THEORETICAL ASPECTS REGARDING THE RELATIVE MOTION OF SEEDS ON AN INCLINED VIBRATING PLANE SURFACE

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Abstract: The trajectories described by the seeds on the planned double inclined surface play an important role in sorting seeds by density on the vibrating tables. The article develops the mathematical models of the particles trajectory supported by friction on double inclined plane surfaces subject to a motion by rectilinear translation harmonic oscillation and it emphasizes the difference between the particles trajectory with various friction coefficients. The values of such friction coefficients are in accordance with the variable density of the seeds which thus entails different places of seeds collection by their density on one of the sides of the surface.

Keywords: cereal seeds, sorting, density, trajectories, mathematical models

1. INTRODUCTION

In order to obtain high quality seeds, the cleaning, sorting and selection of the seeds are operations of utmost importance which have to be carried out to the mass of seeds after harvesting.

One of the various methods used for this purpose is the seeds sorting by density which is the last and compulsory operation in the chain of cleaning and sorting operations. The result of seeds segregation by density consists of the most vigorous material destined either for planting in order to get uniform, high-yield crops or for selection.

Thus, an important category of machines is represented by the pneumatic vibrating machines (or densimetric tables) which assure the separating of the seeds by their density.

As proved by experimental researches, the segregation of the seeds mixture on these machines is performed mainly by density, the resulting fractions differing more by seeds density and less by dimension.

The working process is complex and may be approached as a combination of two separate phenomena: motion on the table and stratification.

The simultaneous action of vibrations and air flow through the seed layer on the table surface is conducive both to the selection by density and also to differentiations between seeds trajectory on the table which provides their separate collection [1, 2].

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The simultaneous action of vibrations and air flow that goes through the seed layer on the table surface brings about both the stratification of particles by density, and the differentiation of their trajectories on the table, thus making their separate collection possible [1, 2].

The seed layers will get trajectories under the significant influence of the friction coefficients on the plane double inclined surfaces and the seeds may be collected separately (the coefficient is in direct connection with particles density in connection with their position along the layer thickness in the stratification process on the machine table). Therefore, this thesis contains a theoretical study on the analysis of the relative motion of particles on a double inclined oscillating surface, establishing motion equations, integrating such equations numerically and highlighting the differentiation of the particles' trajectories according to their friction coefficient.

2. MATERIAL AND METHOD

Experimental results have shown that the essential parameter in the seeds separation process on the pneumatic vibrating surfaces is their density. As a result of the air flow going through the seed layer together with the agitation provoked by the vibration of the surface, the particle mixture on the sorting surface will get loose to an increasing extent and stratification is obtained.

The seeds get in the air flow in suspension condition, the interaction between seeds decreases towards zero which makes motion possible within the mixing layer on its thickness. The heavy particles (with high density) fall to the base of the mixing layer (getting in contact with the segregation surface) and the lighter particles move to the upper part of the layer getting loose, with a more reduced interaction with the lower layers proportional to the one of heavy particles with segregation surface, as shown in Figure 1 [3].

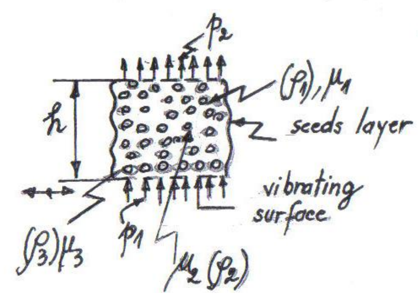
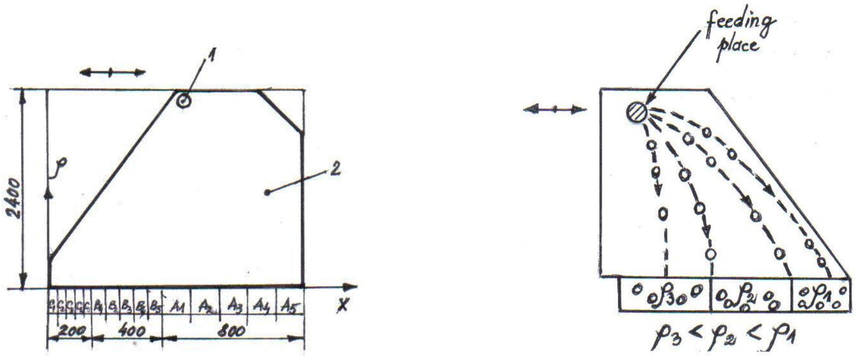


Fig. 1. Section detail, on seeds layer's height (h) at segregation state by particles density, ($\rho_1 < \rho_2 < \rho_3$).

