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Scientific Studies and Research
Series Mathematics and Informatics
Vol. 20 (2010), No. 1, 241 - 252

A NOTE ON ALMOST s -CONTINUITY

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Abstract. T. Noiri, M. B. Ahmad and M. Khan introduced the notion of almost s-continuous functions [20] since then the function studied by various authors [1,4,11] Continuing in the spirit of this papers we obtain several properties and new characterizations of almost s -continuous functions. We improve and strengthen some of the known results. The concept of *co-SR*-closed graph is introduced.

1. Introduction and Preliminaries

Almost s-continuous functions being both quasi-irresolute and almost continuous, introduced by T. Noiri, M. B. Ahmad and M. Khan [20]. In [4] Dontchev, Ganster and Reilly introduced quasi-open sets and they related ultra Hausdorffness and almost s-continuity. Note that ultra Hausdorffness implies totally disconnectedness [25]. So almost s-continuity can be considered as a tool for studying various disconnectedness properties. Almost s-continuity generate clopen sets from semi-regular sets under the inverse image so this function equivalent to γ-continuity introduced by Ganguly and Basu [9] in 1992.

Keywords and phrases: TAlmost s-continuous functions, semi-open, semi-regular sets, clopen sets, quasi-open sets, *ultra Hausdorff spaces*, *co*-SR-closed graphs.
(2000)Mathematics Subject Classification: 54C08

Quasi-open sets can be used for characterizing almost s-continuity. Since every clopen set is quasi-open this improves some of the almost s-continuity characterizations. In [1] Cho, studied net characterizations of almost s-continuity, we strengthened and extended his results using clopen sets. In the same manner using co-SR-closed graphs instead of almost s-closed graphs some results of Jafari and Noiri [11] are improved. In addition using almost s-continuity and co-SR-closed graphs, ultra Hausdorff spaces are characterized.

Throughout this paper (X, τ) and (Y, σ) (or simply X and Y) represent nonempty topological spaces on which no separation axioms are assumed, unless otherwise mentioned. For a subset S of (X, τ), $cl(S)$ and $int(S)$ represent the closure of S and the interior of S, respectively. A subset S of a space (X, τ) is said to be semi-open [16] if $S \subset cl(int(S))$. The family of all semi-open sets of X is denoted by $SO(X)$. The complement of a semi-open set is said to be semi-closed. The semiclosure of S, denoted by $scl(S)$, is the intersection of all semi closed sets containing S. The family of all semi-closed sets of X is denoted by $SC(X)$. A subset S of a space (X, τ) is said to be semi-regular [14] if it is both semiopen as well as semi-closed. The family of all semi-regular sets of a space X and that containing a point x of X are respectively denoted by $SR(X)$ and $SR(X, x)$. A point $x \in X$ is said to be in the semi-θ-closure [14] of A, denoted by $scl_\theta(A)$, if $A \cap scl(V) \neq \emptyset$ for every $V \in SO(X, x)$. If $scl_\theta(A) = A$, then A is said to be semi-θ-closed. The complement of a semi-θ-closed set is said to be semi-θ-open.

The quasi-component [25] of a point $x \in X$ is the intersection of all clopen subsets of X which contain the point x. The quasi-topology τ_q on X is the topology having as base clopen subsets of (X, τ). The closure of each point in quasi-topology is precisely the quasi-component of that point. The open (resp. closed) subsets of the quasi-topology is called quasi-open [4] (resp. quasi-closed [4]). For a space (X, τ) the space (X, τ_q) is called by Staum [25] the ultraregular kernel of X and denoted by X_q for simplicity. A space (X, τ) is called ultraregular [25] if $\tau = \tau_q$.For a subset A of a space X, we define the quasi-interior (resp. quasi-closure) of A, denoted by $int_q(A)$ (resp. $cl_q(A)$), defined by $int_q(A) = \cup\{U$ is quasi-open:$U \subset A\}$,(resp. $cl_q(A) = \cap\{F$ is quasi-closed:$A \subset F\}$).

Lemma 1. *[9] Let A be a subset of a space X.*

(a): *If* $A \in SO(X)$ *then* $scl(A) \in SR(X)$ *and* $scl(A) = scl_\theta(A)$*;*
(b): *If* $A \in SR(X)$ *then* $X - A \in SR(X)$;

Lemma 2. *[6,14,20] Let* A *be a subset of a space* X*, the following statements are equivalent:*

(a): $A \in SR(X)$;
(b): $A = scl(sint(A))$;
(c): $A = sint(scl(A))$;
(d): A is semi-θ-closed and semi-θ-open.

