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GENERALIZED VERSION OF FUZZY δ-SEMICLOSED SET

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Abstract. The notions of fuzzy δ-semiopen and fuzzy δ-semiclosed set have been introduced in [5]. Taking this idea as a basic tool, we introduce the notion of fuzzy generalized δ-semiclosed set ($fg\delta$-semiclosed set, for short). Then the mutual relationships between this set with fg-closed set [2, 3], fgs-closed set [3], fsg-closed set [3], $fg\beta$-closed set [3], $f\beta g$-closed set [3] are established. Afterwards, we introduce and characterize $fg\delta$-semiclosed function. In Section 4, a new type of idempotent operator, viz., generalized δ-semiclosure operator is introduced and studied some of its properties. Next we introduce and characterize fuzzy generalized δ-semicontinuous function and show that the composition of two fuzzy generalized δ-semicontinuous functions may not be so. In Section 5, we introduce and characterize fuzzy generalized δ-semiregular and fuzzy generalized δ-seminormal spaces and also we prove the invariance of the propery of a fuzzy topological space of being generalized δ-seminormal, under fuzzy generalized δ-semiirresolute function. In the last section, we first introduce fuzzy generalized δ-semi T_2-space and then three different types of fuzzy continuous-like functions are introduced and establish that the inverse image of fuzzy generalized δ-semi T_2-space under these functions are fuzzy T_2-spaces [13].

Keywords and phrases: $fg\delta$-semiclosed set, $fg\delta$-semiclosed function, $fg\delta$-semicontinuous function, $fg\delta$-semiregular (normal) space, $fg\delta$-semi T_2-space.
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1. Introduction and Preliminaries

Fuzzy δ-open set is introduced in [9]. Using this idea, in [5], fuzzy δ-semiopen set is introduced and studied. Different types of generalized version of fuzzy closed sets are defined in [2, 3, 6]. Also in [3, 4], several types of generalized version of fuzzy continuous-like functions are introduced and studied. In this way, here we introduce a new type of generalized version of fuzzy closed set and using this concept a new version of fuzzy continuous-like function is introduced and studied.

Throughout this paper (X, τ) or simply by X we shall mean a fuzzy topological space (fts, for short) in the sense of Chang [7]. A fuzzy set [16] A in an fts X, denoted by $A \in I^X$, is defined to be a mapping from a non-empty set X into the closed interval $I = [0, 1]$. The support [16] of a fuzzy set A, denoted by $suppA$ [16] and is defined by $suppA = \{x \in X : A(x) \neq 0\}$. The fuzzy set with the singleton support $\{x\} \subseteq X$ and the value t $(0 < t \leq 1)$ will be denoted by x_t [16]. 0_X and 1_X are the constant fuzzy sets taking values 0 and 1 respectively in X. The complement [16] of a fuzzy set A in X is denoted by $1_X \setminus A$ and is defined by $(1_X \setminus A)(x) = 1 - A(x)$, for each $x \in X$. For any two fuzzy sets A, B in X, $A \leq B$ means $A(x) \leq B(x)$, for all $x \in X$ [16] while AqB means A is quasi-coincident (q-coincident, for short) [11] with B, i.e., there exists $x \in X$ such that $A(x) + B(x) > 1$. The negation of these two statements will be denoted by $A \not\leq B$ and $A \not{q} B$ respectively. For a fuzzy set A, clA and $intA$ will stand for fuzzy closure [7] and fuzzy interior [7] respectively. A fuzzy set A in an fts X is called fuzzy regular open [1] if $A = intclA$. A fuzzy set A is called a fuzzy neighbourhood (nbd, for short) of a fuzzy point x_t if there exists a fuzzy open set G in X such that $x_t \leq G \leq A$ [11]. If, in addition, A is open, then A is called a fuzzy open nbd [11] of x_t. A fuzzy set A in X is called a q-neighbourhood (q-nbd, for short) [11] of a fuzzy point x_t if there is a fuzzy open set U in X such that $x_t q U \leq A$. If, in addition, A is fuzzy open (resp., fuzzy regular open), then A is called fuzzy open q-nbd [11] (resp., fuzzy regular open q-nbd [1]) of x_t. A fuzzy point x_α is said to be a fuzzy δ-cluster point of a fuzzy set A in an fts X if every fuzzy regular open q-nbd U of x_α is q-coincident with A [9]. The union of all fuzzy δ-cluster points of A is called the fuzzy δ-closure of A, denoted by δclA [9]. A fuzzy set A is called fuzzy δ-closed if $A = \delta clA$ [9] and the complement of a fuzzy δ-closed set is called fuzzy δ-open [9]. The union of all fuzzy δ-open sets contained in a fuzzy set A is called fuzzy δ-interior of A and is denoted by $\delta intA$ [9]. For a fuzzy set A in an fts (X, τ), $\delta cl(1_X \setminus A) = 1_X \setminus \delta intA$ [9]. A fuzzy set A in an fts X is called fuzzy semiopen [1] (respectively, fuzzy β-open [8]) if $A \leq clintA$ (respectively, $A \leq clintclA$). The complement of a fuzzy semiopen (respectively, fuzzy β-open) set is called fuzzy semiclosed [1] (respectively, fuzzy β-closed [8]). The intersection of all fuzzy semiclosed (respectively, fuzzy β-closed) sets containing a fuzzy set A is called fuzzy semiclosure [1] (respectively, fuzzy β-closure [8]) of A, denoted by $sclA$ (respectively, βclA). The collection of all fuzzy semiopen (respectively, fuzzy β-open, fuzzy δ-open) sets in an fts X is denoted by $FSO(X)$ (respectively, $F\beta O(X)$, $F\delta O(X)$) and that of fuzzy semiclosed (respectively, fuzzy β-closed, fuzzy δ-closed) sets is denoted by $FSC(X)$ (respectively, $F\beta C(X)$, $F\delta C(X)$).

