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FLOW OF AN UNSTEADY DUSTY FLUID BETWEEN TWO OSCILLATING PLATES UNDER VARYING PULSATILE PRESSURE GRADIENT

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Abstract. An analytical study of unsteady dusty fluid flow between two oscillating plates has been considered. The flow is due to influence of non-torsional oscillations of plates and pulsatile pressure gradient. Flow analysis is carried out using differential geometry techniques and exact solutions of the problem are obtained using Laplace Transform technique. Further graphs drawn for different values of Reynolds number and on basis of these the conclusions are given. Finally, the expressions for skin-friction are obtained at the boundaries.

1. INTRODUCTION

The fluid flow embedded with dust particles is encountered in a wide variety of engineering problems concerned with atmospheric fallout, dust collection, nuclear reactor cooling, powder technology, acoustics, sedimentation, performance of solid fuel rock nozzles, rainerosion, guided missiles and paint spraying etc.

P.G.Saffman [17] has discussed the stability of the laminar flow of a dusty gas in which the dust particles are uniformly distributed. Michael and Miller [12] investigated the motion of dusty gas with uniform distribution of the dust particles occupied in the semi-infinite space above a rigid plane boundary. Marble [11] has applied techniques of fluid mechanics for investigation of two-phase flow of gas and solid particles.

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He introduced the concept of temperature and diameter of the solid particles in his analysis. P.Mitra et.al. [13, 14] have made investigation of various aspects of hydromagnetic dusty fluid flows between two parallel plates. T.M.Nabil [15] studied the effect of couple stresses on pulsatile hydromagnetic poiseuille flow, N.Datta [5] obtained the solutions for pulsatile flow of heat transfer of a dusty fluid through an infinitely long annular pipe. Later, A.Eric [6] has studied the quantitative assessment of steady and pulsatile flow fields in a parallel plate flow chamber. Thierry Feraille and Gregoire Casalis [18] discussed the channel flow induced by wall injection of fluid and particles. These authors studied the different type of flows in only Cartesian and polar coordinate system.

During the second part of 20th century, some researchers like Kanwal [10], Trusdell [19], Indrasena [9], Purushotham [16], Bagewadi, Shantharajappa and Gireesha [1, 2, 3] have applied differential geometry techniques to investigate the kinematical properties of fluid flows in the field of fluid mechanics.

Further, the authors [2,3] have studied two-dimensional dusty fluid flow in Frenet frame field system. Recently the authors [7,8] have studied the flow of unsteady dusty fluid under varying different pressure gradients like constant, periodic and exponential. This paper involves study of flow of dusty fluid between two infinite oscillating plates in anholonomic co-ordinate system. Here we consider as the flow is due to influence of non-torsional oscillations of plates and time dependent pressure gradient. The exact solutions for the fluid and particle velocities are determined by the Laplace Transform method. The graphs drawn for different values of Reynolds number and on basis of these the conclusions are given. Finally, the skin-friction on the plates are then obtained in the closed form.

2. EQUATIONS OF MOTION

The governing equations of motion of unsteady viscous incompressible fluid with uniform distribution of dust particles are [17]:

For fluid phase

(2.1) $\nabla.\overleftarrow{u} = 0$ (Continuity)

(2.2) $$\frac{\partial \overleftarrow{u}}{\partial t} + (\overleftarrow{u}.\nabla)\overleftarrow{u} = -\rho^{-1}\nabla p + \overleftarrow{g} + \nu\nabla^2\overleftarrow{u} + \frac{kN}{\rho}(\overleftarrow{v} - \overleftarrow{u}) \quad \text{(Linear Momentum)}$$

For dust phase

(2.3) $\nabla.\overleftarrow{v} = 0$ (Continuity)

(2.4) $$\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla)\vec{v} = \frac{k}{m}(\vec{u} - \vec{v}) \text{ (Linear Momentum)}$$

We have following nomenclature:

$\overleftarrow{u}$ – velocity of the fluid phase, $\overleftarrow{v}$ – velocity of dust phase, ρ – density of the gas, $\overleftarrow{g} = -\nabla\varphi$, φ – gravitational potential, p – pressure of the fluid, N – number density of dust particles, ν – kinematic viscosity, $k = 6\pi a\mu$ – Stoke's resistance (drag coefficient), a – spherical radius of dust particle, m – mass of the dust particle, μ – the co-efficient of viscosity of fluid particles, t – time.

Let $\overleftarrow{s}, \overleftarrow{n}, \overleftarrow{b}$ be triply orthogonal unit vectors tangent, principal normal, binormal respectively to the spatial curves of congruences formed by fluid phase velocity and dusty phase velocity lines respectively as shown in the figure-1.

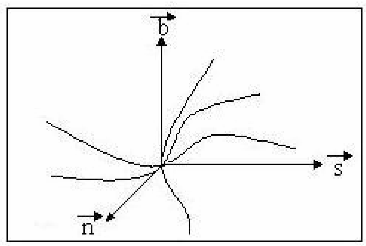

Figure1: Frenet Frame Field System

Geometrical relations are given by Frenet formulae [4]

(2.5)

$$i) \quad \frac{\partial \overleftarrow{s}}{\partial s} = k_s \overleftarrow{n}, \frac{\partial \overleftarrow{n}}{\partial s} = \tau_s \overleftarrow{b} - k_s \overleftarrow{s}, \frac{\partial \overleftarrow{b}}{\partial s} = -\tau_s \overleftarrow{n}$$

$$ii) \quad \frac{\partial \overleftarrow{n}}{\partial n} = k_n' \overleftarrow{s}, \frac{\partial \overleftarrow{b}}{\partial n} = -\sigma_n' \overleftarrow{s}, \frac{\partial \overleftarrow{s}}{\partial n} = \sigma_n' \overleftarrow{b} - k_n' \overleftarrow{n}$$

$$iii) \quad \frac{\partial \overleftarrow{b}}{\partial b} = k_b'' \overleftarrow{s}, \frac{\partial \overleftarrow{n}}{\partial b} = -\sigma_b'' \overleftarrow{s}, \frac{\partial \overleftarrow{s}}{\partial b} = \sigma_b'' \overleftarrow{n} - k_b'' \overleftarrow{b}$$

$$iv) \quad \nabla.\overleftarrow{s} = \theta_{ns} + \theta_{bs}; \nabla.\overleftarrow{n} = \theta_{bn} - k_s; \nabla.\overleftarrow{b} = \theta_{nb}$$

where $\partial/\partial s$, $\partial/\partial n$ and $\partial/\partial b$ are the intrinsic differential operators along fluid phase velocity (or dust phase velocity) lines, principal normal and binormal. The functions (k_s, k_n', k_b'') and $(\tau_s, \sigma_n', \sigma_b'')$ are the curvatures and torsion of the above curves and θ_{ns} and θ_{bs} are normal deformations of these spatial curves along their principal normal and binormal respectively.

