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ON $(\Lambda, mn^\star)$-CLOSED SETS IN IDEAL BI m-SPACES

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Abstract. The notions of mn-$\mathcal{I}_g$-closed sets and mn-$\mathcal{T}_I$ spaces in an ideal bi m-space are introduced and investigated by Sanabria et al. [18]. In this paper, we introduce the notion of $(\Lambda, mn^\star)$-closed sets and obtain a decomposition of $n^\star$-closed sets and a characterization of mn-$\mathcal{T}_I$ spaces [18] by using mn-$\mathcal{I}_g$-closed sets and $(\Lambda, mn^\star)$-closed sets.

Dedicated to Professor Valeriu Popa on the Occasion of His 80th Birthday

1. Introduction

A subfamily m of the power set $\mathcal{P}(X)$ of a set X is called a minimal structure of X [16] if $\emptyset \in m$ and $X \in m$. Ozbakir and Yildirim [17] introduced and investigated the m-local function and minimal $\star$-closures in a minimal space (X, m) with an ideal $\mathcal{I}$ on X. And they constructed the minimal structure $m^\star$ containing m. The notion of m-$\mathcal{I}_g$-closed sets is defined and investigated in [17]. In a bi m-space (X, m, n) with an ideal $\mathcal{I}$, Sanabria al. [18] introduced and investigated the notion of mn-$\mathcal{I}_g$-closed sets which is a generalization of m-$\mathcal{I}_g$-closed sets. And also they defined mn-$\mathcal{T}_I$ spaces and obtained a characterization of mn-$\mathcal{T}_I$ spaces.

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In this paper, we introduce the notion of $(\Lambda, mn^\star)$-closed sets and obtain decompositions of $n^\star$-closed sets and a characterization of mn-$\mathcal{T}_I$ spaces by using mn-$\mathcal{I}_g$-closed sets and $(\Lambda, mn^\star)$-closed sets.

2. Preliminaries

Definition 2.1. A subfamily m of the power set $\mathcal{P}(X)$ of a nonempty set X is called a *minimal-structure* (briefly *m-structure*) [16] on X if m satisfies the following properties: $\emptyset \in m$ and $X \in m$.

By (X, m), we denote a nonempty set X with a minimal structure m on X and call it an *m-space*. Each member of m is said to be *m-open* and the complement of an m-open set is said to be *m-closed*.

Definition 2.2. A minimal structure m of a set X is said to have:
(1) *property* $\mathcal{B}$ [13] if the union of any collection of elements of m is an element of m,
(2) *property* $\mathcal{F}$ [18] any finite intersection of sets belonging to m belongs to m.

Definition 2.3. Let (X, m) be an m-space and A a subset of X. The *m-closure* $\mathrm{mCl}(A)$ of A [13] is defined as follows: $\mathrm{mCl}(A) = \cap\{F : A \subset F, X \setminus F \in m\}$.

Lemma 2.1. (Popa and Noiri [16]). *Let (X, m) be an m-space and m have property $\mathcal{B}$. For a subset A of X, the following properties hold:*
(1) A is m-closed if and only if $\mathrm{mCl}(A) = A$,
(2) $\mathrm{mCl}(A)$ *is m-closed.*

Definition 2.4. A nonempty subfamily $\mathcal{I}$ of $\mathcal{P}(X)$ is called an *ideal* on X [8] if it satisfies the following properties:
(1) $A \in \mathcal{I}$ and $B \subset A$ imply $B \in \mathcal{I}$,
(2) $A \in \mathcal{I}$ and $B \in \mathcal{I}$ imply $A \cup B \in \mathcal{I}$.

A topological space (X, τ) with an ideal $\mathcal{I}$ on X is called an ideal topological space and is denoted by $(X, \tau, \mathcal{I})$. For an ideal topological space $(X, \tau, \mathcal{I})$ and a subset A of X, $A^\star(\mathcal{I})$ is defined as follows:

$$A^\star(\mathcal{I}) = \{x \in X : U \cap A \notin \mathcal{I} \text{ for every open set } U \text{ containing } x\}.$$

In case there is no chance for confusion, $A^\star(\mathcal{I})$ is simply written as $A^\star$. In [10], $A^\star$ is called the *local function* of A with respect to $\mathcal{I}$ and τ and $\mathrm{Cl}^\star(A) = A^\star \cup A$ defines a Kuratowski closure operator for a topology $\tau^\star(\mathcal{I})$ which is finer than τ. A subset A is said to be $\star$-closed [8] if $A^\star \subset A$, that is, $\mathrm{Cl}^\star(A) = A$. If $\mathcal{I} = \{\emptyset\}$, then $A^\star = \mathrm{Cl}(A)$ and hence $\mathrm{Cl}^\star(A) = \mathrm{Cl}(A)$.