2. Some Well-Known Definitions

In this section we first recall some definitions from [2, 3, 10, 12, 15] for ready references.

Definition 2.1. A fuzzy set A in an fts (X, τ) is called
(i) fg-closed [2, 3] if $clA \leq U$ whenever $A \leq U$ where U is fuzzy open in X,
(ii) fgs-closed [3] if $sclA \leq U$ whenever $A \leq U$ where U is fuzzy open in X,
(iii) fsg-closed [3] if $sclA \leq U$ whenever $A \leq U$ where $U \in FSO(X)$,
(iv) $fg\beta$-closed [3] if $\beta clA \leq U$ whenever $A \leq U$ where U is fuzzy open in X,
(v) $f\beta g$-closed [3] if $\beta clA \leq U$ whenever $A \leq U$ where $U \in F\beta O(X)$.

Definition 2.2. A function $f : X \to Y$ is called
(i) fuzzy closed [15] if $f(U)$ is fuzzy closed in Y for every fuzzy closed set U in X,
(ii) fg-closed [3] if $f(U)$ is fg-closed in Y for every fuzzy closed set U in X,
(iii) fgs-closed [3] if $f(U)$ is fgs-closed in Y for every fuzzy closed set U in X,
(iv) fuzzy continuous [12] if $f^{-1}(U)$ is fuzzy open in X for every fuzzy open set U in Y,
(v) fg-continuous [3] if $f^{-1}(U)$ is fg-closed in X for every fuzzy closed set U in Y.

Definition 2.3 [10]. An fts (X, τ) is said to be fuzzy normal if for any two fuzzy closed sets A, B in X with $A \not{q} B$, there exist two fuzzy open sets U, V in X such that $A \leq U, B \leq V$ and $U \not{q} V$.

3. Fuzzy Generalized δ-semiopen Set : Some Properties

In this section we first recall the definition of fuzzy δ-semiopen set from [5] and then establish the mutual relationships between this set with the sets mentioned in Section 2. Afterwards, we introduce and study $fg\delta$-semiopen set and $fg\delta$-semiclosed function.

Definition 3.1 [5]. A fuzzy set A in an fts (X,τ) is called fuzzy δ-semiopen if $A \leq cl(\delta intA)$. The complement of fuzzy δ-semiopen set is called fuzzy δ-semiclosed set.

The union (intersection) of all fuzzy δ-semiopen (fuzzy δ-semiclosed) sets contained in (containing) a fuzzy set A is called fuzzy δ-semiinterior (fuzzy δ-semiclosure) of A, denoted by $\delta sintA$ ($\delta sclA$). A fuzzy set A in an fts (X,τ) is fuzzy δ-semiclosed (fuzzy δ-semiopen) iff $A = \delta sclA$ ($A = \delta sintA$).

The collection of all fuzzy δ-semiopen (fuzzy δ-semiclosed) sets in X is denoted by $F\delta SO(X)$ ($F\delta SC(X)$).

Remark 3.2 (i). It is clear from definition that $\delta intA \leq intA$ for any fuzzy set A in an fts (X,τ) and so fuzzy δ-semiopen set is fuzzy semiopen. But the converse is not true, in general, follows from next example.
(ii) The collection of all fuzzy closed sets in $X$and $F\delta SC(X)$ are independent concepts follows from the next example.
(iii) Fuzzy δ-open set is fuzzy δ-semiopen, but not conversely, follows from the next example.
(iv) For any fuzzy set A in an fts (X,τ), $A \in F\delta SO(X)$ implies $A \leq cl(\delta intA) \leq cl(intA) \leq cl(int(clA))$ implies $A \in F\beta O(X)$. But not conversely follows from the next example.

Example 3.3. Let $X = \{a,b\}$, $\tau = \{0_X, 1_X, A, B\}$ where $A(a) = 0.5, A(b) = 0.3, B(a) = 0.6, B(b) = 0.4$. Then (X,τ) is an fts. Now $FSO(X) = \{0_X, 1_X, U, V\}$ where $A \leq U \leq 1_X \setminus A, V \geq B$ and $FSC(X) = \{0_X, 1_X, 1_X \setminus U, 1_X \setminus V\}$ where $A \leq 1_X \setminus U \leq 1_X \setminus A, 1_X \setminus V \leq 1_X \setminus B$, $F\delta O(X) = \{0_X, 1_X, A\}$, $F\delta C(X) = \{0_X, 1_X, 1_X \setminus A\}$, $F\delta SO(X) = \{0_X, 1_X, U\}, F\delta SC(X) = \{0_X, 1_X, 1_X \setminus U\}$ where $A \leq U \leq 1_X \setminus A$. Consider the fuzzy set C, defined by $C(a) = 0.4, C(b) = 0.5$. Then $C \in FSC(X)$. But $int(\delta clC) = int(1_X \setminus A) = A \not\leq C \Rightarrow C \notin F\delta SC(X)$.
Now $1_X \setminus B \in \tau^c$. But $1_X \setminus B \notin F\delta SC(X)$ as $int(\delta cl(1_X \setminus B)) = int(1_X \setminus A) = A \not\leq 1_X \setminus B$.