3. FORMULATION AND SOLUTION OF THE PROBLEM

Let us consider an unsteady flow of an incompressible viscous fluid with uniform distribution of dust particles between two oscillating plates separated by a distance h under conservative body forces as shown in the figure 2.

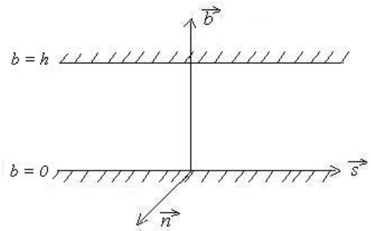

Figure 2: Geometry of the flow.

The flow is due to the influence of non-torsional oscillations of the plates and time dependent pressure gradient. Both the fluid and the dust particle clouds are supposed to be static at the beginning. The dust particles are assumed to be spherical in shape and uniform in size. The number density of the dust particles is taken as a constant throughout the flow. Under these assumptions the flow will be a parallel flow in which the streamlines are along the tangential direction and the velocities are varies along binormal direction and with time t, since we extended the fluid to infinity in the principal normal direction.

For the above described flow the velocities of fluid and dust are of the form

$$\overleftarrow{u} = u_s \overleftarrow{s}, \quad \overleftarrow{v} = v_s \overleftarrow{s}$$

where (u_s, u_n, u_b) and (v_s, v_n, v_b) are velocity components of fluid and dust particles respectively.

By virtue of system of equations (2.5) the intrinsic decomposition of equations (2.2) and (2.4) give the following forms:

$$\frac{\partial u_s}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial s} + \nu\left[\frac{\partial^2 u_s}{\partial b^2} - C_r u_s\right] + \frac{kN}{\rho}(v_s - u_s) \tag{3.1}$$

$$2u_s^2 k_s = -\frac{1}{\rho}\frac{\partial p}{\partial n} + \nu\left[2\sigma_b'' \frac{\partial u_s}{\partial b} - u_s k_s^2\right] \tag{3.2}$$

(3.3) $$0 = -\frac{1}{\rho}\frac{\partial p}{\partial b} + \nu\left[u_s k_s \tau_s - 2k_b''\frac{\partial u_s}{\partial b}\right]$$

(3.4) $$\frac{\partial v_s}{\partial t} = \frac{k}{m}(u_s - v_s)$$

(3.5) $$v_s^2 k_s = 0.$$

where $C_r = (\sigma_n'^2 + k_n'^2 + k_b''^2 + \sigma_b''^2)$ is called curvature number [3].

From equation (3.5) we see that $v_s^2 k_s = 0$ which implies either $v_s = 0$ or $k_s = 0.$ The choice $v_s = 0$ is impossible, since if it happens then $u_s = 0,$ which shows that the flow doesn't exist. Hence $k_s = 0,$ it suggests that the curvature of the streamline along tangential direction is zero. Thus no radial flow exists.

The equations (3.1) and (3.4) are to be solved when subjected to the following initial and boundary conditions;

Initial condition; $at\ t = 0;\ u_s = 0, v_s = 0$

Boundary condition; $for\ t > 0;\ u_s = a_1 e^{i\omega_1 t} + a_2 e^{-i\omega_1 t},\ at\ b = 0\ and$

$$u_s = c_1 e^{i\omega_2 t} + c_2 e^{-i\omega_2 t},\ at\ b = h$$

where a_1, a_2, c_1 and c_2 are complex constants such that u_s becomes real on the plates, ω_1 &ω_2 are frequency of oscillations.

Assumed that a pulsatile pressure gradient is imposed on the system, i.e., for $t > 0$, we can write it as

$$-\frac{1}{\rho}\frac{\partial p}{\partial s} = C + \alpha\cos(\beta t)$$

where C and α are constants and β is the frequency of oscillation.

Let us consider the following non-dimensional flow variables,

$$u_s^* = u_s h/U,\ v_s^* = v_s h/V,\ p^* = ph^2/\rho U^2,\ b^* = b/h,\ t^* = t/(h^2/U);$$

and non dimensional flow parameters,

$$(\sigma_1, \sigma_2) = \frac{h^2}{U}(\omega_1, \omega_2),$$

$$(a_1', a_2', c_1', c_2') = \frac{h}{U}(a_1, a_2, c_1, c_2)$$

where h is the characteristic of length and U is the characteristic of velocity.

Using the above non-dimensional quantities we get the nondimensionalized form of the equations (3.1), (3.4) and the boundary conditions as follows;

$$\frac{\partial u_s}{\partial t} = -\frac{\partial p}{\partial s} + \frac{h}{Re}\frac{\partial^2 u_s}{\partial b^2} - \frac{h^3}{Re}C_r u_s + \frac{kNh^2}{\rho U}(v_s - u_s) \tag{3.6}$$

$$\frac{\partial v_s}{\partial t} = \frac{kh^2}{mU}(u_s - v_s) \tag{3.7}$$

$$u_s = a_1 e^{i\sigma_1 t} + a_2 e^{-i\sigma_1 t}, \text{ at } b = 0 \text{ and}$$

$$u_s = c_1 e^{i\sigma_2 t} + c_2 e^{-i\sigma_2 t}, \text{ at } b = 1, \tag{3.8}$$

where $Re = Uh/\nu$ the Reynold's number.