The air flow that goes through the vibrating seed layer is analogous to the air circulation through granular, porous, stationary beds allowing the use of Ergun equation as a link between pressure downfall ($-\Delta P$) on the thickness of the seeds layer and conventional air speed [4].



a) b)

Fig. 2. Section Scheme of densimetric table machine from a machine in use in working practice:
a) table scheme: 1 - feeding basket; 2 - active table segregation; b) trajectories scheme described by the particles having different densities; ρ_1, ρ_2, ρ_3 - density of the sorting particles.

The seeds movements from the layer thickness follow different trajectories - those from the bottom are moving mainly in the oscillating direction and those from the upper part follow trajectories nearby the line of utmost inclination of the double inclined table allowing their segregated collection on the opposite side of the feeding side of the table (see Figure 2.b) [3].

This is due to the difference between the seeds friction coefficient depending on their position in the layer - lower values for those in the upper layer- as a consequence of stratification and air layer existence between different seeds lines on the layer thickness and higher values for those situated on the bottom of the layer.

The particles get different trajectories depending on the friction coefficients in direct connection with density, assuring their collection in different places: those with lower density closer to the feeding point but those with greater density far from the feeding point, (see Figure 2.b), which explains and theoretically substantiates the geometric shape of the segregation surface from the densimetric machine existing in practice, (see Figure 2.a) where their inactive parts (useless) have been removed and fractions sorted by density are collected (evacuated) on one of the side surface's [3].

3. EQUATION OF THE RELATIVE MOTION OF THE PARTICLES ON A DOUBLE INCLINED OSCILLATING SURFACE

The equations of the relative motion on the plane pneumatic vibrating double inclined segregation table shall be developed.

It was agreed that the inclined plane surface compared to the horizontal plan, is positioned compared to a fix reference system OXYZ by reference system oxyz jointly with mobile surface, through α and β angles between axis of coordinates ox and OX, oy and OY, respectively, Figure 3.

It will be taken into consideration the case of one particle having the mass m (seed) with friction on a plane surface which is set in motion by rectilinear translation harmonic oscillation.

As it is well known [5] in the relative motion of the particle in this case, Coriolis force has no value, remaining only the forces G, N, F_f, and F_i (Figure 3).

In the harmonic rectilinear oscillation motion, inertia can be expressed as follows: $F_i = m \cdot A \cdot \omega^2 \cdot \sin \omega t$ where: m - particle mass, A - oscillating amplitude, ω - oscillating pulsation.

We are taking into consideration the general case where the oscillating direction (Figure 3) which determines the direction of inertia F_i and makes $\varepsilon > \beta$ angle with horizontal axis and it is situated in yoz plan. Consequently, inertia F_i, splits, by oy and oz axis, as follows:

$$F_{iy} = F_i \cos\varepsilon = mA\omega^2 \sin\omega t \cos\varepsilon \tag{1}$$

$$F_{iz} = F_i \sin(\varepsilon - \beta) = mA\omega^2 \sin\omega t \sin(\varepsilon - \beta) \tag{2}$$

As the particle relative motion develops in the surface plan perpendicular on the normal reacting force N, (N//Oz), projecting the vector equation of the relative motion on Oz axis, having in view item (2) above, the reacting force will be:

$$N = G\cos\alpha - F_{iz} = G\cos\alpha - mA\omega^2 \sin\omega t \sin(\varepsilon - \beta) \tag{3}$$

In this case, friction force F_f, is:

$$F_f = \mu N = \mu mg\cos\alpha - \mu mA\omega^2 \sin\omega t \sin(\varepsilon - \beta) \tag{4}$$

where μ represents the friction coefficient of the particle with the surface. The projection equations of relative motion, in a random point of the trajectory, will be:

$$m\ddot{x} = G\sin\alpha - F_f\cos\gamma \qquad m\ddot{y} = F_i\cos\varepsilon - F_f\sin\gamma \tag{5}$$

where γ represents the tangent angle to the trajectory (Γ - F_f direction) with the Ox axis.