Definition 3. *A function* $f : X \to Y$ *is said to be almost* s*-continuous [20] if for each point* $x \in X$ *and each* $V \in SO(Y, f(x))$*, there exists an open set* U *of* X *containing* x *such that* $f(U) \subset scl(V)$*.*

2. Characterizations

Definition 4. *A subfamily* m_X *of the power set* $\wp(X)$ *of a nonempty set* X *is called a minimal structure [24] (briefly* m*-structure) on* X *if* $\varnothing \in m_X$ *and* $X \in m_X$*. By* (X, m_X)*, we denote a nonempty subset* X *with a minimal structure* m_X *on* X*. Each member of* m_X *is said to be* m_X*-open and the complement of* m_X*-open set is said to be* m_X*-closed. For a subset* A *of* X*, the* m_X*-closure of* A *and the* m_X*-interior of* A *are defined in [18] as* $m_X\text{-}Cl(V) = \cap\{F : A \subset F, X - F \in m_X\}$ *and* $m_X\text{-}Int(V) = \cup\{U : U \subset A, U \in m_X\}$*.*

Remark 5. *Let* (X, τ) *be a topological space. Then the families* τ*,* τ_q*,* $SO(X)$*,* $PO(X)$*,* $\alpha(X)$*,* $\beta(X)$ *(=*$\beta O(X)$*),* $SR(X)$*,* $\beta R(X)$ *are all* m*-structures on* X*.*

Definition 6. *A function* $f : (X, m_X) \to (Y, m_Y)$*, where* X *and* Y *are nonempty sets with minimal structures* m_X *and* m_Y*, respectively,is said to be weakly* M*-continuous [22] (*M*-continuous [24]) at* $x \in X$ *if for each* $V \in m_Y$ *containing* $f(x)$ *such that* $f(U) \subset m_X\text{-}Cl(V)$ *(resp.* $f(U) \subset V$*). A function* $f : (X, m_X) \to (Y, m_Y)$ *is said to be weakly* M*-continuous (resp.* M*-continuous) if it has the property at each point* $x \in X$*.*

Theorem 7. *For a function* $f : X \to Y$*, the following are equivalent:*

(a): f is almost s-continuous.
(b): For each $x \in X$ and each $V \in SR(Y, f(x))$, there exists a clopen set U containing x such that $f(U) \subset V$;

- **(c):** For each $x \in X$ and each $V \in SR(Y, f(x))$, there exists an quasi-open set U of X containing x such that $f(U) \subset V$;
- **(d):** $f : (X, \tau_q) \to (Y, SO(Y))$ is weakly M-continuous.
- **(e):** $f^{-1}(V) \subset int_q(f^{-1}(scl(V)))$ for every $V \in SO(Y)$;
- **(f):** $cl_q(f^{-1}(sint(F))) \subset f^{-1}(F)$ for every $F \in SC(Y)$;
- **(g):** $cl_q(f^{-1}(V)) \subset f^{-1}(scl(V))$ for every $V \in SO(Y)$.
- **(h):** $f(cl_q(A)) \subset scl_\theta(f(A))$ for each subset A of X.
- **(i):** $cl_q(f^{-1}(B)) \subset f^{-1}(scl_\theta(B))$ for each subset B of Y.

Proof. (a)⇒(b): This is known by Theorem 3.3 of [20].

(b)⇒(c)⇒(a): These implications are clear from the definition of quasi topology.

(c)⇒(d)Let $x \in X$ and $V \in SR(Y, f(x))$.Then by (c) there exists a quasi-open set U containing x such that $f(U) \subset V$. Since every semi-regular set is semi-open, f is M-continuous, hence weakly M-continuous.

(d)⇒(a) Let $x \in X$ and $V \in SO(Y, f(x))$ then there exists a quasi-open set U containing x such that $f(U) \subset scl(V)$. Since U is quasi open there exists an open set W in U containing x such that $f(W) \subset scl(V)$ and by Definition 3 f is almost s-continuous.

(c)⇒(e): Let $V \in SO(Y)$ and $x \in f^{-1}(V)$. Then $f(x) \in V$ and $scl(V) \in SR(Y, f(x))$ hence by (c), there exists a quasi-open set U of X containing x such that $f(U) \subset scl(V)$. Then $x \in U \subset f^{-1}(scl(V))$ and hence $x \in int_q(f^{-1}(scl(V)))$.