We define Laplace transformations of u_s and v_s as

$$U_s = \int_0^\infty e^{-xt} u_s dt \text{ and } V_s = \int_0^\infty e^{-xt} v_s dt \tag{3.9}$$

Applying the Laplace transform to equations (3.6), (3.7) and to (3.8), then by using initial conditions one obtains

$$xU_s = \left[\frac{C}{x} + \frac{\alpha x}{x^2+\beta^2}\right] + \frac{h}{Re}\frac{d^2 U_s}{db^2} - \frac{h^3 C_r}{Re}U_s + \frac{h^2 l}{U\tau}(V_s - U_s) \tag{3.10}$$

$$xV_s = \frac{h^2}{U\tau}(U_s - V_s) \tag{3.11}$$

$$U_s = \frac{a_1}{x - i\sigma_1} + \frac{a_2}{x + i\sigma_1} \text{ at } b = 0 \text{ and}$$

$$U_s = \frac{c_1}{x - i\sigma_2} + \frac{c_2}{x + i\sigma_2} \text{ at } b = 1. \tag{3.12}$$

where $l = \frac{mN}{\rho}$ and $\tau = \frac{m}{k}$. Equation (3.11) implies

$$V_s = \frac{h^2}{(h^2 + xU\tau)}U_s \tag{3.13}$$

Eliminating V_s from (3.10) and (3.13) we obtain the following equation

$$\frac{d^2 U_s}{db^2} - Q^2 U_s = -\left[\frac{C}{x} + \frac{\alpha x}{x^2+\beta^2}\right] \tag{3.14}$$

where $Q^2 = h^2 C_r + \frac{xRe}{h}\left[1 + \frac{h^2 l}{(xU\tau + h^2)}\right]$.

The velocities of fluid and dust particle are obtained by solving the equation (3.14) and satisfying the boundary conditions (3.12) as

$$U_s = \frac{1}{Q^2}\left[\frac{c}{s} + \frac{\alpha s}{s^2 + \beta^2}\right]\left\{\frac{\sinh Q(b-h) - \sinh(Qb)}{\sinh(Qh)} + 1\right\} +$$

$$+ \frac{\sinh(Qb)}{\sinh(Q)}\left[\frac{(x + i\sigma_2)c_1 + (x - i\sigma_2)c_2}{x^2 + \sigma_2^2}\right] +$$

$$+ \frac{\sinh Q(b-1)}{\sinh(Q)}\left[\frac{(x + i\sigma_1)a_1 + (x - i\sigma_1)a_2}{x^2 + \sigma_1^2}\right]$$

Using U_s in (3.13) we obtain V_s as

$$V_s = \frac{h^2}{Q^2(h^2 + xU\tau)}\left[\frac{c}{s} + \frac{\alpha s}{s^2 + \beta^2}\right]\left\{\frac{\sinh Q(b-h) - \sinh(Qb)}{\sinh(Qh)} + 1\right\} +$$

$$+ \frac{h^2 \sinh(Qb)}{(h^2 + xU\tau)\sinh(Q)}\left[\frac{(x + i\sigma_2)c_1 + (x - i\sigma_2)c_2}{x^2 + \sigma_2^2}\right] +$$

$$+ \frac{h^2 \sinh Q(b-1)}{(h^2 + xU\tau)\sinh(Q)}\left[\frac{(x + i\sigma_1)a_1 + (x - i\sigma_1)a_2}{x^2 + \sigma_1^2}\right]$$

By taking inverse Laplace transform to U_s and V_s, one can obtain u_s and v_s, as

$$u_s = \left[\frac{c_1[(\psi_1 \cos\sigma_2 t - \psi_2 \sin\sigma_2 t) + i(\psi_2 \cos\sigma_2 t + \psi_1 \sin\sigma_2 t)]}{(A'^2 + B'^2)}\right]$$

$$+ \left[\frac{c_2[(\psi_1 \cos\sigma_2 t - \psi_2 \sin\sigma_2 t) - i(\psi_2 \cos\sigma_2 t + \psi_1 \sin\sigma_2 t)]}{(A'^2 + B'^2)}\right]$$

$$
\begin{aligned}
&- \left[\frac{a_1[(\phi_1\cos\sigma_1 t-\phi_2\sin\sigma_1 t)+i(\phi_2\cos\sigma_1 t+\phi_1\sin\sigma_1 t)]}{(E'^2+F'^2)}\right] \\
&- \left[\frac{a_2[(\phi_1\cos\sigma_1 t-\phi_2\sin\sigma_1 t)-i(\phi_2\cos\sigma_1 t+\phi_1\sin\sigma_1 t)]}{(E'^2+F'^2)}\right] \\
&- \frac{2h\pi}{R_e}\sum_{r=0}^{\infty}(-1)^r r\sin(r\pi b)\left[\frac{e^{x_1 t}[(x_1+i\sigma_2)c_1+(x_1-i\sigma_2)c_2](h^2+x_1U\tau)^2}{(x_1^2+\sigma_2^2)[(h^2+x_1U\tau)^2+lh^3]}\right. \\
&\quad \left.+\frac{e^{x_2 t}[(x_2+i\sigma_2)c_1+(x_2-i\sigma_2)c_2](h^2+x_2U\tau)^2}{(x_2^2+\sigma_2^2)[(h^2+x_2U\tau)^2+lh^3]}\right] \\
&+ \frac{2h\pi}{R_e}\sum_{r=0}^{\infty}(-1)^r r\sin((b-1)r\pi)\left[\frac{e^{x_1 t}[(x_1+i\sigma_1)a_1+(x_1-i\sigma_1)a_2](h^2+x_1U\tau)^2}{(x_1^2+\sigma_1^2)[(h^2+x_1U\tau)^2+lh^3]}\right. \\
&\quad \left.+\frac{e^{x_2 t}[(x_2+i\sigma_1)a_1+(x_2-i\sigma_1)a_2](h^2+x_2U\tau)^2}{(x_2^2+\sigma_1^2)[(h^2+x_2U\tau)^2+lh^3]}\right] \\
&+ \left[\frac{\alpha[X_1\cos\beta t-X_2\sin\beta t]}{(k_5^2+k_6^2)(G'^2+H'^2)}\right]+\frac{4\alpha h}{\pi R_e}\sum_{r=0}^{\infty}\frac{\sin(2r+1)\pi b}{2r+1} \\
&\quad \times\left[\frac{x_1 e^{x_1 t}(x_1U\tau+h^2)^2}{(x_1^2+\beta^2)((x_1U\tau+h^2)^2+h^3 l)}+\frac{x_2 e^{x_2 t}(x_2U\tau+h^2)^2}{(x_2^2+\beta^2)((x_2U\tau+h^2)^2+h^3 l)}\right] \\
&+ \frac{4Ch}{\pi R_e}\sum_{r=0}^{\infty}\frac{\sin(2r+1)\pi b}{2r+1}\left[\frac{e^{x_1 t}(x_1U\tau+h^2)^2}{x_1((x_1U\tau+h^2)^2+h^3 l)}+\frac{e^{x_2 t}(x_2U\tau+h^2)^2}{x_2((x_2U\tau+h^2)^2+h^3 l)}\right] \\
&+ \frac{C}{M^2}\left[\frac{\sinh M(b-1)-\sinh Mb+\sinh M}{\sinh M}\right]
\end{aligned}
$$