Similarly, Ozbakir and Yildirim [17] constructed a new m-structure $n^\star$ on X containing an m-structure n from an m-space (X, n) with an ideal $\mathcal{I}$ on X.

Definition 2.5. Let (X, n) be an m-space with an ideal $\mathcal{I}$. For a subset A of X, the *minimal local function* $A^\star_n$ of A [17] is defined as follows:

$$A^\star_n = \{x \in X : U \cap A \notin \mathcal{I} \text{ for every } U \in n(x)\}, \text{ where}$$
$$n(x) = \{U : x \in U \in n\}.$$

Moreover, the minimal $\star$-closure $\mathrm{nCl}^\star(A) = A \cup A^\star_n$ is defined and the m-structure $n^\star$ generated by $\mathrm{nCl}^\star(A)$ is defined as follows: $n^\star = \{U \subset X : \mathrm{nCl}^\star(X \setminus U) = X \setminus U\}$.

Lemma 2.2. (Al-Omari and Noiri [1]). *Let (X, n) be an m-space with an ideal $\mathcal{I}$ and n have property $\mathcal{F}$. Then the operator* $\mathrm{nCl}^\star(.)$ *is a Kuratowski closure operator and $n^\star$ is a topology for X containing n.*

Lemma 2.3. (Ozbakir and Yildirim [17]). *Let $(X, n, \mathcal{I})$ be an ideal m-space. For subsets A and B of X, the following properties hold:*

(1) If $A \subset B$, then $A^\star_n \subset B^\star_n$,
(2) $A^\star_n = \mathrm{nCl}(A^\star_n) \subset \mathrm{nCl}(A)$,
(3) $A^\star_n \cup B^\star_n \subset (A \cup B)^\star_n$,
(4) $(A^\star_n)^\star_n \subset A^\star_n$.

Lemma 2.4. *Let $(X, n, \mathcal{I})$ be an ideal m-space and A a subset of X. Then the following properties hold:*

(1) $(A \cup A^\star_n)^\star_n = A^\star_n$,
(2) $\mathrm{nCl}^\star(\mathrm{nCl}^\star(A)) = \mathrm{nCl}^\star(A)$, that is, $\mathrm{nCl}^\star(A)$ is $n^\star$-closed.

Proof. (1) Since $A \subset (A \cup A^\star_n)$, by Lemma 2.3(1), $A^\star_n \subset (A \cup A^\star_n)^\star_n$. Suppose that $x \notin A^\star_n$. Then there exists $U \in n(x)$ such that $U \cap A \in \mathcal{I}$. Hence $U \cap A^\star_n = \emptyset$. Because, if $(U \cap A^\star_n)$ is not empty, there exists $u \in U \cap A^\star_n$ and hence $U \in n(u)$ and $u \in A^\star_n$. Thus $U \cap A \notin \mathcal{I}$. This is a contradiction. Hence $U \cap A^\star_n$ is empty. Now, we have $U \cap (A \cup A^\star_n) = (U \cap A) \cup (U \cap A^\star_n) = U \cap A \in \mathcal{I}$. Therefore, $x \notin (A \cup A^\star_n)^\star_n$ and $(A \cup A^\star_n)^\star_n \subset A^\star_n$. Therefore, we have $(A \cup A^\star_n)^\star_n = A^\star_n$.

(2) $\mathrm{nCl}^\star(\mathrm{nCl}^\star(A)) = \mathrm{nCl}^\star(A \cup A^\star_n) = (A \cup A^\star_n) \cup (A \cup A^\star_n)^\star_n = (A \cup A^\star_n) \cup A^\star_n = \mathrm{nCl}^\star(A)$. By the definition of $n^\star$, $\mathrm{nCl}^\star(A)$ is $n^\star$-closed.

Remark 2.1. It is pointed out in Remark 2.2 of [17] that Lemma 2.4(2) holds under the assumption that m has property $\mathcal{F}$. The proof is not given. However, this is valid without the assumtion.