(e)⇔(a): It follows from Theorem 3.2 of [22].

(f)⇒(g): Let $F \in SC(Y)$, then $Y - F \in SO(Y)$ and by (e), we have $f^{-1}(Y - F) \subset int_q(f^{-1}(scl(Y - F)))$ i.e., $X - f^{-1}(F) \subset int_q(f^{-1}(scl(Y - F))) = int_q(f^{-1}(Y - sint(F))) = X - cl_q(f^{-1}(sint(F)))$ Hence we obtain $cl_q(f^{-1}(sint(F))) \subset f^{-1}(F)$.

(f)⇔(a): It follows from Theorem 2.1 of [23].

(f)⇒(g): Let $V \in SO(Y)$. Then $scl(V)$ is semi-closed, by (e) $cl_q(f^{-1}(V)) \subset cl_q(f^{-1}(scl(V))) = cl_q(f^{-1}(sint(scl(V)))) \subset f^{-1}(scl(V))$.

(g)⇔(a) It follows from Theorem 3.4 of [22].

(a)⇒(h)⇒(i)⇒(a): It follows from Theorem 3.3 of [22]. □

Definition 8. *A filter base $\mathcal{F}$ is said to be;*

- **(a):** *s-θ-convergent [1] to a point x in X, if for any semi-open set U containing x there exist $B \in \mathcal{F}$ such that $B \subset scl(U)$;*

(b): *clopen convergent to a point x in X, if for any clopen set U containing x, there exist $B \in \mathcal{F}$ such that $B \subset U$.*

Theorem 9. *A function $f : X \to Y$ is almost s-continuous if and only if for each point $x \in X$ and each filter base $\mathcal{F}$ in X clopen converging to x the filter base $f(\mathcal{F})$ is s-θ-convergent to $f(x)$.*

Proof. Suppose that $x \in X$ and $\mathcal{F}$ is any filter base in X clopen converges to x. Since f is almost s-continuous for any semi-open set V containing $f(x)$ $scl(V) \in SR(Y, f(x))$ and by Theorem 7, there exists a clopen set U containing x in X such that $f(U) \subset scl(V)$. Since $\mathcal{F}$ is clopen convergent to x in X then there exists $B \in \mathcal{F}$ such that $B \subset U$. It follows that $f(B) \subset scl(V)$. This means that $f(\mathcal{F})$ is s-θ-convergent to $f(x)$.

Conversely, let x be a point in X and V be a semi-open set containing $f(x)$. If we set $\mathcal{F} = \{U : U \text{ is clopen and } x \in U\}$, then $\mathcal{F}$ will be a filter base which clopen converges to x. So there exists $U \in \mathcal{F}$ such that $f(U) \subset scl(V)$. This completes the proof. □

Definition 10. *A net (x_i) in a space X, θ-converges (resp. clopen converges [10], s-θ-converges [1]) to x if and only if for each open (resp. clopen, semi-open,) set U containing x, there exists i_0 such that $x_i \in cl(U)$ (resp. $x_i \in U$, $x_i \in scl(U)$) for all $i \geqq i_0$.*

Lemma 11. *For a net (x_i) in a space X;*

- **(a):** *[1] if (x_i) s-θ-converges to x, then (x_i) θ-converges to x;*
- **(b):** *[2] if (x_i) converges to x, then (x_i) θ-converges to x ;*
- **(c):** *[10] if (x_i) converges or θ-converges to x, then (x_i) clopen converges to x.*

Theorem 12. *For a function $f : X \to Y$, the following statements are equivalent:*

- **(a):** *f is almost s-continuous;*
- **(b):** *For each $x \in X$ and each net (x_i) in X which clopen converges to x, the net $(f(x_i))$ s-θ-converges to $f(x)$;*
- **(c):** *For each $x \in X$ and each net (x_i) in X which θ-converges to x, the net $(f(x_i))$ s-θ-converges to $f(x)$;*
- **(d):** *For each $x \in X$ and each net (x_i) in X which converges to x, the net $(f(x_i))$ s-θ-converges to $f(x)$.*

Proof. $(a) \Rightarrow (b)$ Let $x \in X$ and let (x_i) be a net in X such that (x_i) clopen converges to x. Let V be a semi-open set containing $f(x)$. Since f is almost s-continuous and $scl(V) \in SR(Y)$, there exists a clopen set U containing x such that $f(U) \subset scl(V)$. Since (x_i) clopen converges to x, there exists i_0 such that $x_i \in U$ for all $i \geqq i_0$. Hence $f(x_i) \in scl(V)$ for all $i \geqq i_0$.