$$
\begin{aligned}
v_s \;=\; & \left[\frac{h^2 c_1[(\theta_1 \cos\,\sigma_2 t - \theta_2 \sin\,\sigma_2 t) + i(\theta_2 \cos\,\sigma_2 t + \theta_1 \sin\,\sigma_2 t)]}{(A'^2 + B'^2)(h^2 + (\sigma_2 U\tau)^2)}\right] \\
& + \left[\frac{h^2 c_2[(\theta_1 \cos\,\sigma_2 t - \theta_2 \sin\,\sigma_2 t) - i(\theta_2 \cos\,\sigma_2 t + \theta_1 \sin\,\sigma_2 t)]}{(A'^2 + B'^2)(h^2 + (\sigma_2 U\tau)^2)}\right] \\
& - \left[\frac{h^2 a_1[(\lambda_1 \cos\sigma_1 t - \lambda_2 \sin\sigma_1 t) + i(\lambda_2 \cos\sigma_1 t + \lambda_1 \sin\sigma_1 t)]}{(E'^2 + F'^2)(h^2 + (\sigma_1 U\tau)^2)}\right] \\
& - \left[\frac{h^2 a_2[(\lambda_1 \cos\sigma_1 t - \lambda_2 \sin\sigma_1 t) - i(\lambda_2 \cos\sigma_1 t + \lambda_1 \sin\sigma_1 t)]}{(E'^2 + F'^2)(h^2 + (\sigma_1 U\tau)^2)}\right] \\
& - \frac{2h^3\pi}{R_e}\sum_{r=0}^{\infty}(-1)^r r\sin(r\pi b)\left[\frac{e^{x_1 t}[(x_1 + i\sigma_2)c_1 + (x_1 - i\sigma_2)c_2](h^2 + x_1 U\tau)}{(x_1^2 + \sigma_2^2)[(h^2 + x_1 U\tau)^2 + lh^3]}\right. \\
& \left. + \frac{e^{x_2 t}[(x_2 + i\sigma_2)c_1 + (x_2 - i\sigma_2)c_2](h^2 + x_2 U\tau)}{(x_2^2 + \sigma_2^2)[(h^2 + x_2 U\tau)^2 + lh^3]}\right] \\
& + \frac{2h^3\pi}{R_e}\sum_{r=0}^{\infty}(-1)^r r\sin((b-1)r\pi)\left[\frac{e^{x_1 t}[(x_1 + i\sigma_1)a_1 + (x_1 - i\sigma_1)a_2](h^2 + x_1 U\tau)}{(x_1^2 + \sigma_1^2)[(h^2 + x_1 U\tau)^2 + lh^3]}\right. \\
& \left. + \frac{e^{x_2 t}[(x_2 + i\sigma_1)a_1 + (x_2 - i\sigma_1)a_2](h^2 + x_2 U\tau)^2}{(x_2^2 + \sigma_1^2)[(h^2 + x_2 U\tau)^2 + lh^3]}\right] \\
& + \left[\frac{\alpha[X_1 \cos\beta t - X_2 \sin\beta t]}{(k_5^2 + k_6^2)(G'^2 + H'^2)}\right] + \frac{4\alpha h}{\pi R_e}\sum_{r=0}^{\infty}\frac{\sin(2r+1)\pi b}{2r+1} \\
& \times\left[\frac{x_1 e^{x_1 t}(x_1 U\tau + h^2)^2}{(x_1^2 + \beta^2)((x_1 U\tau + h^2)^2 + h^3 l)} + \frac{x_2 e^{x_2 t}(x_2 U\tau + h^2)^2}{(x_2^2 + \beta^2)((x_2 U\tau + h^2)^2 + h^3 l)}\right] \\
& + \frac{4Ch}{\pi R_e}\sum_{r=0}^{\infty}\frac{\sin(2r+1)\pi b}{2r+1}\left[\frac{e^{x_1 t}(x_1 U\tau + h^2)^2}{x_1((x_1 U\tau + h^2)^2 + h^3 l)} + \frac{e^{x_2 t}(x_2 U\tau + h^2)^2}{x_2((x_2 U\tau + h^2)^2 + h^3 l)}\right]
\end{aligned}
$$

$$+\frac{C}{M^2}\left[\frac{\sinh M(b-1)-\sinh Mb+\sinh M}{\sinh M}\right]$$

$$v_s = \left[\frac{h^2 c_1[(\theta_1\cos\sigma_2 t-\theta_2\sin\sigma_2 t)+i(\theta_2\cos\sigma_2 t+\theta_1\sin\sigma_2 t)]}{(A'^2+B'^2)(h^2+(\sigma_2 U\tau)^2)}\right]$$

$$+\left[\frac{h^2 c_2[(\theta_1\cos\sigma_2 t-\theta_2\sin\sigma_2 t)-i(\theta_2\cos\sigma_2 t+\theta_1\sin\sigma_2 t)]}{(A'^2+B'^2)(h^2+(\sigma_2 U\tau)^2)}\right]$$

$$-\left[\frac{h^2 a_1[(\lambda_1\cos\sigma_1 t-\lambda_2\sin\sigma_1 t)+i(\lambda_2\cos\sigma_1 t+\lambda_1\sin\sigma_1 t)]}{(E'^2+F'^2)(h^2+(\sigma_1 U\tau)^2)}\right]$$

$$-\left[\frac{h^2 a_2[(\lambda_1\cos\sigma_1 t-\lambda_2\sin\sigma_1 t)-i(\lambda_2\cos\sigma_1 t+\lambda_1\sin\sigma_1 t)]}{(E'^2+F'^2)(h^2+(\sigma_1 U\tau)^2)}\right]$$