Consider the fuzzy set D, defined by $D(a) = D(b) = 0.5$. Then $D \notin \tau^c$, but $int(\delta clD) = int(1_X \setminus A) = A \leq D$ and so $D \in F\delta SC(X)$. Again consider the fuzzy set E, defined by $E(a) = 0.5, E(b) = 0.6$. Then $E \in F\delta SO(X)$, but $E \notin F\delta O(X)$.
Also $int(cl(intC)) = 0_X \leq C$ and so $C \in F\beta C(X)$, but $C \notin F\delta SC(X)$.

Definition 3.4. A function $f : X \to Y$ is called fuzzy δ-semiclosed (resp., fuzzy δ-semiopen) if $f(F) \in F\delta SC(Y)$ (resp., $f(F) \in F\delta SO(Y)$) in Y for each $F \in F\delta SC(X)$ (resp., $F \in F\delta SO(X)$).

Proposition 3.5. If a function $f : X \to Y$ is fuzzy δ-semiclosed, injective, then for each $B \in I^Y$ and each $V \in F\delta SO(X)$ with $f^{-1}(B) \leq V$, there exists $U \in F\delta SO(Y)$ such that $B \leq U$ and $f^{-1}(U) \leq V$.
Proof. Let $B \in I^Y$ and $V \in F\delta SO(X)$ with $f^{-1}(B) \leq V$. Then $1_X \setminus V \leq 1_X \setminus f^{-1}(B) \Rightarrow f(1_X \setminus V) \leq f(1_X \setminus f^{-1}(B)) \leq 1_Y \setminus B$ (as f is injective). Since f is fuzzy δ-semiclosed, $f(1_X \setminus V) \in F\delta SC(Y)$. Let $U = 1_Y \setminus f(1_X \setminus V)$. Then $U \in F\delta SO(Y)$ and $B \leq U$. Again $f^{-1}(U) = f^{-1}(1_Y \setminus f(1_X \setminus V)) \leq 1_X \setminus (1_X \setminus V) = V$.

Definition 3.6. A fuzzy set A in an fts (X, τ) is said to be fuzzy generalized δ-semiclosed ($fg\delta$-semiclosed, for short) if $\delta sclA \leq U$ whenever $A \leq U$ where U is fuzzy open in X.

The complement of a fuzzy generalized δ-semiclosed set is called fuzzy generalized δ-semiopen ($fg\delta$-semiopen, for short).

Definition 3.7. A fuzzy set A is called a fuzzy generalized δ-semiopen neighbourhood ($fg\delta$-semiopen nbd, for short) of a fuzzy point x_α if there is an $fg\delta$-semiopen set U in X such that $x_\alpha \leq U \leq A$.

Remark 3.8. (i) A fuzzy δ-semiclosed set is $fg\delta$-semiclosed, but not conversely follows from the next example.
(ii) Since for any fuzzy set A in an fts (X, τ), $sclA \leq \delta sclA$, $\beta clA \leq \delta sclA$, we conclude that $fg\delta$-semiclosed set is fgs-closed and $fg\beta$-closed. But the converses are not true, in general, follow from the next example.
(iii) In [5], it is shown that a fuzzy point $x_\alpha \in \delta sclA$ for any fuzzy set A in an fts X iff every fuzzy δ-semiopen set U with $x_\alpha qU$, UqA. From

this it is clear that union of two $fg\delta$-semiclosed sets is $fg\delta$-semiclosed. But the intersection of two $fg\delta$-semiclosed sets may not be so, follows from the next example.

Example 3.9 (i). Let $X = \{a,b\}$, $\tau = \{0_X, 1_X, A, B\}$ where $A(a) = 0.5, A(b) = 0.6, B(a) = 0.5, B(b) = 0.55$. Then (X,τ) is an fts. Here $FSO(X) = \{0_X, 1_X, U\}$ where $U \geq B$, $FSC(X) = \{0_X, 1_X, 1_X \setminus U\}$ where $1_X \setminus U \leq 1_X \setminus B$, $F\delta SO(X) = F\delta SC(X) = \{0_X, 1_X\}$. Consider a fuzzy set V, defined by $V(a) = V(b) = 0.6$. Then $V \notin F\delta SC(X)$. But 1_X is the only fuzzy open set in X such that $V < 1_X$ and so $\delta sclV \leq 1_X \Rightarrow V$ is $fg\delta$-semiclosed in X.
Next consider a fuzzy set C, defined by $C(a) = 0.5, C(b) = 0.4$. Then $C \in FSC(X) \Rightarrow C$ is fgs-closed in X. But $C < B (\in \tau)$ and $\delta sclC = 1_X \not\leq B \Rightarrow C$ is not $fg\delta$-semiclosed. Also $C \in F\beta C(X)$ and so C is $fg\beta$-closed in X.
(ii). Consider Example 3.3. Consider two fuzzy sets C and D, defined by $C(a) = 0.55, C(b) = 0.7, D(a) = 0.7, D(b) = 0.4$. Only 1_X is the fuzzy open set in X such that $C < 1_X$, $D < 1_X$ and so $\delta sclC \leq 1_X$ and $\delta sclD \leq 1_X$ imply that C and D are $fg\delta$-semiclosed in X. Let $E = C \bigwedge D$. Then $E(a) = 0.55, E(b) = 0.4$. Then $B \in \tau$ be such that $E < B$. Then $\delta sclE = 1_X \not\leq B$ which implies that E is not $fg\delta$-semiclosed in X

Remark 3.10. $f\beta g$-closedness and $fg\delta$-semiclosedness are independent concepts follows from the next two examples.