3. mn-$\mathcal{I}_g$-CLOSED SETS

A subset X with two m-structures m, n and an ideal $\mathcal{I}$ is called an *ideal bi m-space* and is briefly denoted by $(X, m, n, \mathcal{I})$. First, we shall recall Λ_m-sets in an m-space (X, m).

Definition 3.1. Let (X, m) be an m-space and A a subset of X. The subset $\Lambda_m(A)$ is defined in [4] as follows: $\Lambda_m(A) = \cap\{U : A \subset U, U \in m\}$.

A subset A is called a *Λ_m-set* if $A = \Lambda_m(A)$. The family of all Λ_m-sets on X is denoted by Λ_m. In case $m = \tau$ a subset A is called a *Λ-set* [12] if $A = \Lambda_m(A)$.

Lemma 3.1. (Cammaroto and Noiri [4]). *For any subsets A, B and A_α ($\alpha \in \Delta$) of X, the following properties hold:*
(1) $A \subset \Lambda_m(A)$,
(2) $m \subset \Lambda_m$,
(3) $\Lambda_m(A)$ is a Λ_m-set,
(4) If $A \subset B$, then $\Lambda_m(A) \subset \Lambda_m(B)$,
(5) If A_α is a Λ_m-set for each $\alpha \in \Delta$, then $\cap_{\alpha\in\Delta} A_\alpha$ is a Λ_α-set.

Remark 3.1. In [4], it is assumed that m has property $\mathcal{B}$. However, the above lemma holds without this assumption.

Definition 3.2. Let $(X, m, n, \mathcal{I})$ be an ideal bi m-space. A subset A of X is said to be *mn-$\mathcal{I}_g$*-closed [18] (resp. *mng-closed* [14]) if $A^\star_n \subset U$ (resp. $\mathrm{nCl}(A) \subset U$) whenever $A \subset U$ and $U \in m$.

Remark 3.2. (1) For every subset A of X, $A^\star_n \subset \mathrm{nCl}(A)$ by Lemma 2.3 and hence every mng-closed set is mn-$\mathcal{I}_g$-closed (Proposition 5.1 of [18]). If $\mathcal{I} = \{\emptyset\}$, then $A^\star_n = \mathrm{nCl}(A)$ and mng-closed sets are coincident with mn-$\mathcal{I}_g$-closed sets.
(2) By the definitions, we have the following diagram:

DIAGRAM I

$$\begin{array}{ccc} n\text{-closed} & \Rightarrow & n^\star\text{-closed} \\ \Downarrow & & \Downarrow \\ mng\text{-closed} & \Rightarrow & mn\text{-}\mathcal{I}_g\text{-closed} \end{array}$$

The following theorem is a slight modification of the results due to [18].

Theorem 3.1. *Let $(X, m, n, \mathcal{I})$ be an ideal bi m-space. For a subset A of X, the following properties are equivalent:*

(1) A *is* mn-$\mathcal{I}_g$*-closed;*
(2) $\mathrm{nCl}^\star(A) \subset U$ *whenever* $A \subset U$ *and* $U \in m$*;*
(3) $\mathrm{nCl}^\star(A) \cap F = \emptyset$ *whenever* $A \cap F = \emptyset$ *and* F *is* m*-closed;*
(4) $\mathrm{nCl}^\star(A) \subset \Lambda_m(A)$.

Proof. (1) $\Rightarrow$ (2): Suppose that $A \subset U$ and $U \in m$. By (1), $A_n^\star \subset U$ and $\mathrm{nCl}^\star(A) = A \cup A_n^\star \subset U$.

(2) $\Rightarrow$ (3): Suppose that $A \cap F = \emptyset$ and F is m-closed. Then $A \subset X \setminus F \in m$ and by (2) $\mathrm{nCl}^\star(A) \subset X \setminus F$. Therefore, we have $\mathrm{nCl}^\star(A) \cap F = \emptyset$.

(3) $\Rightarrow$ (4): Suppose that $x \notin \Lambda_m(A)$. Then there exists $U \in m$ such that $x \notin U$ and $A \subset U$. Hence $A \cap (X \setminus U) = \emptyset$ and $X \setminus U$ is m-closed. By (3), we have $\mathrm{nCl}^\star(A) \cap (X \setminus U) = \emptyset$ and $\mathrm{nCl}^\star(A) \subset U$. Hence $x \notin \mathrm{nCl}(^\star(A)$. This implies that $\mathrm{nCl}^\star(A) \subset \Lambda_m(A)$.