$$-\frac{2h^3\pi}{R_e}\sum_{r=0}^{\infty}(-1)^r r\sin(r\pi b)\left[\frac{e^{x_1 t}[(x_1+i\sigma_2)c_1+(x_1-i\sigma_2)c_2](h^2+x_1U\tau)}{(x_1^2+\sigma_2^2)[(h^2+x_1U\tau)^2+lh^3]}\right.$$

$$\left.+\frac{e^{x_2 t}[(x_2+i\sigma_2)c_1+(x_2-i\sigma_2)c_2](h^2+x_2U\tau)}{(x_2^2+\sigma_2^2)[(h^2+x_2U\tau)^2+lh^3]}\right]$$

$$+\frac{2h^3\pi}{R_e}\sum_{r=0}^{\infty}(-1)^r r\sin((b-1)r\pi)\left[\frac{e^{x_1 t}[(x_1+i\sigma_1)a_1+(x_1-i\sigma_1)a_2](h^2+x_1U\tau)}{(x_1^2+\sigma_1^2)[(h^2+x_1U\tau)^2+lh^3]}\right.$$

$$\left.+\frac{e^{x_2 t}[(x_2+i\sigma_1)a_1+(x_2-i\sigma_1)a_2](h^2+x_2U\tau)}{(x_2^2+\sigma_1^2)[(h^2+x_2U\tau)^2+lh^3]}\right]$$

$$+\left[\frac{\alpha h^2[(X_1h^2+X_2\beta U\tau)\cos\beta t-(X_2h^2-X_1\beta U\tau)\sin\beta t]}{(k_5^2+k_6^2)(G'^2+H'^2)(h^4+(\beta U\tau)^2)}\right]+\frac{4\alpha h^3}{\pi R_e}\sum_{r=0}^{\infty}\frac{\sin(2r+1)\pi b}{2r+1}$$

$$\times\left[\frac{x_1e^{x_1 t}(x_1U\tau+h^2)}{(x_1^2+\beta^2)((x_1U\tau+h^2)^2+h^3 l)}+\frac{x_2e^{x_2 t}(x_2U\tau+h^2)}{(x_2^2+\beta^2)((x_2U\tau+h^2)^2+h^3 l)}\right]$$

$$+\frac{4Ch^3}{\pi R_e}\sum_{r=0}^{\infty}\frac{\sin(2r+1)\pi b}{2r+1}\left[\frac{e^{x_1 t}(x_1U\tau+h^2)}{x_1((x_1U\tau+h^2)^2+h^3 l)}+\frac{e^{x_2 t}(x_2U\tau+h^2)}{x_2((x_2U\tau+h^2)^2+h^3 l)}\right]$$

$$+\frac{C}{M^2}\left[\frac{\sinh M(b-1)-\sinh Mb+\sinh M}{\sinh M}\right].$$

The results of u_s and v_s describe the fluid and particle velocities respectively for the general cases. For instance,

• if $a_1 = a_2 = 0$ & $c_1 = c_2 = u_1/2$ and $\omega_2 = \omega$ the we can obtain the results for the lower plate is fixed and upper plate is moving $(u_1 \cos\omega t)$ i.e., the generalized Couette flow,

• if $a_1 = a_2 = 0$ & $(c_1, c_2) = (u_1/2i, -u_1/2i)$ and $\omega_2 = \omega$ then one can obtain the velocity profile for the flow due to the movement of upper plate $(u_1 \sin\omega t$, lower plate is fixed) i.e., the generalized Couette flow,

• if $(a_1, a_2) = (u_0/2i, -u_0/2i)$ & $c_1 = c_2 = 0$ and $\omega_1 = \omega$ then we obtain the velocity profile for the flow due to the movement of lower plate $(u_0 \sin\omega t$, upper plate is fixed) i.e., the generalized Couette flow,

• if $(a_1, a_2) = (u_0/2i, -u_0/2i)$ & $(c_1, c_2) = (u_1/2i, -u_1/2i)$ and $\omega_1 = \omega_2 = \omega$ i.e., both lower and upper plates are moving with the oscillations $u_0 \sin\omega t,\ u_1 \sin\omega t$ respectively

• if $\omega_1 = \omega_2 = 0$ then both lower and upper plates are moving with uniform velocity.

All the above results are very similar with those of Mitra et. al. [13,14].

Shearing Stress (Skin Friction): The Shear stress at the boundaries $b = 0$ and $b = 1$ are given by

$$D_0 = \mu c_1\left[\frac{\cos\sigma_2 t[(A'\alpha_1 + B'\beta_1) + i(A'\beta_1 - B'\alpha_1)] - \sin\sigma_2 t[(A'\beta_1 - B'\alpha_1) - i(A'\alpha_1 + B'\beta_1)]}{(A'^2 + B'^2)}\right]$$

$$+\mu c_2\left[\frac{\cos\sigma_2 t[(A'\alpha_1 + B'\beta_1) - i(A'\beta_1 - B'\alpha_1)] - \sin\sigma_2 t[(A'\beta_1 - B'\alpha_1) + i(A'\alpha_1 + B'\beta_1)]}{(A'^2 + B'^2)}\right]$$

$$-\mu a_1\left[\frac{\cos\sigma_1 t[(E'\delta_1 + F'\delta_2) + i(E'\delta_2 - F'\delta_1)] - \sin\sigma_1 t[(E'\delta_2 - F'\delta_1) - i(E'\delta_1 + F'\delta_2)]}{(E'^2 + F'^2)}\right]$$

$$-\mu a_2\left[\frac{\cos\sigma_1 t[(E'\delta_1 + F'\delta_2) - i(E'\delta_2 - F'\delta_1)] - \sin\sigma_1 t[(E'\delta_2 - F'\delta_1) + i(E'\delta_1 + F'\delta_2)]}{(E'^2 + F'^2)}\right]$$

$$-\frac{2\mu h\pi^2}{R_e}\sum_{r=0}^{\infty} r^2\left[\frac{e^{x_1 t}[(x_1 + i\sigma_2)c_1 + (x_1 - i\sigma_2)c_2](h^2 + x_1 U\tau)^2}{(x_1^2 + \sigma_2^2)[(h^2 + x_1 U\tau)^2 + lh^3]}\right.$$

$$\left.+\frac{e^{x_2 t}[(x_2 + i\sigma_2)c_1 + (x_2 - i\sigma_2)c_2](h^2 + x_2 U\tau)^2}{(x_2^2 + \sigma_2^2)[(h^2 + x_2 U\tau)^2 + lh^3]}\right]$$