$(b) \Rightarrow (a)$ Suppose that f is not *almost s-continuous*. Then there exists $x \in X$ and $V \in SO(Y, f(x))$ such that $f(U) \nsubseteq scl(V)$ for all clopen neighborhood U of x. Thus for every clopen neighborhood U of x we can find $x_U \in U$ such that $f(x_U) \notin scl(V)$. Let $\mathcal{N}(x)$ be the set of clopen neighborhoods of x in X. The set $\mathcal{N}(x)$ with the relation of inverse inclusion (that is $U_1 \leq U_2$ if and only if $U_2 \subseteq U_1$) form a directed set (Theorem 1.1 of [10]). Clearly the net $\{x_U : U \in \mathcal{N}(x)\}$ clopen converges to x in X but $(f(x_U))_{U \in \mathcal{N}(x)}$ does not s-θ-converge to $f(x)$.

$(b) \Rightarrow (c)$ Let $x \in X$ and let (x_i) be a net in X such that (x_i) θ-converges to x. By Lemma 11 (x_i) clopen converges to x. By (b), $(f(x_i))$ s-θ-converges to $f(x)$.

For the proof of the other implications see [1]. □

By Lemma 11 and Theorem 12 we have the following as corollary. This is an improvement of Corollary 3.1 of [1].

Corollary 13. *If a function $f : X \to Y$ is almost s-continuous then, for each $x \in X$ and each net (x_i) in X which clopen converges to x, the net $(f(x_i))$ θ-converges to $f(x)$.*

Proposition 14. *[1] A net (x_i) in a space X, s-θ-converges to x if and only if for each semiregular set U containing x, there exists i_0 such that $x_i \in U$ for all $i \geqq i_0$.*

By Theorem 12 and Proposition 14 we have the following extension of Corollary 3.2 of [1].

Theorem 15. *For a function $f : X \to Y$, the following are equivalent:*

- **(a):** *f is almost s-continuous;*
- **(b):** *If for each $x \in X$ and, a net (x_i) in X clopen converges to x then for each $V \in SR(Y, f(x))$, there exists i_0 such that $f(x_i) \in V$ for all $i \geqq i_0$;*

(c): *If for each $x \in X$ and, a net (x_i) in X θ-converges to x then for each $V \in SR(Y, f(x))$, there exists i_0 such that $f(x_i) \in V$ for all $i \geqq i_0$;*
(d): *If for each $x \in X$ and, a net (x_i) in X converges to x then for each $V \in SR(Y, f(x))$, there exists i_0 such that $f(x_i) \in V$ for all $i \geqq i_0$.*

3. Separation axioms and co-SR-closed graphs

Definition 16. *A space X is said to be*

(a): *ultra Hausdorff [25] if every two distinct points of X can be separated by disjoint clopen sets.*
(b): *semi-T_2 [17] if for each pair of distinct points x and y in X, there exist semi-open sets U and V of X containing x and y, respectively, such that $U \cap V = \varnothing$* (or equivalently $scl(U) \cap scl(V) = \varnothing$ [14]).
(c): clopen T_1 [8] *(*$\equiv$ultra T_1 [13])if for each pair of distinct points x and y of X, there exist clopen sets U and V containing x and y respectively such that $y \notin U$ and $x \notin V$.
(d): ultra T_0 [13] if for each pair of distinct points x and y of X, there exist a clopen set U containing one of the points x and y but not the other.

Remark 17. *Kohli and Singh proved that [13] ultra Hausdorff, clopen T_1, and ultra T_0 axioms are all equivalent.*

Definition 18. *[22] A nonempty set X is with a minimal structure m_X, (X, m_X), is said to be m-Hausdorff if for each distinct points x, $y \in X$, there exist U, $V \in m_X$ containing x and y, respectively, such that $U \cap V = \varnothing$.*

Theorem 19. *[26] If $f : (X, \tau_q) \to (Y, m_Y)$ is a weakly M-continuous function and (Y, m_Y) is m-Hausdorff, then f has quasi-closed point inverses in X.*

Corollary 20. *If $f : (X, \tau) \to (Y, \sigma)$ is almost s-continuous and (Y, σ) is semi-T_2 then f has quasi-closed point inverses in X.*

Recall that for a function $f : X \to Y$, the subset $\{(x, f(x)) : x \in X\} \subset X \times Y$ is called the graph of f and is denoted by $G(f)$.