(4) $\Rightarrow$ (1): Suppose that $A \subset U$ and $U \in m$. By (4) and Lemma 3.1, $\mathrm{nCl}^\star(A) \subset \Lambda_m(A) \subset \Lambda_m(U) = U$ and $A_n^\star \subset \mathrm{nCl}^\star(A) \subset U$. This shows that A is mn-$\mathcal{I}_g$-closed.

Definition 3.3. Let (X, τ) be a topological space with an ideal $\mathcal{I}$. A subset A of X is said to be $\mathcal{I}$-mg-closed [15] if $A^\star \subset U$ whenever $A \subset U$ and $U \in m$.

If we set $n = \tau$ in Theorems 3.1, then we obtain the following corollary.

Corollary 3.1. (Noiri and Popa [15]) *Let* (X, τ) *be a topological space with an ideal* $\mathcal{I}$*. For a subset* A *of* X*, the following properties are equivalent:*
(1) A *is* $\mathcal{I}$-mg*-closed;*
(2) $\mathrm{Cl}^\star(A) \subset U$ *whenever* $A \subset U$ *and* $U \in m$*;*
(3) $\mathrm{Cl}^\star(A) \cap F = \emptyset$ *whenever* $A \cap F = \emptyset$ *and* F *is* m*-closed;*
(4) $\mathrm{Cl}^\star(A) \subset \Lambda_m(A)$.

4. $(\Lambda, mn^\star)$-closed sets

A subset A of a topological space is said to be *locally closed* [3], [7] (resp. λ-closed [2]) if it is the intersection of an open set (resp. a Λ-set) and a closed set. In this section, we investigate some generalizations of locally closed sets and λ-closed sets.

Definition 4.1. Let $(X, m, n, \mathcal{I})$ be an ideal bi m-space. A subset A of X is said to be:

(1) *locally* $mn^\star$*-closed* if $A = U \cap F$, where U is an m-open set and F is a $\star$-closed set,

(2) *$(\Lambda, mn^\star)$-closed* if $A = U \cap F$, where U is a Λ_m-set and F is $n^\star$-closed.

Remark 4.1. For a subset of an ideal bi m-space $(X, m, n, \mathcal{I})$, we obtain the following implications.

DIAGRAM II

$$n^\star\text{-closed} \Rightarrow \text{locally } mn^\star\text{-closed} \Rightarrow (\Lambda, mn^\star)\text{-closed}$$

Proposition 4.1. *Let $(X, m, n, \mathcal{I})$ be an ideal bi m-space and n have property $\mathcal{F}$. If A_α is $(\Lambda, mn^\star)$-closed for each $\alpha \in \Delta$, then $\cap_{\alpha\in\Delta}A_\alpha$ is $(\Lambda, mn^\star)$-closed.*

Proof. Let A_α be $(\Lambda, mn^\star)$-closed for each $\alpha \in \Delta$. Then $A_\alpha = U_\alpha \cap F_\alpha$, where U_α is a Λ_m-set and F_α is an $n^\star$-closed set for each $\alpha \in \Delta$. Then $\cap_{\alpha\in\Delta}A_\alpha = (\cap_{\alpha\in\Delta}U_\alpha) \cap (\cap_{\alpha\in\Delta}F_\alpha)$. By Lemma 3.1, $\cap_{\alpha\in\Delta}U_\alpha$ is a Λ_m-set. Since n has property $\mathcal{F}$, by Lemma 2.2 $\cap_{\alpha\in\Delta}F_\alpha$ is $n^\star$-closed. Therefore, $\cap_{\alpha\in\Delta}A_\alpha$ is $(\Lambda, mn^\star)$-closed.

Theorem 4.1. *Let $(X, m, n, \mathcal{I})$ be an ideal bi m-space. For a subset A of X, the following properties are equivalent:*
(1) A is $(\Lambda, mn^\star)$-closed;
(2) $A = U \cap \mathrm{nCl}^\star(A)$ for some $U \in \Lambda_m$;
(3) $A = \Lambda_m(A) \cap \mathrm{nCl}^\star(A)$.