$$+\frac{2\mu h\pi^2}{R_e}\sum_{r=0}^{\infty} r^2\left[\frac{e^{x_1 t}[(x_1+i\sigma_1)a_1+(x_1-i\sigma_1)a_2](h^2+x_1U\tau)^2}{(x_1^2+\sigma_1^2)[(h^2+x_1U\tau)^2+lh^3]}\right.$$

$$\left.+\frac{e^{x_2 t}[(x_2+i\sigma_1)a_1+(x_2-i\sigma_1)a_2](h^2+x_2U\tau)^2}{(x_2^2+\sigma_1^2)[(h^2+x_2U\tau)^2+lh^3]}\right]$$

$$+\frac{\mu\omega h}{(k_5^2+k_6^2)(G'^2+H'^2)}[-(k_5M_3+K_6M_4)\cos\beta t+(k_5M_4-k_6M^3)\sin\beta t]$$

$$+\frac{4\mu\omega h}{R_e}\sum_{r=0}^{\infty}\left[\frac{e^{x_1 t}(x_1U\tau+h^2)^2}{(x_1^2+\beta^2)((x_1U\tau+h^2)^2+h^3l)}+\frac{e^{x_2 t}(x_2U\tau+h^2)^2}{(x_2^2+\beta^2)((x_2U\tau+h^2)^2+h^3l)}\right]$$

$$+\frac{4\mu Ch}{R_e}\sum_{r=0}^{\infty}\left[\frac{e^{x_1 t}(x_1U\tau+h^2)^2}{x_1((x_1U\tau+h^2)^2+h^3l)}+\frac{e^{x_2 t}(x_2U\tau+h^2)^2}{x_2((x_2U\tau+h^2)^2+h^3l)}\right]+\mu C\left[\frac{(\cosh M-1)}{M\sinh M}\right]$$

$$D_1=\frac{\mu c_1}{(A'^2+B'^2)}\left[-\cos\sigma_2 t[(A'M_5+B'M_6)+i(A'M_6-B'M_5)]\right.$$

$$\left.-\sin\sigma_2 t[(A'M_6-B'M_5)-i(A'M_5+B'M_6)]-\right]$$

$$+\frac{\mu c_2}{(A'^2+B'^2)}\left[-\cos\sigma_2 t[(A'M_5+B'M_6)-i(A'M_6-B'M_5)]\right.$$

$$\left.-\sin\sigma_2 t[(A'M_6-B'M_5)+i(A'M_5+B'M_6)]-\right]$$

$$-\mu a_1\left[\frac{\cos\sigma_1 t[(E'\delta_1+F'\delta_2)+i(E'\delta_2-F'\delta_1)]-\sin\sigma_1 t[(E'\delta_2-F'\delta_1)-i(E'\delta_1+F'\delta_2)]}{(E'^2+F'^2)}\right]$$

$$-\mu a_2\left[\frac{\cos\sigma_1 t[(E'\delta_1+F'\delta_2)-i(E'\delta_2-F'\delta_1)]-\sin\sigma_1 t[(E'\delta_2-F'\delta_1)+i(E'\delta_1+F'\delta_2)]}{(E'^2+F'^2)}\right]$$

$$-\frac{2\mu h\pi^2}{R_e}\sum_{r=0}^{\infty} r^2\left[\frac{e^{x_1 t}[(x_1+i\sigma_2)c_1+(x_1-i\sigma_2)c_2](h^2+x_1U\tau)^2}{(x_1^2+\sigma_2^2)[(h^2+x_1U\tau)^2+lh^3]}\right.$$

$$+\frac{e^{x_2 t}[(x_2+i\sigma_2)c_1+(x_2-i\sigma_2)c_2](h^2+x_2U\tau)^2}{(x_2^2+\sigma_2^2)[(h^2+x_2U\tau)^2+lh^3]}\Bigg]$$

$$+\frac{2\mu h\pi^2}{R_e}\sum_{r=0}^{\infty}r^2\Bigg[\frac{e^{x_1 t}[(x_1+i\sigma_1)a_1+(x_1-i\sigma_1)a_2](h^2+x_1U\tau)^2}{(x_1^2+\sigma_1^2)[(h^2+x_1U\tau)^2+lh^3]}$$

$$+\frac{e^{x_2 t}[(x_2+i\sigma_1)a_1+(x_2-i\sigma_1)a_2](h^2+x_2U\tau)^2}{(x_2^2+\sigma_1^2)[(h^2+x_2U\tau)^2+lh^3]}\Bigg]$$

$$+\frac{\mu\alpha h}{(k_5^2+k_6^2)(G'^2+H'^2)}[-(k_5M_3+K_6M_4)\cos\beta t+(k_5M_4-k_6M^3)\sin\beta t]$$

$$-\frac{4\mu\alpha h}{R_e}\sum_{r=0}^{\infty}\left[\frac{e^{x_1 t}(x_1U\tau+h^2)^2}{(x_1^2+\beta^2)((x_1U\tau+h^2)^2+h^3l)}+\frac{e^{x_2 t}(x_2U\tau+h^2)^2}{(x_2^2+\beta^2)((x_2U\tau+h^2)^2+h^3l)}\right]$$

$$-\frac{4\mu Ch}{R_e}\sum_{r=0}^{\infty}\left[\frac{e^{x_1 t}(x_1U\tau+h^2)^2}{x_1((x_1U\tau+h^2)^2+h^3l)}+\frac{e^{x_2 t}(x_2U\tau+h^2)^2}{x_2((x_2U\tau+h^2)^2+h^3l)}\right]+\mu C\left[\frac{(\cosh M-1)}{M\sinh M}\right]$$