Example 3.11. Not every $f\beta g$-closed set is $fg\delta$-semiclosed set
Consider Example 3.9(i). Here any fuzzy set $W \not\leq 1_X \setminus B$ is fuzzy β-open in X. Consider a fuzzy set T such that $T > 1_X \setminus B$. Now $C < T$ and $\beta clC = C < T$ and so C is $f\beta g$-closed set in (X,τ), though C is not $fg\delta$-semiclosed in X.

Example 3.12. Not every $fg\delta$-semiclosed set is $f\beta g$-closed set
Let $X = \{a,b\}$, $\tau = \{0_X, 1_X, A\}$ where $A(a) = 0.5, A(b) = 0.6$. Then (X,τ) is an fts. $F\beta O(X) = \{0_X, 1_X, U\}$ where $U \not\leq 1_X \setminus A$ and $F\beta C(X) = \{0_X, 1_X, 1_X \setminus U\}$ where $1_X \setminus U \not\geq A$. $F\delta SO(X) = F\delta SC(X) = \{0_X, 1_X\}$. Consider the fuzzy set B defined by $B(a) = 0.5, B(b) = 0.7$. Then $B \in F\beta O(X)$ and $B \leq B$. Now $\beta clB = 1_X \not\leq B \Rightarrow B$ is not $f\beta g$-closed set. But 1_X is the only

fuzzy open set in X such that $B < 1_X$ and so $\delta sclB = 1_X \leq 1_X$ which shows that B is $fg\delta$-semiclosed in X.

Remark 3.13. fsg-closedness and $fg\delta$-semiclosedness are independent concepts follows from the next two examples.

Example 3.14. Not every fsg-closed set is $fg\delta$-semiclosed set Consider Example 3.9(i). Now $C < B \in FSO(X)$ and $sclC = C < B$ which implies that C is fsg-closed set in X. But C is not $fg\delta$-semiclosed as shown in Example 3.9(i).

Example 3.15. Not every $fg\delta$-semiclosed set is fsg-closed set Consider Example 3.12. Here B is $fg\delta$-semiclosed. Now $B \in FSO(X) = \{0_X, 1_X, U\}$ where $U \geq A$. Then $FSC(X) = \{0_X, 1_X, 1_X \setminus U\}$ where $1_X \setminus U \leq 1_X \setminus A$. So $sclB = 1_X \not\leq B$ and so B is not fsg-closed set.

Remark 3.16. fg-closedness and $fg\delta$-semiclosedness are independent concepts follows from the next two examples.

Example 3.17. Not every fg-closed set is $fg\delta$-semiclosed set Consider Example 3.14. Here C is not $fg\delta$-semiclosed. Now $C < B(\in \tau)$ and $clC = 1_X \setminus A = C < B$ and so C is fg-closed set.

Example 3.18. Not every $fg\delta$-semiclosed set is fg-closed set Consider Example 3.3. Here $1_X \setminus E$ being fuzzy δ-semiclosed is $fg\delta$-semiclosed. Now $1_X \setminus E \leq B(\in \tau)$. But $cl(1_X \setminus E) = 1_X \setminus A \not\leq B$ and so $1_X \setminus E$ is not fg-closed set.

Definition 3.19. A function $f : X \to Y$ is called $fg\delta$-semiclosed if $f(U)$ is $fg\delta$-semiclosed in Y for each fuzzy closed set U in X.

Note 3.20. $fg\delta$-semiclosed function and fg-closed function are independent concepts follow from the next two examples.

Example 3.21. ot every $fg\delta$-semiclosed function is fg-closed function
Let $X = \{a, b\}$, $\tau = \{0_X, 1_X, C\}$, $\tau_1 = \{0_X, 1_X, A, B\}$ where $C(a) = 0.5, C(b) = 0.6, A(a) = 0.5, A(b) = 0.3, B(a) = 0.6, B(b) = 0.4$. Then (X, τ) and (X, τ_1) are fts's. Now $F\delta SC(X, \tau_1) = \{0_X, 1_X, U\}$ where

$A \leq U \leq 1_X \setminus A$. Consider the identity function $i : (X, \tau) \to (X, \tau_1)$. Now $1_X \setminus C \in \tau^c$, $i(1_X \setminus C) = 1_X \setminus C \leq B(\in \tau_1)$. Now $\delta scl_{\tau_1}(1_X \setminus C) = 1_X \setminus C \leq B$ implies that $1_X \setminus C$ is $fg\delta$-semiclosed in (X, τ_1) and so i is $fg\delta$-semiclosed function. But $cl_{\tau_1}(1_X \setminus C) = 1_X \setminus A \not\leq B$ implies that $1_X \setminus C$ is not fg-closed in $(X.\tau_1)$ and so i is not fg-closed function.

Example 3.22. Not every fg-closed function is $fg\delta$-semiclosed function
Let $X = \{a, b\}$, $\tau = \{0_X, 1_X, A\}$, $\tau_1 = \{0_X, 1_X, A, B\}$ where $A(a) = 0.5, A(b) = 0.6, B(a) = 0.5, B(b) = 0.55$. Then (X, τ) and (X, τ_1) are fts's. Consider the identity function $i : (X, \tau) \to (X, \tau_1)$. Now $F\delta SC(X, \tau_1) = \{0_X, 1_X\}$. Now $1_X \setminus A \in \tau^c$, $i(1_X \setminus A) = 1_X \setminus A < B(\in \tau_1)$. So $cl_{\tau_1}(1_X \setminus A) = 1_X \setminus A < B$ and so $1_X \setminus A$ is fg-closed in (X, τ_1) which shows that i is fg-closed function. But $\delta scl_{\tau_1}(1_X \setminus A) = 1_X \not\leq B$ and so $1_X \setminus A$ is not $fg\delta$-semiclosed set in (X, τ_1). Hence i is not $fg\delta$-semiclosed function.