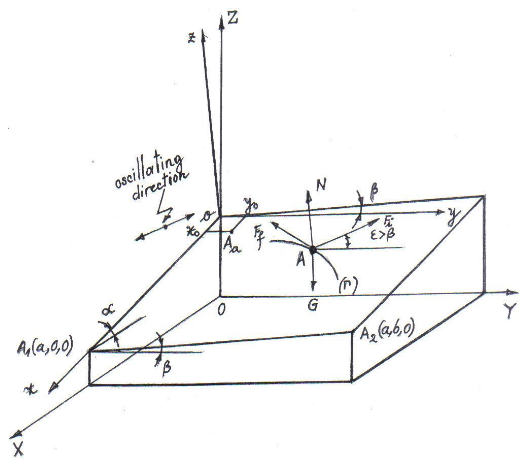


Fig. 3. Scheme for pneumatic vibrating segregation surface by seeds density together with the forces acting on the particle; for ε > β.

Considering the formulae of F_i and F_f forces and making the relevant calculation, we obtain the relative motion equations:

$$\ddot{x} = g\sin\alpha - \mu\left[g\cos\alpha - A\omega^2\sin\omega t\sin(\varepsilon-\beta)\right]\cos\gamma \tag{6}$$

$$\ddot{y} = A\omega^2\sin\omega t\cos\varepsilon - \mu\left[g\cos\alpha - A\omega^2\sin\omega t\sin(\varepsilon-\beta)\right]\sin\gamma \tag{7}$$

Taking into account the following formula:

$$\sin\gamma = \frac{\dot{y}}{\sqrt{\dot{x}^2+\dot{y}^2}}; \qquad \cos\gamma = \frac{\dot{x}}{\sqrt{\dot{x}^2+\dot{y}^2}} \tag{8}$$

we get the final relation of the relative motion equation:

$$\ddot{x} + \mu\left[g\cos\alpha - A\omega^2\sin\omega t\sin(\varepsilon-\beta)\right]\cdot\frac{\dot{x}}{\sqrt{\dot{x}^2+\dot{y}^2}} - g\sin\alpha = 0 \tag{9}$$

$$\ddot{y} + \mu\left[g\cos\alpha - A\omega^2\sin\omega t\sin(\varepsilon-\beta)\right]\cdot\frac{\dot{y}}{\sqrt{\dot{x}^2+\dot{y}^2}} - A\omega^2\sin\omega t\cdot\cos\varepsilon = 0 \tag{10}$$

Differential equations (9) and (10) of the particle motion on the surface of the table are general.

4. THEORETICAL DETERMINATION OF THE TRAJECTORIES OF THE PARTICLES RELATIVE MOTION ON THE SURFACE

We will give values to the geometric dimensions: α, β; physical characteristics: μ, g and kinematic dimensions: A, ω which compound particles motion equations and we obtain the curves - particles trajectories (seeds) described on the vibrating surface until seeds leave the surface, depending on its dimension, in order to evaluate the extent to which these trajectories differ.

The research will examine the influence on differentiating the trajectories, on the difference between frictions coefficient due to the particles stratification by density on thickness of the layer situated on vibrating surface, the particles on the bottom have high density and high values of friction coefficients while the upper particles have low density and low friction coefficient values.

The equation of the relative motion (9) and (10) has been numerical integrated using the Runge-Kutta fourth rank method.

Thus, the second rank differential equation system (9) and (10) will be converted into a first rank differential equation system, making the following notation: $\dot{x} = z_1$ and $\dot{y} = z_2$, as follows, [6]:

$$\dot{z}_1 = g\sin\alpha - \mu\left[g\cos\alpha - A\omega^2\sin\omega t\sin(\varepsilon - \beta)\right]\cdot\frac{z_1}{\sqrt{z^2{}_1 + z^2{}_2}} \tag{11}$$

$$\dot{z}_2 = A\omega^2\sin\omega t\cos\alpha - \mu\left[g\cos\alpha - A\omega^2\sin\omega t\sin(\varepsilon - \beta)\right]\cdot\frac{z_2}{\sqrt{z^2{}_1 + z^2{}_2}} \tag{12}$$

with the initial conditions:

$$x(0) = 0;\ \dot{x}(0) = V_{ox};\qquad y(0) = 0;\ \dot{y}(0) = V_{oy} \tag{13}$$

The TURBO-PASCAL program has been used, using the METODNUM program library (numerical methods), presented in the study [6].