Definition 21. *A function $f:(X,m_X)\to(Y,m_Y)$ is said to has a strongly M-closed graph [22] if and only if for each $(x,y)\in(X\times Y)-G(f)$ there exists an m_X-open set U containing x and an m_Y-open set V containing y such that $(U\times m_Y\text{-}Cl(V))\cap G(f)=\varnothing$.*

Lemma 22. *[22] A function $f:(X,m_X)\to(Y,m_Y)$ has a strongly M-closed graph if and only if for each $(x,y)\in(X\times Y)-G(f)$ there exists an m_X-open set U containing x and m_Y-open set V containing y such that $f(U)\cap m_Y\text{-}Cl(V)=\varnothing$.*

Definition 23. *A graph $G(f)$ of a function $f:X\to Y$ is said to be co-SR-closed if for each $(x,y)\in(X\times Y)-G(f)$, there exists an clopen set U in X containing x and $V\in SR(Y,y)$ such that $(U\times V)\cap G(f)=\varnothing$.*

Remark 24. *If a function $f:(X,m_X)\to(Y,m_Y)$ has the strongly M-closed graph, then for the special case $m_X=\tau_q$ and $m_Y=SO(Y)$, $G(f)$ has co-SR-closed graph and we may state the following.*

Theorem 25. *The following properties are equivalent for a graph $G(f)$ of a function:*

- **(a):** $G(f)$ is *co*-SR-closed.
- **(b):** for each $(x,y)\in(X\times Y)-G(f)$, there exists a clopen set U containing x in X and $V\in SR(Y,y)$ such that $f(U)\cap V=\varnothing$.
- **(c):** for each point $(x,y)\in(X\times Y)-G(f)$, there exists a clopen set U containing x in X and $V\in SO(Y,y)$ such that. $f(U)\cap scl(V)=\varnothing$.
- **(d):** for each point $(x,y)\in(X\times Y)-G(f)$, there exists a quasi-open set U containing x in X and $V\in SO(Y,y)$ such that $f(U)\cap scl(V)=\varnothing$

Theorem 26. *If $f:X\to Y$ is almost s-continuous function and Y is semi-T_2 , then $G(f)$ is co-SR-closed in $X\times Y$.*

Proof. First suppose Y is *semi*-T_2. Let $(x,y)\in(X\times Y)-G(f)$. It follows that $f(x)\neq y$. Since Y is *semi*-T_2, there exist $V\in SO(Y,f(x))$ and $W\in SO(Y,y)$ such that $scl(V)\cap scl(W)=\varnothing$. Since f is almost s-continuous, there exists an clopen set $U=f^{-1}(scl(V))$ in X containing x such that $f(U)\subset scl(V)$. Therefore $f(U)\cap scl(W)=\varnothing$ and $G(f)$ is *co*-SR-closed with respect to $X\times Y$. □

Theorem 27. *Let $f:X\to Y$ have a co-SR-closed graph then the following properties hold:*

(a): *if f is injective then X is ultra Hausdorff;*
(b): *if f is surjective then X is semi-T_2.*

Proof. (a) Suppose that x and y are any two distinct points of X by the injectivity of f, $(x, f(y)) \notin G(f)$. Since $G(f)$ is *co*-SR-closed, by Theorem 25, there exist a clopen set U containing x and $V \in SO(Y, f(y))$ such that $f(U) \cap scl(V) = \varnothing$. We have $U \cap f^{-1}(scl(V)) = \varnothing$. Therefore $y \notin U$. Then U and $X - U$ are disjoint clopen sets containing x and y, respectively. Hence X is ultra Hausdorff.

(b) Let y_1 and y_2 be any two distinct points of Y. Since f is surjective there exists a point $x \in X$ such that $f(x) = y_2$. Since $G(f)$ is *co*-SR-closed and $(x, y_1) \notin G(f)$ there exists a clopen set U containing x and $V \in SR(Y, y_1)$ such that $f(U) \cap V = \varnothing$. Therefore we have $y_2 \in f(U) \subset Y - V \in SR(Y)$ and hence Y is semi-T_2. □

Note that since *ultra Hausdorff spaces are totally disconnected [25] first part of the theorem characterizes totally disconnectednes.*