Theorem 3.23. An injective function $f : X \to Y$ is $fg\delta$-semiclosed if and only if for each $S \in I^Y$ and each fuzzy open set U in X with $f^{-1}(S) \leq U$, there exists $fg\delta$-semiopen set V in Y such that $S \leq V$ and $f^{-1}(V) \leq U$.
Proof. Let f be $fg\delta$-semiclosed function. Let $S \in I^Y$ and U be a fuzzy open set in X such that $f^{-1}(S) \leq U$. Then $1_X \setminus f^{-1}(S) \geq 1_X \setminus U \Rightarrow f(1_X \setminus U) \leq f(1_X \setminus f^{-1}(S)) \leq 1_Y \setminus f(f^{-1}(S)) = 1_Y \setminus S$ (as f is injective). Now $1_X \setminus U$ is fuzzy closed in X. Then $f(1_X \setminus U)$ is $fg\delta$-semiclosed in Y. Let $V = 1_Y \setminus f(1_X \setminus U)$. Then V is $fg\delta$-semiopen in Y. Now $S \leq 1_Y \setminus f(1_X \setminus U) = V$ and $f^{-1}(V) = f^{-1}(1_Y \setminus f(1_X \setminus U)) = 1_X \setminus f^{-1}(f(1_X \setminus U)) \leq U$.
Conversely, let F be a fuzzy closed set in X and O be a fuzzy open set in Y such that

$$f(F) \leq O....(i)$$

Then $f^{-1}(1_Y \setminus f(F)) = 1_X \setminus f^{-1}(f(F)) \leq 1_X \setminus F$ which is fuzzy open in X. By hypothesis, there exists an $fg\delta$-semiopen set V in Y such that

$$1_Y \setminus f(F) \leq V...(ii)$$

and

$$f^{-1}(V) \leq 1_X \setminus F...(iii)$$

Therefore, $F \leq 1_X \setminus f^{-1}(V)$ implies that $f(F) \leq f(1_X \setminus f^{-1}(V)) \leq 1_Y \setminus V$ (as f is injective) and so

$$V \leq 1_Y \setminus f(F)...(iv)$$

From (i), $1_Y \setminus O \leq 1_Y \setminus f(F)$, $f^{-1}(1_Y \setminus O) \leq f^{-1}(1_Y \setminus f(F)) \leq f^{-1}(V)$ (by (ii)) $\leq 1_X \setminus F$ (by (iii)). Then $F \leq 1_X \setminus f^{-1}(V) \leq 1_X \setminus f^{-1}(1_Y \setminus f(F))$ (by (ii) $\leq 1_X \setminus f^{-1}(1_Y \setminus O)$ which shows that $f(F) \leq f(1_X \setminus f^{-1}(1_Y \setminus O)) \leq 1_Y \setminus f(f^{-1}(1_Y \setminus O)) = O$ (as f is injective). As $1_Y \setminus V$ is $fg\delta$-semiclosed in Y, $\delta scl(f(F)) \leq \delta scl(1_Y \setminus V)$ (by (iv)) $= 1_Y \setminus V \leq f(F)$ (by (ii)) $\leq O$ (by (i))and so $f(F)$ is $fg\delta$-semiclosed in Y. Consequently, f is $fg\delta$-semiclosed function.

Now we recall the next two definitions from [8, 14] for ready references.

Definition 3.24 [14]. A function $f : X \rightarrow Y$ is said to be fuzzy presemiopen (resp., fuzzy presemiclosed) function if $f(V)$ is fuzzy semiopen (resp., fuzzy semiclosed) in Y for every fuzzy semiopen (resp., fuzzy semiclosed) set V in X.

Definition 3.25 [8]. A function $f : X \rightarrow Y$ is said to be fuzzy β-open (resp., fuzzy β-closed) function if $f(V)$ is fuzzy β-open (resp., fuzzy β-closed) in Y for every fuzzy β-open (resp., fuzzy β-closed) set V in X.

Theorem 3.26. If a function $f : X \rightarrow Y$ is fuzzy presemiclosed, continuous and $fg\delta$-semiclosed function and $A(\in I^X)$ is $fg\delta$-semiclosed set in X, then $f(A)$ is fgs-closed set in Y.

Proof. Let O be any fuzzy set in Y such that $f(A) \leq O$. Then $A \leq f^{-1}(f(A)) \leq f^{-1}(O)$ which is fuzzy open in X as f is continuous. Since A is $fg\delta$-semiclosed, $sclA \leq \delta sclA \leq f^{-1}(O)$. $sclA$ being fuzzy semiclosed in X, $f(sclA)$ is fuzzy semiclosed in Y as f is fuzzy presemiclosed and so $scl(f(sclA)) = f(sclA) \leq f(\delta sclA) \leq f(f^{-1}(O)) \leq O$. Now $f(A) \leq scl(f(A)) \leq scl(f(sclA)) \leq O$ which implies that $scl(f(A)) \leq O$ and so $f(A)$ is fgs-closed in Y.

Theorem 3.27. If a function $f : X \rightarrow Y$ is fuzzy β-closed, continuous and $fg\delta$-semiclosed function and $A(\in I^X)$ is $fg\delta$-semiclosed set in X, then $f(A)$ is $fg\beta$-closed set in Y.

Proof. The proof is same as that of the proof of Theorem 3.26.

Remark 3.28. Composition of two $fg\delta$-semiclosed functions may not be so as it seen from the following example.

