watts and H. Strogatz developed a one-parameter model in which the introduction of few long range connections leads to good global properties,

without altering the high clustering property, typical of regular networks. The model starts from an initial condition of a regular ring lattice with N nodes and k edges per vertex. Then a random rewiring procedure is implemented: each link is rewired with probability p ; i.e. with probability p , in proximity of one end, an edge is cut and randomly reattached to another vertex.

The effect of rewiring is that of substituting some short range connections with long range ones. By varying p , it is possible to tune the graph from the regular configuration ($p = 0$) to the random one ($p = 1$) and for $0 < p \ll 1$ the small-world behavior appears: a few rewired edges are sufficient to obtain small L and high C , i.e. efficient local and global communication [15, 16]. Under the assumption of low cost (a few edges in the initial condition), economic small-worlds networks are obtained.

3. MATERIALS AND METHODS

The method used in the analysis of these networks was based on primal approach. A primal graph is a spatial graph in which zero dimensional geographic entities (intersections) are turned into zero dimensional graph entities (nodes) placed in two dimensional Euclidean space. One dimensional graph entities such as streets were also turned into one dimensional graph entities (edges/links). Primal graphs were constructed by following a road-centerline-between-nodes rule in ArcGIS environment. Real intersections were converted to graph nodes and real streets were converted to graph edges. All graph edges were defined by two nodes regarded as the endpoints of an arc. Edges followed the footprint of real streets as they appeared on the source map while all distances were calculated metrically as they corresponded to the length values of the streets in the networks.

Three sections (each of 1 square mile) were sampled from Akure road map (with scale of 1:20000) obtained from the Ondo state Ministry of work and developed into graphs as shown in figure 3 using the primal approach.

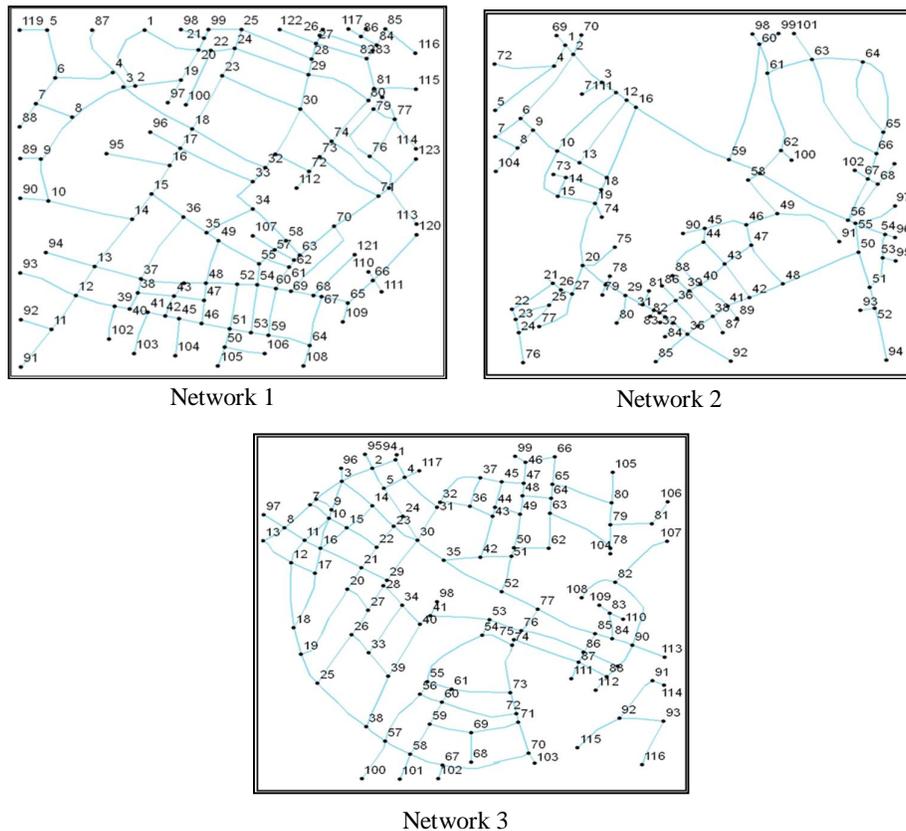


Fig. 3. Primal graphs of the networks.

All information was obtained from the map. Edges were assigned with weights which represented the distance values of the street lengths obtained from the road networks studied. Each graph G , was described by the adjacency matrix, $A = \{a_{ij}\}$ which is an $N \times N$ square matrix whose element a_{ij} is equal to 1 if (i, j) belongs to the set of links, and zero otherwise. It was also described by an $N \times N$ matrix, $L = \{l_{ij}\}$ whose entry l_{ij} is the distance value associated to each edge which in this case is given by the metric length of the street connecting i and j . The typical separation between two generic nodes in each graph G was then determined by using equation 1.

4. RESULTS AND DISCUSSION

The three areas studied within Akure showed striking differences in the activities that took place within them as well as in the level of land development. The basic structure observed over the planar road network (network embedded in real space) in Akure is branching network (with no form of circuit network) distinguished by the presence of tree-like structure which consists of connected line without any complete circuit.

The graphs studied were weighted with weights corresponding to the distances between geodesic nodes, sparse (meaning that $k < N(N-1)/2$) and connected (meaning that there existed at least one path connecting any couple of nodes). In addition, dead ends were considered as nodes since travel paths from such an area makes use of the nearest or neighboring nodes.

4.1. Length distribution of the graphs/networks under study

Basic properties of the three primal graphs (network case studies) considered are as indicated in table 1 where N is the number of nodes, $N(L)$ is the number of edges and $\langle l \rangle$ is the average edge length.

Table 1. Basic properties of the primal graphs.