where

$$\begin{aligned}
A &= \sinh\alpha_1 b\cos\beta_1 b,\ B=\cosh\alpha_1 b\sin\beta_1 b,\ A'=\sinh\alpha_1\cos\beta_1\\
B' &= \cosh\alpha_1\sin\beta_1,\ E=\sinh\delta_1(b-1)\cos\delta_2(b-1),\\
F &= \cosh\delta_1(b-1)\sin\delta_2(b-1),\ E'=\sinh\delta_1\cos\delta_2,\ F'=\cosh\delta_1\sin\delta_2\\
\psi_1 &= AA'+BB',\ \psi_2=BA'-AB',\ \phi_1=EE'+FF',\ \phi_2=FE'-EF',\\
\theta_1 &= h^2\psi_1+\sigma_2U\tau\psi_2,\ \theta_2=h^2\psi_2-\sigma_2U\tau\psi_1,\\
\lambda_1 &= h^2\phi_1+\sigma_1U\tau\phi_2,\ \lambda_2=h^2\phi_2-\sigma_2U\tau\phi_1,\\
k_1 &= h^2C_r+\left[\frac{\sigma_2^2R_e hU\tau l}{h^4+(\sigma_2U\tau)^2}\right],\ k_2=\frac{\sigma_2R_e(h^4(1+l)+\sigma_2^2U^2\tau^2)}{h(h^4+(\sigma_2U\tau)^2)},\\
\alpha_1 &= \frac{\sqrt{k_1+\sqrt{k_1^2+k_2^2}}}{2},\ \beta_1=\frac{\sqrt{-k_1+\sqrt{k_1^2+k_2^2}}}{2},\ k_3=h^2C_r+\left[\frac{\sigma_1^2R_e hU\tau l}{h^4+(\sigma_1U\tau)^2}\right],
\end{aligned}$$

$$k_4 = \frac{\sigma_1 R_e (h^4(1+l) + \sigma_1^2 U^2 \tau^2)}{h(h^4 + (\sigma_1 U\tau)^2)}, \delta_1 = \frac{\sqrt{k_3 + \sqrt{k_3^2 + k_4^2}}}{2}, \delta_2 = \frac{\sqrt{-k_3 + \sqrt{k_3^2 + k_4^2}}}{2},$$

$$k_5 = h^2 C_r + \left[\frac{\beta^2 R_e h U\tau l}{h^4 + (\beta U\tau)^2}\right], k_6 = \frac{\beta R_e (h^4(1+l) + \beta^2 U^2 \tau^2)}{h(h^4 + (\beta U\tau)^2)},$$

$$\mu_1 = \frac{\sqrt{k_5 + \sqrt{k_5^2 + k_6^2}}}{2}, \mu_2 = \frac{\sqrt{-k_5 + \sqrt{k_5^2 + k_6^2}}}{2},$$

$$G = \sinh(b-1)\mu_1 \cos(b-1)\mu_2 - \sinh\mu_1 b \cos\mu_2 b + \sinh\mu_1 \cos\mu_2,$$

$$H = \cosh(b-1)\mu_1 \sin(b-1)\mu_2 - \cosh\mu_1 b \sin\mu_2 b + \cosh\mu_1 \sin\mu_2,$$

$$G' = \sinh\mu_1 \cos\mu_2, H' = \cosh\mu_1 \sin\mu_2, M = \sqrt{h^2 C_r}$$

$$X_1 = k_5(GG' + HH') + k_6(G'H - H'G), X_2 = k_5(HG' - H'G) - k_6(GG' + HH')$$

$$x_1 = -\frac{1}{2R_e U\tau}\left((h^2 C_r + r^2\pi^2)hU\tau + R_e(1+l)h^2\right) + \frac{1}{2R_e U\tau}\sqrt{\left((h^2 C_r + r^2\pi^2)hU\tau + R_e(1+l)h^2\right)^2 - 4R_e U\tau h^3\left(h^2 C_r + r^2\pi^2\right)},$$

$$x_2 = -\frac{1}{2R_e U\tau}\left((h^2 C_r + r^2\pi^2)hU\tau + R_e(1+l)h^2\right) - \frac{1}{2R_e U\tau}\sqrt{\left((h^2 C_r + r^2\pi^2)hU\tau + R_e(1+l)h^2\right)^2 - 4R_e U\tau h^3\left(h^2 C_r + r^2\pi^2\right)}$$

CONCLUSION

The figures 3 to 12 represents the velocity profiles for the fluid and dust particles respectively, which are parabolic in nature. It is observed that the path of fluid particles much steeper than that of dust particles. Further one can observe that if the dust is very fine i.e., mass of the dust particles is negligibly small then the relaxation time of dust particle decreases and ultimately as $\tau \to 0$ the velocities of fluid and dust particles will be the same. Also we see that the fluid particles will reach the steady state earlier than the dust particles. We know that Reynolds number (Re) means the inertial force to viscous force. From graphs one can observe the impressive effect of Reynolds number on the velocity field. It is seen that the Reynolds number is favorable to the velocity fields i.e., for a constant value $t,$ the velocity profiles for both fluid and dust particles increases as Reynolds number increases.

The graphs are drawn for the following values

$h=1,\ \beta=1,\ C=2,\ \alpha=2,\ \tau=0.5,\ C_r=1,\ u_0=1,\ \sigma_1,\sigma_2=2,\ U=1,$
$l=1,\ t=0.2.$

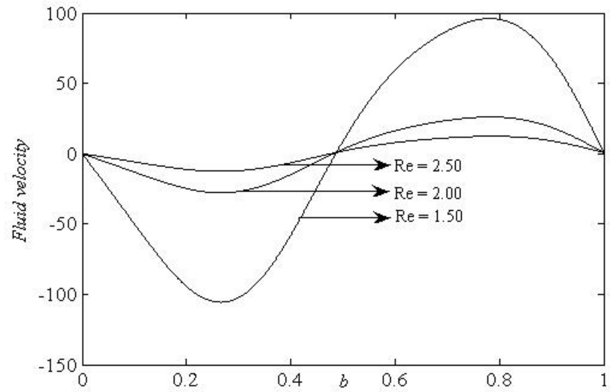

Figure 3: Variation of fluid velocity with b for $a_1=a_2=0$ & $c_1=c_2=u_1/2$

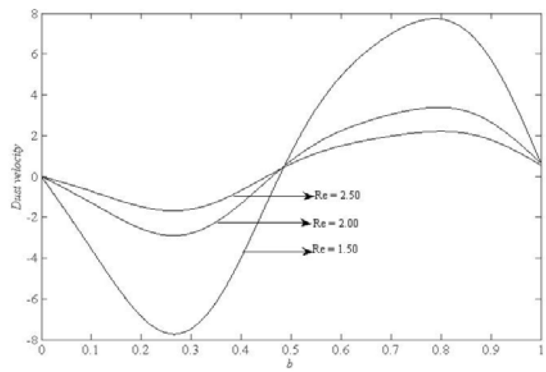

Figure 4: Variation of particle velocity with b for
$a_1=a_2=0$ & $c_1=c_2=u_1/2$