The equations mentioned above have been numerically integrated for usually existing data for machines used in the work practice: n=450 rot/min ($\omega = 15\pi = 47.1\ s^{-1}$); A = 5 mm = $5\cdot10^{-3}$ m; $\alpha = 8^0$; $\beta = 3^0$; $\varepsilon = 28^0$.

Three friction coefficient values: μ = 0.05; μ = 0.1 and μ = 0.15 have been numerically integrated in equations. Using such data, together with the solutions of the differential equations (9) and (10) and the three friction coefficient values, the relative trajectories of the seeds on the sorting machine table have been represented graphically.

The six variants have been examined and recorded in Table 1 [3]. These variants emphasize the influence of the initial speed at the feeding point of the seeds on the table machine both on the relative motion speed and on their trajectories and consequently on the segregation.

Table 1. Data variants for integrating the differential equations ($\varepsilon = 28^0$).

Variant No.	x(0)	y(0)	$\dot{x}(0)$ (m/s)	$\dot{y}(0)$ (m/s)	μ_1	μ_2	μ_3
1.	0	0	0.25	0.25	0.05	0.1	0.15
2.	0	0	0.5	0.5	0.05	0.1	0.15
3.	0	0	0	0.25	0.05	0.1	0.15
4,	0	0	0.05	0.25	0.05	0.1	0.15
5.	0	0	0.1	0.35	0.05	0.1	0.15
6.	0	0	0.15	0.4	0.05	0.1	0.15

5. RESULTS AND DISCUSSION

A synthetic and selective presentation of the data sheets (Table 2) has been made, as well as the graphic representation (Figure 4) obtained as a result of integration, for the first variant from sheet 1 which will be analyzed and construed.

As for the others five variants of Table 1, related to the particles trajectories and speed on densimetric table, no significant differences have been recorded from the point of view of the relative motion, only the first variant is hereby presented.

Table 2. Speed projections variation and trajectories coordinates in relation with time for $\varepsilon = 28^0$ and three friction coefficient values μ (selective values).

t(s)	X (mm)	Vx (m/s)	Y (mm)	Vy (m/s)	-Y*) (mm)
μ = 0.05					
0	0	0.25	0	0.25	0
0.1	0.029	0.348	0.023	0.22	-0.023
0.5	0.243	0.713	0.096	0.154	-0.096
0.9	0.599	1.067	0.151	0.124	-0.151
1.3	1.096	1.419	0.197	0.106	-0.197
1.7	1.734	1.772	0.237	0.094	-0.237
1.9	2.106	1.948	0.255	0.089	-0.255
2.0	2.306	2.036	0.264	0.087	-0.264
μ = 0.1					
0.2	0.062	0.359	0.039	0.147	-0.039
0.6	0.239	0.528	0.077	0.062	-0.077
1.0	0.483	0.686	0.095	0.033	-0.095
1.4	0.789	0.844	0.106	0.019	-0.106
1.7	1.060	0.962	0.111	0.014	-0.111
1.9	1.260	1.041	0.113	0.012	-0.113
2.0	1.367	1.081	0.114	0.011	-0.114
μ = 0.15					
0.1	0.026	0.271	0.020	0.161	-0.020
0.5	0.134	0.257	0.048	0.020	-0.048
0.9	0.229	0.221	0.051	0.002	-0.051
1.3	0.311	0.184	0.052	0.0001	-0.052
1.7	0.377	0.147	0.052	0.000003	-0.052
1.9	0.405	0.129	0.052	0	-0.052
2.0	0.417	0.119	0.052	0	-0.052

*) In order to be able to draw the trajectories symmetric curve against axis Ox the values -Y values have been taken into account, which means that the physical position of the machine segregation densimetric table will match with the field of axis of coordinates, where origin O (0.0) represents the feeding point, (see scheme in Figure 2.a)

The data in the above table have been used to represent the graphics in Figure 4 regarding the variation of projection on axis of speed V_x and V_y and trajectories coordinates x and y, for the three values taken into consideration for the friction coefficient μ.