Example 3.29. Let $X = \{a, b\}$, $\tau_1 = \{0_X, 1_X, A\}$, $\tau_2 = \{0_X, 1_X\}$, $\tau_3 = \{0_X, 1_X, A, B\}$ where $A(a) = 0.5, A(b) = 0.6, B(a) = 0.5, B(b) = 0.55$. Then (X, τ_1), (X, τ_2) and (X, τ_3) are fts's. Consider two identity functions $i_1 : (X, \tau_1) \to (X, \tau_2)$ and $i_2 : (X, \tau_2) \to (X, \tau_3)$. Clearly i_1 and i_2 are $fg\delta$-semiclosed functions. Now $1_X \setminus A \in \tau_1^c$. Then $(i_2 \circ i_1)(1_X \setminus A) = 1_X \setminus A < B(\in \tau_3)$. Now $F\delta SC(X, \tau_3) = \{0_X, 1_X\}$ and so $\delta scl_{\tau_3}(1_X \setminus A) = 1_X \not\leq B$ and so $1_X \setminus A$ is not $fg\delta$-semiclosed in (X, τ_3) and so $i_2 \circ i_1$ is not $fg\delta$-semiclosed function.

Theorem 3.30. If $f : X \to Y$ is fuzzy closed function and $g : Y \to Z$ is $fg\delta$-semiclosed function, then $g \circ f : X \to Z$ is $fg\delta$-semiclosed function.
Proof. Let U be a fuzzy closed set in X. As f is fuzzy closed function $f(U)$ is fuzzy closed set in Y. Again g is $fg\delta$-semiclosed function, $g(f(U)) = (g \circ f)(U)$ is $fg\delta$-semiclosed set in Z. Hence $g \circ f$ is $fg\delta$-semiclosed function.

Definition 3.31. An fts (X, τ) is called fuzzy δ-seminormal if for any two fuzzy closed sets A, B in X with $A \not q B$, there exist two $fg\delta$-semiopen sets U, V in X such that $A \leq U, B \leq V$ and $U \not q V$.

Theorem 3.32. If $f : X \to Y$ is $fg\delta$-semiclosed, continuous, bijective function from a fuzzy normal space X onto an fts Y, then Y is fuzzy δ-seminormal.
Proof. Let A, B be two fuzzy closed sets in Y with $A \not q B$, Then $f^{-1}(A), f^{-1}(B)$ are fuzzy closed sets in X with $f^{-1}(A) \not q f^{-1}(B)$ (as f is fuzzy continuous function). Since X is fuzzy normal, there exist two fuzzy open sets U, V in X such that $f^{-1}(A) \leq U, f^{-1}(B) \leq V$ and $U \not q V$. By Theorem 3.23, there are $fg\delta$-semiopen sets G, H in Y such that $A \leq G, B \leq H$ and $f^{-1}(G) \leq U, f^{-1}(H) \leq V$. We claim that $G \not q H$. Indeed, if GqH, then there exists $y \in Y$ such that $G(y) + H(y) > 1$ and so $[f(U)](y) + [f(V)](y) > 1$ (as f is bijective) which implies that $U(f^{-1}(y)) + V(f^{-1}(y)) > 1$ and so UqV, a contradiction. Hence Y is fuzzy δ-seminormal space.

4. Fuzzy Generalized δ-Semiclosure Operator and Fuzzy Generalized δ-Semicontinuous Function

In this section we first introduce and study fuzzy generalized δ-semiclosure operator and then introduce fuzzy generalized δ-semiopen function. Afterwards, fuzzy generalized δ-semicontinuous function is introduced and studied.

Definition 4.1. The intersection of all $fg\delta$-semiclosed sets containing a fuzzy set A in an fts (X, τ) is called fuzzy generalized δ-semiclosure of A, denoted by $g\delta scl(A)$, i.e., $g\delta scl(A) = \bigwedge\{F : A \leq F$ and F is $fg\delta$-semiclosed set in $X\}$.

Remark 4.2. For any fuzzy set A in an fts (X, τ), we have $A \leq g\delta scl(A)$. If A is $fg\delta$-semiclosed, then $A = g\delta scl(A)$. But $g\delta scl$ may not be $fg\delta$-semiclosed follows from the fact that intersection of two $fg\delta$-semiclosed sets need not be so, as it is seen in Example 3.9(ii).

Proposition 4.3. Let (X, τ) be an fts and $A \in I^X$. Then for a fuzzy point x_t in X, $x_t \in g\delta scl(A)$ if and only if every $fg\delta$-seniopen set U, x_tqU implies UqA.

Proof. Let $x_t \in g\delta scl(A)$ for any $A \in I^X$ and U be any $fg\delta$-semiopen set in X with x_tqU. Now $x_t \in g\delta scl(A) \Rightarrow x_t \in F$, for all $fg\delta$-semiclosed sets $F \geq A$. Now $U(x) + t > 1$ implies that $t > 1 - U(x)$ and so $x_t \notin 1_X \setminus U$ which is $fg\delta$-semiclosed in X. Then by definition, $A \not\leq 1_X \setminus U$ and so there exists $y \in X$ such that $A(y) > (1_X \setminus U)(y) = 1 - U(y)$. Hence AqU.

Conversely, let for every $fg\delta$-semiopen set U in X, x_tqU imply UqA. We have to prove that $x_t \in F$, for all $fg\delta$-semiclosed set $F \geq A$. Let F be $fg\delta$-semiclosed set in X with $F \geq A$. If possible, let $x_t \notin F$. Then $F(x) < t$ and so $1 - F(x) > 1 - t$ which implies that $x_tq(1_X \setminus F)$ where $1_X \setminus F$ is $fg\delta$-semiopen in X. By hypothesis, $(1_X \setminus F)qA$. As $1_X \setminus F \leq 1_X \setminus A$, $(1_X \setminus A)qA$, a contradiction. The claim follows.