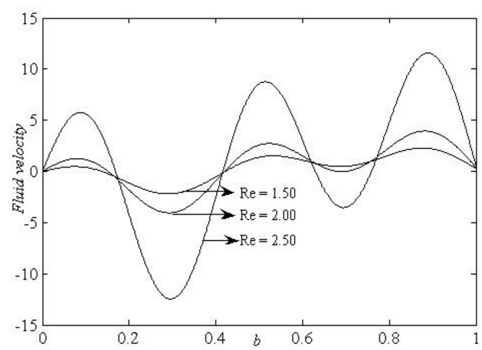

Figure 5: Variation of fluid velocity with b for
$a_1=a_2=0$ & $(c_1,c_2)=(u_1/2i,-u_1/2i)$

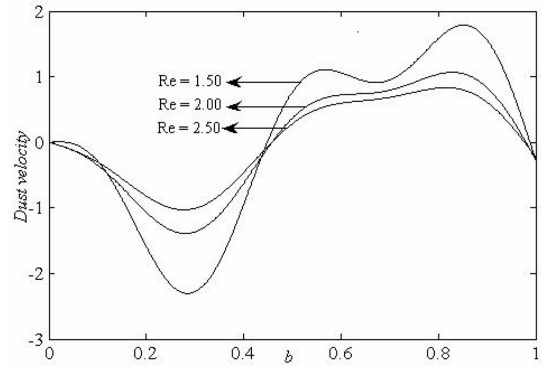

Figure 6: Variation of particle velocity with b for $a_1 = a_2 = 0$ & $(c_1, c_2) = (u_1/2i, -u_1/2i)$

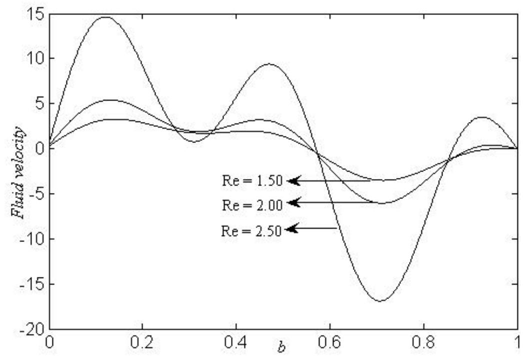

Figure 7: Variation of fluid velocity with b for $(a_1, a_2) = (u_0/2i, -u_0/2i)$ & $c_1 = c_2 = 0$

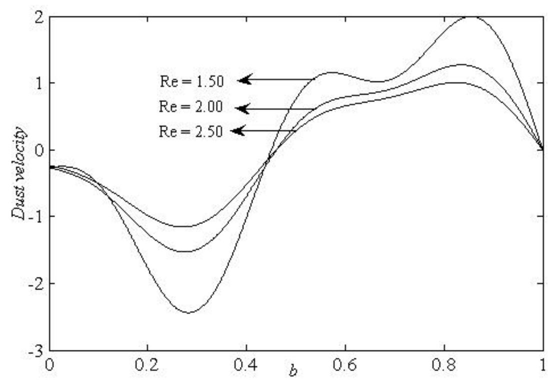

Figure 8: Variation of particle velocity with b for $(a_1, a_2) = (u_0/2i, -u_0/2i)$ & $c_1 = c_2 = 0$

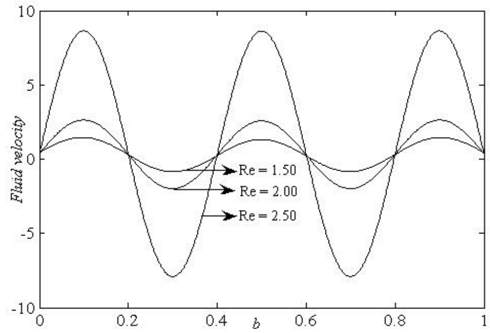

Figure 9: Variation of fluid velocity with b for
$(a_1, a_2) = (u_0/2i, -u_0/2i)$ & $(c_1, c_2) = (u_1/2i, -u_1/2i)$

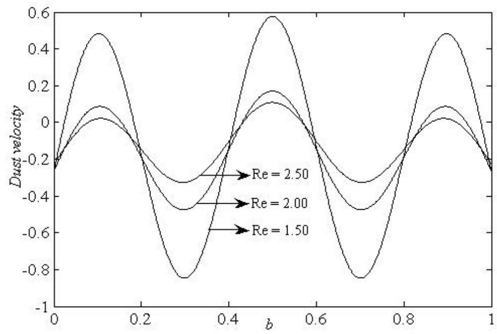

Figure 10: Variation of particle velocity with b for
$(a_1, a_2) = (u_0/2i, -u_0/2i)$ & $(c_1, c_2) = (u_1/2i, -u_1/2i)$

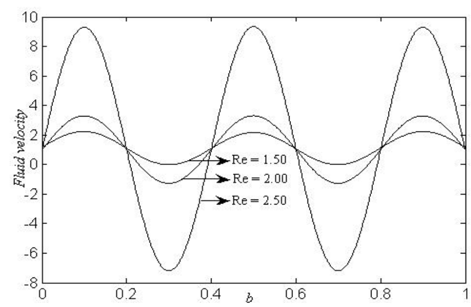

Figure 11: Variation of fluid velocity with b for
$a_1 = a_2 = u_0/2$ & $c_1 = c_2 = u_1/2$

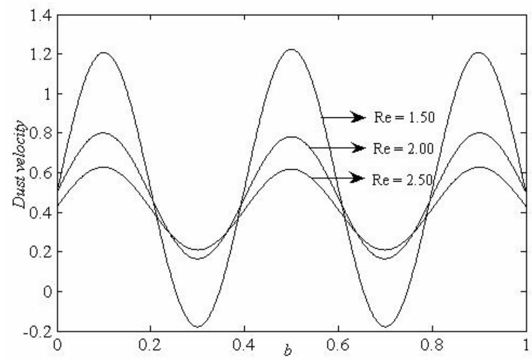

Figure 12: Variation of particle velocity with b for
$a_1 = a_2 = u_0/2$ & $c_1 = c_2 = u_1/2$

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