Definition 28. *A subset K of a nonempty set X with a minimal structure m_X is said to be m-compact [21] (m-closed [21]) relative to (X, m_X) if any cover $\{U_i : i \in I\}$ of K by m_X-open sets, there exists a finite subset I_0 of I such that $K \subseteq \cup\{U_i : i \in I_0\}$ ($K \subseteq \cup\{m_X$ - $Cl(U_i) : i \in I_0\}$). (X, m_X) is m-closed if X is m-closed relative to (X, m_X).*

Definition 29. *A subset K of a space X is said to be s-closed [14] relative to X if for every cover $\{V_\alpha : \alpha \in I\}$ of K by semi-open sets of X, there exists a finite subset I_0 of I such that $K \subset \cup\{scl(V_\alpha) : \alpha \in I_0\}$.*

Theorem 30. *[19] Let $f : (X, m_X) \to (Y, m_Y)$ be a function. Assume that m_X is a base for a topology. If the graph $G(f)$ is strongly M-closed, then m_X-$Cl(f^{-1}(K)) = f^{-1}(K)$ whenever the set $K \subseteq Y$ is m-closed relative to (Y, m_Y).*

Corollary 31. *[26] If a function $f : (X, \tau_q) \to (Y, m_Y)$ has a strongly M-closed graph, then $f^{-1}(K)$ is quasi-closed in (X, τ_q) for each set K which is m-closed relative to (Y, m_Y).*

Corollary 32. *If a function $f : X \to Y$ has co-SR-closed graph, then $f^{-1}(K)$ is quasi-closed in X for every subset K which is s-closed relative to Y.*

Theorem 33. *If a function $f : X \to Y$ has a co-SR-closed graph and Y is s-closed then f is almost s-continuous.*

Proof. Let $V \in SR(Y)$, then by Lemma 1, $Y - V \in SR(Y)$. By the s-closedness of Y it follows from Theorem 1 of [15], $Y - V$ is s-closed. By Corollary 32, $f^{-1}(Y - V) = X - f^{-1}(V)$ is quasi-closed, hence $f^{-1}(V)$ is quasi open. Set $U = f^{-1}(V)$, then $f(U) \subset V$, and by Theorem 7, f is almost s-continuous. □

Corollary 34. *Let Y be an s-closed semi-T_2 space. The following are equivalent for a function $f : X \to Y$:*

- **(a):** f is almost s-continuous;
- **(b):** $G(f)$ is *co*-SR-closed;
- **(c):** for each K, s-closed relative to Y, $f^{-1}(K)$ is quasi-closed in X.

Proof. This is a direct consequence of Theorems 26, 33 and Corollary 32. □

Definition 35. *A topological space (X, τ) is called countably rs-compact [5] (resp. countably S-closed [3], mildly countably compact [25]) if every countable cover of X by semi-regular (resp. regular closed, clopen) sets has a finite subcover.*

Definition 36. *A topological space (X, τ) is called rs-Lindelöf [7] (resp. rc-Lindelöf [12]) if every cover of X by semi-regular (resp. regular closed) sets has a countable subcover.*

Definition 37. *For any infinite cardinal κ, a topological space (X, τ) is called κ-extremally disconnected [5] (κ-e.d.) if the boundary of every regular open set has cardinality (strictly) less than κ.*

Theorem 38. *Let $f\colon (X, \tau) \to (Y, \sigma)$ be an almost s-continuous surjection, then the following properties hold:*

- **(a):** *If* X is mildly countably compact, then Y is countably S-closed and $\aleph_0$-e.d. (almost extremally disconnected).
- **(b):** *If* X is mildly Lindelöf, then Y is rc-Lindelöf and ω_1-e.d. (the boundary of every regular open set is at most countable).

Proof. (a) Let $f\colon (X, \tau) \to (Y, \sigma)$ be an almost s-continuous surjection. If X is mildly countably compact, then Y is countably rs-compact by Theorem 2.6 of [4]. Then Y is both countably S-closed and $\aleph_0$-e.d. (almost extremally disconnected) by Theorem 3.13 of [5].

(b) Let $f\colon (X,\tau) \to (Y,\sigma)$ be an almost s-continuous surjection. If X is mildly Lindelöf, then Y is rs-Lindelöf. by Theorem 2.6 of [4]. Then Y is rc-Lindelöf and ω_1-e.d. by Theorem 3.14 of [5]. □

Acknowledgement. The author wishes to thank Professor Valeriu Popa from the Department of Mathematics and Informatics, Faculty of Sciences, "Vasile Alecsandri" University of Bacău, Romania for valuable comments on an earlier version of this paper.

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