Theorem 4.4. Let (X, τ) be an fts and $A, B \in I^X$. Then the following statements are true :

(i) $g\delta scl(0_X) = 0_X$,

(ii) $g\delta scl(1_X) = 1_X$,

(iii) if $A \leq B$, then $g\delta scl(A) \leq g\delta scl(B)$,

(iv) $g\delta scl(A \bigvee B) = g\delta scl(A) \bigvee g\delta scl(B)$,

(v) $g\delta scl(A \bigwedge B) \leq g\delta scl(A) \bigwedge g\delta scl(B)$, equality does not hold, in

general, follows from Remark 4.2,
(vi) $g\delta scl(g\delta scl(A)) = g\delta scl(A)$.
Proof. (i), (ii) and (iii) are obvious.
(iv) By (iii), $g\delta scl(A) \bigvee g\delta scl(B) \leq g\delta scl(A \bigvee B)$.
To prove the converse, let $x_t \in g\delta scl(A \bigvee B)$. Then by Result 4.3, for any $fg\delta$-semiopen set U in X, $x_t qU$ implies $Uq(A \bigvee B)$. Then there exists $y \in X$ such that $U(y) + max\{A(y), B(y)\} > 1$ which implies that either $U(y) + A(y) > 1$ or $U(y) + B(y) > 1$ and so either UqA or UqB. Then either $x_t \in g\delta scl(A)$ or $x_t \in g\delta scl(B)$. So $x_t \in g\delta scl(A) \bigvee g\delta scl(B)$.
(v) Follows from (iii).
(vi) From (iii) as $A \leq g\delta scl(A)$, $g\delta scl(A) \leq g\delta scl(g\delta scl(A))$.
Conversely, let $x_t \in g\delta scl(g\delta scl(A)) = g\delta scl(B)$ where $B = g\delta scl(A)$. Let U be any $fg\delta$-semiopen set in X with $x_t qU$. Then UqB which implies that there exists $y \in X$ such that $U(y)+B(y) > 1$. Let $B(y) = s$. Then $y_s \in B = g\delta scl(A)$. Now $y_s qU$ where U is $fg\delta$-semiopen in X and so UqA and so $x_t \in g\delta scl(A)$ and so $g\delta scl(g\delta scl(A)) \leq g\delta scl(A)$. The claim follows.

Theorem 4.5. If $f : X \to Y$ is $fg\delta$-semiclosed function, then $g\delta scl(f(A)) \leq f(clA)$, for all $A \in I^X$.
Proof. Let $A \in I^X$. Then clA is fuzzy closed in X. As f is $fg\delta$-semiclosed function, $f(clA)$ is $fg\delta$-semiclosed in Y. Now $f(A) \leq f(clA)$. So $g\delta scl(f(A)) \leq g\delta scl(f(clA)) = f(clA)$.

Definition 4.6. The union of all $fg\delta$-semiopen sets contained in a fuzzy set A in an fts X is called fuzzy $g\delta$-semiinterior of A, denoted by $g\delta sint(A)$.

Lemma 4.7. For a fuzzy set A in an fts (X, τ), the following statements are true:
(i) $g\delta scl(1_X \setminus A) = 1_X \setminus g\delta sint(A)$
(ii) $g\delta sint(1_X \setminus A) = 1_X \setminus g\delta scl(A)$.
Proof (i). Let $x_t \in g\delta scl(1_X \setminus A)$. If possible, let $x_t \notin 1_X \setminus g\delta sint(A)$. Then $1 - (g\delta sint(A))(x) < t$ which implies that $[g\delta sint(A)](x) + t > 1$ and so $g\delta sint(A)qx_t$. Then there exists at least one $fg\delta$-semiopen set $F \leq A$ with $x_t qF$ which shows that $x_t qA$. As $x_t \in g\delta scl(1_X \setminus A), Fq(1_X \setminus A)$ and so $Aq(1_X \setminus A)$, a contradiction. Hence

$$g\delta scl(1_X \setminus A) \leq 1_X \setminus g\delta sint(A)...(1)$$

.
Conversely, let $x_t \in 1_X \setminus g\delta sint(A)$. Then $1 - [(g\delta sint(A)](x) \geq t$ which implies that $x_t \not{q}(g\delta sint(A))$ and so $x_t \not{q}F$ where F is $fg\delta$-semiopen set in X contained in A ... (2).
Let U be any $fg\delta$-semiclosed set in X such that $1_X \setminus A \leq U$. Then $1_X \setminus U \leq A$. Now $1_X \setminus U$ is $fg\delta$-semiopen set in X contained in A. By (2), $x_t \not{q}(1_X \setminus U)$. Then $x_t \in U$ and so $x_t \in g\delta scl(1_X \setminus A)$ which implies that

$$1_X \setminus g\delta sint(A) \leq g\delta scl(1_X \setminus A)...(3).$$

Combining (1) and (3), (i) follows.
(ii) Putting $1_X \setminus A$ for A in (i), we get $g\delta scl(A) = 1_X \setminus g\delta sint(1_X \setminus A)$ which implies that $g\delta sint(1_X \setminus A) = 1_X \setminus g\delta scl(A)$.

Definition 4.8. A function $f : X \to Y$ is called $fg\delta$-semiopen if for each fuzzy open set U in X, $f(U)$ is $fg\delta$-semiopen in Y.

The next theorem characterizes $fg\delta$-semiopen function.