The three values taken into consideration for μ will point out the differentiation between trajectories of different layers by density on the surface of the segregation table of the machine, different densities in accordance with different friction (as shown in the segregation process, on the machine segregation surface).

As per Figure 4 (a, b), considering μ = 0.05 (the lowest value of the ones considered to be plausible for the upper layer with minimum density).

It is noted that as particles move during the segregation process on the densimetric table, the speed decreases from 0.25 m/s to 0.087 m/s remaining stable at this value, and the trajectory components increase more on the Oy axis and remain close to it which means that particles will arrange in proximity of this axis.

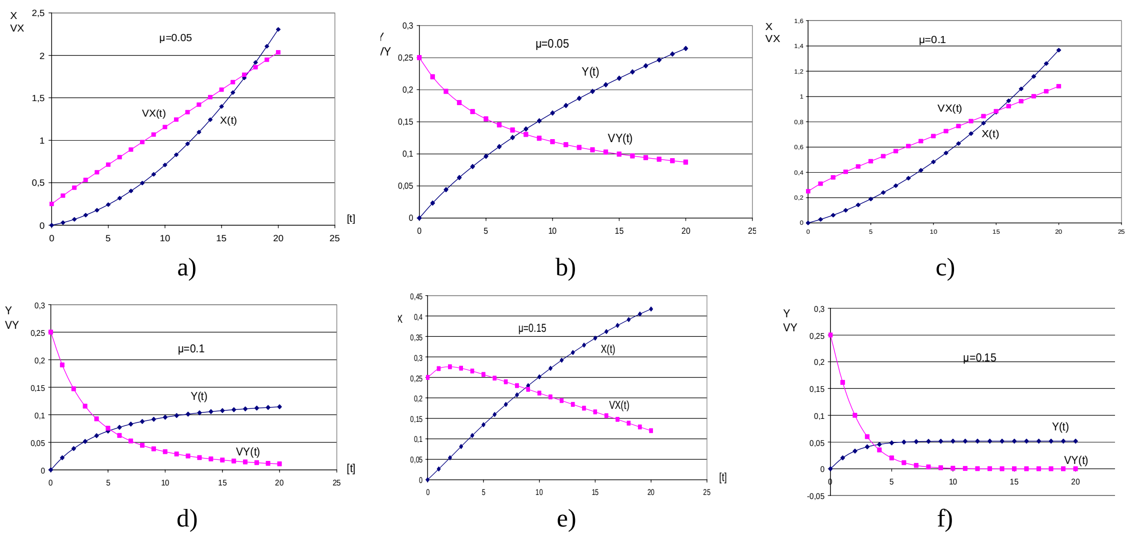


Fig. 4. Particles relative speed variation in relation with time and trajectories projection for:
a) μ = 0.05 on Ox axis; b) μ = 0.05 on Oy axis; c) μ = 0.1 on Ox axis; d) μ = 0.1 on Oy;
e) μ = 0.15 on Ox axis; f) μ = 0.15 on Oy.

An analysis of Figure 4 (c, d) for μ = 0.1 and Figure 4 (e, f) for μ = 0.15, it is found that the projection on Oy axis of the particles relative speed decrease, while the components on Ox increase, which means that their trajectories deviate from Oy axis.