Proof. (1) $\Rightarrow$ (2): Let A be a $(\Lambda, mn^\star)$-closed set. Then $A = U \cap F$, where $U \in \Lambda_m$ and F is $n^\star$-closed. Then, we have $A \subset U \cap \mathrm{nCl}^\star(A) \subset U \cap \mathrm{nCl}^\star(F) = U \cap F = A$. Therefore, we have $A = U \cap \mathrm{nCl}^\star(A)$ for $U \in \Lambda_m$.

(2) $\Rightarrow$ (3): Let $A = U \cap \mathrm{nCl}^\star(A)$ for some $U \in \Lambda_m$. Since $A \subset U$, by Lemma 3.1 $\Lambda_m(A) \subset \Lambda_m(U) = U$ and hence $A \subset \Lambda_m(A) \cap \mathrm{nCl}^\star(A) \subset U \cap \mathrm{nCl}^\star(A) = A$. Therefore, we obtain $A = \Lambda_m(A) \cap \mathrm{nCl}^\star(A)$.

(3) $\Rightarrow$ (1): Let $A = \Lambda_m(A) \cap \mathrm{nCl}^\star(A)$. By Lemma 3.1, $\Lambda_m(A)$ is a Λ_m-set. It follows from Lemma 2.4 that $\mathrm{nCl}^\star(A)$ is $n^\star$-closed. Therefore, A is $(\Lambda, mn^\star)$-closed.

Theorem 4.2. *Let $(X, m, n, \mathcal{I})$ be an ideal bi m-space. For a subset A of X, the following properties are equivalent:*
(1) A is $n^\star$-closed;
(2) A is locally $mn^\star$-closed set and mn-$\mathcal{I}_g$-closed;
(3) A is $(\Lambda, mn^\star)$-closed and mn-$\mathcal{I}_g$-closed.

Proof. (1) $\Rightarrow$ (2) $\Rightarrow$ (3): The proofs are obvious by DIAGRAMs I and II.

(3) $\Rightarrow$ (1): Suppose that A is $(\Lambda, mn^\star)$-closed and mn-$\mathcal{I}_g$-closed. Then $A = U \cap F$, where U is a Λ_m-set and F is $n^\star$-closed. Since $A \subset U$ and A is mn-$\mathcal{I}_g$-closed, by Theorem 3.1 and Lemma 3.1, $\mathrm{nCl}^\star(A) \subset \Lambda_m(A) \subset \Lambda_m(U) = U$. On the other hand. since $A \subset F$, by Proposition 2.1 of [17], $\mathrm{nCl}^\star(A) \subset \mathrm{nCl}^\star(F) = F$. Hence $\mathrm{nCl}^\star(A) \subset U \cap F = A$ and $\mathrm{nCl}^\star(A) = A$. Therefore, A is $n^\star$-closed.

Remark 4.2. Both locally $mn^\star$-closedness and $(\Lambda, mn^\star)$-closedness are independent of mn-$\mathcal{I}_g$-closedness as shown by the following examples.

Example 4.1. Let $X = \{a, b, c\}, m = \{\emptyset, X, \{a\}, \{b\}\}, n = \{\emptyset, X, \{a\}, \{c\}\}$ and $\mathcal{I} = \{\emptyset, \{b\}, \{c\}\}, \{b, c\}\}$. Let $A = \{a\}$. Then A is not mn-$\mathcal{I}_g$-closed by Example 5.1 of [18]. However, since A is m-open and X is $m^\star$-closed, A is locally $mn^\star$-closed.

Example 4.2. Let $X = \{a, b, c, d\}, m = \{\emptyset, X, \{a, b\}, \{b, c\}\}, n = \{\emptyset, X, \{a\}, \{a, c\}, \{c, d\}\}$ and $\mathcal{I} = \{\emptyset, \{b\}\}$. Let $A = \{a, c\}$. Then A is mn-$\mathcal{I}_g$-closed by Example 5.6 of [18]. However, since $\Lambda_m(A) = X$ and A is not $n^\star$-closed, A is not $(\Lambda, mn^\star)$-closed.

Definition 4.2. Let $(X, \tau, \mathcal{I})$ be an ideal topological space. A subset A of X is said to be:

(1) *$\mathcal{I}_g$-closed* [5] (resp. g-closed [11]) if $A^\star \subset U$ (resp. $\mathrm{Cl}(A) \subset U$) whenever $A \subset U$ and U is an open set,

(2) *weakly $\mathcal{I}$-locally closed* [9] (resp. *$(\Lambda, \star)$-closed*) if $A = U \cap F$, where U is an open set (resp. a Λ-set) and F is a $\star$-closed set.