Network	Numbers of Nodes (N)	Number of Edges $N(L)$	Average Edge Length $\langle l \rangle$	Average Path Length (L)
1	123	155	156.774	0.066
2	104	126	144.603	0.088
3	117	159	146.835	0.062

Table 1 indicates that the basic properties of the three networks (primal graphs) considered were different (despite the fact that the same amount of land was considered) except in the case of the average edge lengths, $\langle l \rangle$ whose values were close with only about 8.4 % difference between the highest and lowest values. The table also indicates that the average path lengths obtained over the three networks are quite small while the smallest or most conservative value of 0.062 was obtained over network case study 3 which incidentally has the highest number of edges.

Also, the three areas studied within Akure indicated single peaked edge length distributions as shown in figure 4 with $P(l_1)$, $P(l_2)$, and $P(l_3)$ representing the length distributions of networks or primal graphs 1, 2 and 3 respectively. The edge length distribution $P(l)$ is defined by equation 3.

$$P(l) = \frac{N(L)}{n} \quad (3)$$

where $N(L)$ is the number of edges/links with a particular length and n is the number of links/edges in the network/graph under consideration.

Figure 4 indicates that the edge length distributions $P(l)$ showed peak values of 0.135, 0.103 and 0.150 which corresponded to edge lengths of 80 m, 60 m and 120 m for networks 1, 2 and 3 respectively. Such peaked distributions are typical of cities/networks with a growth pattern over a period of time which is out of the control of any central agency. This suggests that the city of Akure is not a planned city evidenced by the peak distributions. These can be attributed to the presence of star like pattern which are characterized by hub like junction with roughly parallel links (roads) with a set of links (road) crossing the hub (i.e. node with high degree) since a map/ road network can be viewed as a unique collection of pattern.

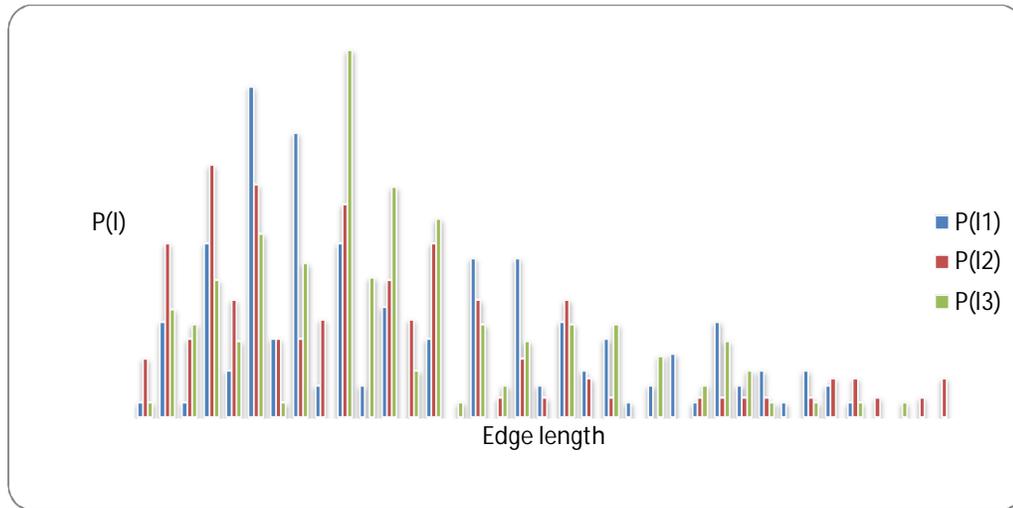


Fig. 4. Probability distribution $P(l)$ of edge lengths over the cases of urban street networks studied.

However, in some cases, there are sets of intersections in a small area instead of a single hub like intersections. These types of hub like intersections (nodes) are considered as complex intersections but the second property changes accordingly with roads converging at a set of intersections rather than at a single intersection. Akure street network exhibits self-organized patterns, in that they 'spontaneously' emerged from a historical process outside of any central coordination. This can be linked to the scale free properties (exponential power) exhibited by the degree distribution.

4.2. Efficiency of the networks based on the shortest paths

The efficiencies of the three networks studied alongside their other properties are as shown in table 2.

Table 2. Efficiency of Networks.

Networks	Number of Edges $N(L)$	Average Edge Length $\langle L \rangle$	Efficiency $E_{glob}[G]$
1	155	156.774	0.00304
2	126	144.603	0.00463
3	159	146.835	0.00231

Based on table 2, it can be observed that the highest network efficiency of 0.00463 was obtained over network case study 2 which incidentally has the lowest number of edges. Conversely, the lowest network efficiency of 0.00231 was recorded over network case study 3 which possessed the highest number of edges.

5. CONCLUSION

It can be concluded that the city of Akure has small world properties based on the fact that the network case studies considered have small path lengths of 0.066, 0.088 and 0.062. This means that the city can be easily traversed and different points can be easily reached by road.

In addition, that Akure is a self-organized city i.e. a city that grows out of the control of any central planning agencies evidenced by distributions with peak values of 0.135, 0.103 and 0.150 corresponding to 120 m, 80 m and 80 m respectively for the three network cases studied.

In terms of the network efficiencies, it seems that they tend to increase as the number of edges decreases while they tend to decrease as the number of edges increase. Furthermore, it seems that the small world effect is greatly enhanced in networks with high number of edges.

The graph network approach considered in this study can be used by the transportation agency of Akure to plan the future growth of the city's road network so as to ensure that it maintains its small world property since a network with this property is considered to be very efficient. This will require that once new edges (roads) and nodes (intersections) are introduced into the network at the design stage, the average path length and efficiency of the network are recalculated and checked to ensure that they converge to the initial value. It will also require use of powerful software capable of analyzing large networks which will give the path lengths and efficiencies quickly.

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