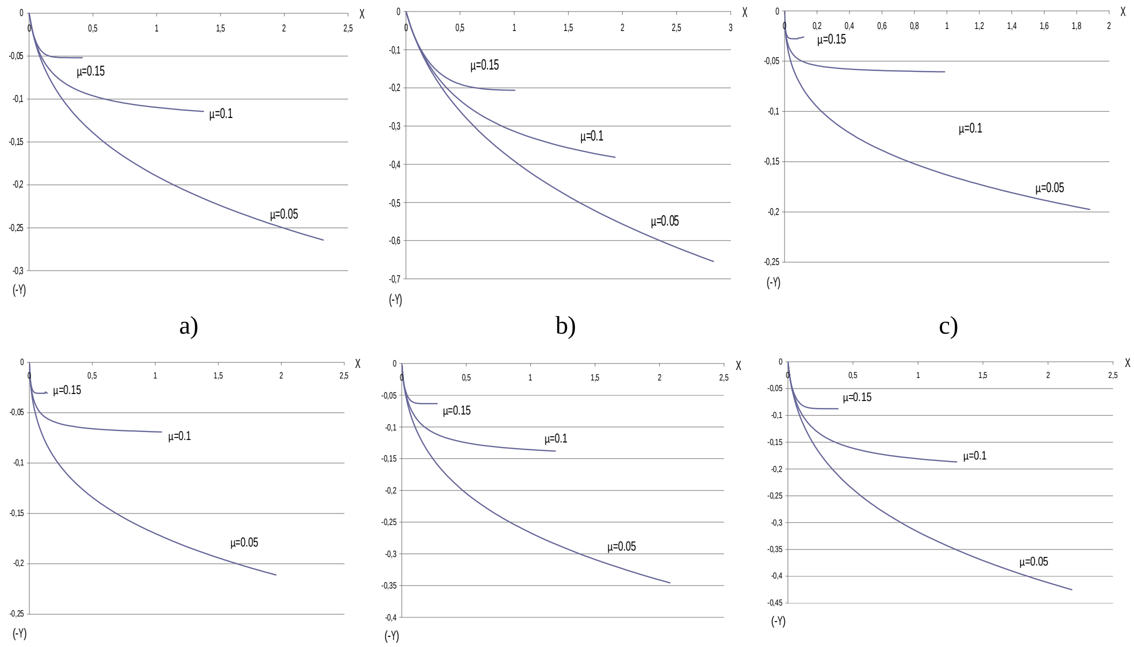
The study of the six graphic representations of Figure 4 indicates that motions and speed decrease simultaneously with a higher friction coefficient in correlation with an increased particles density.

Consequently, the particles with a high friction coefficient, meaning high density, would cover less space on the densimetric table and they are collected closer by the feeding point, whereas particles with lesser friction coefficient, that is lower density, would cover bigger distances on the table and consequently, they are collected far away from the feeding point (Figure 2.b).

The particle trajectories on surface (in their six variants) have been represented in Figure 5 (a-f) [3].

One may notice the particles trajectories corresponding to the three distinct values of the friction coefficients: μ_1=0.05, μ_2=0.1, μ_3=0.15, for all six data variants for integrating differential equations (ε=28^0); the more the friction coefficient increases, to a greater extent the particles trajectory deviates from the Oy axis and they are getting closer to the abscissa which coincides with the high density seeds collection closer to the feeding point, on the right side of the densimetric table, (see Figure 2.b).

Further to an analysis of the six variants by comparison, it has been ascertained that in *c* case and *d* respectively (variant 3 and 4 Table 1), trajectories move away from Oy axis and get closer to the Ox abscissa.



a) b) c) d) e) f)

Fig. 5. Particles relative trajectories on the machine table depending on the friction coefficient μ quality correlated with particles density (small densities-lower friction coefficient; high densities- high friction coefficient) - for $\varepsilon=28^0$; a, b, c, d, e, f, different data variants for integrating differential equations.

6. CONCLUSIONS

The thesis has developed the particles relative motion equations, situated on plane, oscillating, double inclined surfaces (equation 9 and 10) which were arranged in order to be numerically integrated.

Such equations have been solved considering several values, as presented in Table 1.

It is ascertained a net differentiation of the trajectories depending on the differences between the particles' friction coefficients. If we correlate the different values of friction coefficients with the particles density obtained by segregation, then we could correlate collection with density, as follows: seeds with lesser density matching a lower friction coefficient will fall to the base of the table surface, on the left side of the machine, and as density increases, along with the table friction coefficient, the seeds slide on the upper point, on the right side of the machine, following the shortest trajectory on the table (see Figure 1.a).

This differentiation is the main criteria in choosing the place for the collection of the highest density seeds used for sowing, the medium density seeds used for bakery and low density seeds used for livestock. This phenomenon finds practical illustration in experimental researches.

This theoretical analysis has a significant importance in substantiating an appropriate choice of seeds, as well as with regard to the material for pneumatic vibrating table used for segregation, with a practical demonstration by means of experiments performed directly on densimetric machines.

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