In Theorem 4.2, we put $m = n = \tau$ (topology), then we obtain the following corollary.

Corollary 4.1. *Let $(X, \tau, \mathcal{I})$ be an ideal topological space. For a subset A of X, the following properties are equivalent:*

(1) A is $\star$-closed;

(2) A is weakly $\mathcal{I}$-locally closed and $\mathcal{I}_g$-closed;

(3) A is $(\Lambda, \star)$-closed and $\mathcal{I}_g$-closed.

Proof. It is shown in Theorem 2.5 of [6] that (1) and (2) are equivalent.

In Corollary 4.1, we put $\mathcal{I} = \{\emptyset\}$, then $A^\star = \mathrm{Cl}(A)$ and hence every $\star$-closed set is closed. Therefore, we obtain the following corollary.

Corollary 4.2. (Arenas et al. [2]). *Let (X, τ) be a topological space. For a subset A of X, the following properties are equivalent:*

(1) A is closed;
(2) A is locally closed and g-closed;
(3) A is λ-closed and g-closed.

Theorem 4.3. *Let $(X, m, n, \mathcal{I})$ be an ideal bi m-space. Then, the following properties are equivalent:*
(1) Every singleton of X is $n^\star$-open or m-closed;
(2) Every subset A of X is $(\Lambda, mn^\star)$-closed.

Proof. (1) $\Rightarrow$ (2): Let every singleton of X be $n^\star$-open or m-closed. Suppose that there exists a subset A of X such that it is not $(\Lambda, mn^\star)$-closed. Then, by Theorem 4.1, $A \neq \Lambda_m(A) \cap \mathrm{nCl}^\star(A)$. Since $A \subset \Lambda_m(A) \cap \mathrm{nCl}^\star(A)$, there exists a point $x \in \Lambda_m(A) \cap \mathrm{nCl}^\star(A)$ such that $x \notin A$. Since $\{x\}$ is $n^\star$-open or m-closed, we consider two cases.

(i) In case $\{x\}$ is m-closed, since $x \notin A$, $A \subset X \setminus \{x\} \in m$ and $A \subset \Lambda_m(A) \subset X \setminus \{x\}$. However, $x \in \Lambda_m(A)$ and hence $x \in X \setminus \{x\}$. This is a contradiction.

(ii) In case $\{x\}$ is $n^\star$-open, since $x \notin A$, $A \subset X \setminus \{x\}$ and $X \setminus \{x\}$ is $n^\star$-closed. Therefore, $\mathrm{nCl}^\star(A) \subset \mathrm{nCl}^\star(X \setminus \{x\}) = X \setminus \{x\}$. However, $x \in \mathrm{nCl}^\star(A)$ and $x \in X \setminus \{x\}$. This is a contradiction.

Consequently, we obtain that every subset A of X is $(\Lambda, mn^\star)$-closed.

(2) $\Rightarrow$ (1): Suppose that $x \in X$ and $\{x\}$ is not m-closed. Then $X \setminus \{x\}$ is not m-open and the m-open set which contains $X \setminus \{x\}$ is only X. Therefore, $\mathrm{nCl}^\star(X \setminus \{x\}) \subset X$ and by Theorem 3.1 $X \setminus \{x\}$ is mn-$\mathcal{I}_g$-closed. By (2), $X \setminus \{x\}$ is mn-$\mathcal{I}_g$-closed and $(\Lambda, mn^\star)$-closed. By Theorem 4.2, $X \setminus \{x\}$ is $n^\star$-closed and $\{x\}$ is $n^\star$-open.

Definition 4.3. An ideal bi m-space $(X, m, n, \mathcal{I})$ is said to be mn-$\mathcal{T}_I$ [18] if every mn-$\mathcal{I}_g$-closed set of X is $n^\star$-closed.

Corollary 4.3. *Let $(X, m, n, \mathcal{I})$ be an ideal bi m-space. Then, the following properties are equivalent:*
(1) $(X, m, n, \mathcal{I})$ is mn-$\mathcal{T}_I$;
(2) Every singleton of X is $n^\star$-open or m-closed;
(3) Every subset A of X is $(\Lambda, mn^\star)$-closed.

Proof. (1) $\Leftrightarrow$ (2): This follows from Theorem 5.5 of [18].
(2) $\Leftrightarrow$ (3): This follows from Theorem 4.3

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