Theorem 4.9. For a bijective function $f : X \to Y$, the following statements are equivalent:
(i) f is $fg\delta$-semiopen,
(ii) $f(intA) \leq g\delta sint(f(A))$, for all $A \in I^X$,
(iii) for each fuzzy point x_t in X and each fuzzy open set U in X containing x_t, there exists an $fg\delta$-semiopen set V containing $f(x_t)$ such that $V \leq f(U)$.
Proof (i) $\Rightarrow$ (ii). Let $A \in I^X$. Then $intA$ is fuzzy open in X. By (i), $f(intA)$ is $fg\delta$-semiopen in Y. Since $f(intA) \leq f(A)$ and $g\delta sint(f(A))$ is the union of all $fg\delta$-semiopen sets contained in $f(A)$, we have $f(intA) \leq g\delta sint(f(A))$.
(ii) $\Rightarrow$ (i). Let U be a fuzzy open set in X. Then $f(U) = f(intU) \leq g\delta sint(f(U))$ (by (ii)) and so $f(U)$ is $fg\delta$-semiopen in Y.
(ii) $\Rightarrow$ (iii). Let x_t be a fuzzy point in X and U, a fuzzy open set in X such that $x_t \in U$. Then $f(x_t) \in f(U) = f(intU) \leq g\delta sint(f(U))$ (by (ii)). Then $f(U)$ is $fg\delta$-semiopen set in Y. Let $V = f(U)$. Then $f(x_t) \in V$ and $V \leq f(U)$.
(iii) $\Rightarrow$ (i). Let U be any fuzzy open set in X and y_t be any fuzzy point in $f(U)$, i.e., $y_t \in f(U)$. Then there exists $x \in X$ such that $f(x) = y$ (as f is bijective). Then $[f(U)](y) \geq t$ and so $U(f^{-1}(y)) \geq t$. Then $U(x) \geq t$ which implies that $x_t \in U$. By (iii), there exists an $fg\delta$-semiopen set V in Y such that $f(x_t) \in V$ and

$V \leq f(U)$. Then $f(x_t) \in V = g\delta sint(V) \leq g\delta sint(f(U))$. Since x_t is taken arbitrarily and $f(U)$ is the union of all fuzzy points in $f(U)$, $f(U) \leq g\delta sint(f(U))$ and so $f(U)$ is $fg\delta$-semiopen in Y. Hence f is $fg\delta$-semiopen function.

Theorem 4.10. If $f : X \to Y$ is $fg\delta$-semiopen bijective function, then the following statements are true :
(i) for each fuzzy point x_t in X and each fuzzy open set U with $x_t qU$, there exists $fg\delta$-semiopen set V with $f(x_t)qV$ such that $V \leq f(U)$,
(ii) $f^{-1}(g\delta scl(B)) \leq cl(f^{-1}(B))$, for all $B \in I^Y$.
Proof (i). Let x_t be any fuzzy point in X and U be any fuzzy open set in X with $x_t qU = intU$ which implies that $f(x_t)qf(intU) \leq g\delta sint(f(U))$ (by Theorem 4.9). Hence $f(x_t)qg\delta sint(f(U))$ and so there exists $fg\delta$-semiopen set V in Y such that $f(x_t)qV$ and $V \leq f(U)$.
(ii) Let x_t be any fuzzy point in X such that $x_t \notin cl(f^{-1}(B))$ for any $B \in I^Y$. Then there exists a fuzzy open set U in X with $x_t qU$, $U \not q f^{-1}(B)$. Now

$$f(x_t)qf(U)...(i)$$

where $f(U)$ is $fg\delta$-semiopen in Y (as f is $fg\delta$-semiopen function). Now $f^{-1}(B) \leq 1_X \setminus U$. Then $B \leq f(1_X \setminus U) \leq 1_Y \setminus f(U)$ and so $B \ /qf(U)$. Let $V = 1_Y \setminus f(U)$. Then V is $fg\delta$-semiclosed in Y with $B \leq V$. We claim that $f(x_t) \notin V$. If possible, let $f(x_t) \in V = 1_Y \setminus f(U)$. Then $1-[f(U)](f(x)) \geq t$ and so $f(U) \not q f(x_t)$, contradicts (i). So $f(x_t) \notin V$, then $f(x_t) \notin g\delta scl(B)$ which implies that $x_t \notin f^{-1}(g\delta scl(B))$ and hence $f^{-1}(g\delta scl(B)) \leq cl(f^{-1}(B))$.

Theorem 4.11. An injective function $f : X \to Y$ is $fg\delta$-semiopen if and only if for each $B \in I^Y$ and F, a fuzzy closed set in X with $f^{-1}(B) \leq F$, there exists an $fg\delta$-semiclosed set V in Y such that $B \leq V$ and $f^{-1}(V) \leq F$.
Proof. The proof is same as that of the proof of Theorem 3.23.

Definition 4.12. A function $f : X \to Y$ is called $fg\delta$-semicontinuous if $f^{-1}(V)$ is $fg\delta$-semiclosed in X for every fuzzy closed set V in Y.

Remark 4.13. fg-continuity and $fg\delta$-semicontinuity are independent concepts follows from next two examples.

Example 4.14. Not every $fg\delta$-semicontinuity is fg-continuity
Let $X = \{a, b\}$, $\tau = \{0_X, 1_X, A, B\}$, $\tau_1 = \{0_X, 1_X, C\}$ where $A(a) = 0.5, A(b) = 0.3, B(a) = 0.6, B(b) = 0.4, C(a) = 0.5, C(b) = 0.6$. Then (X, τ) and (X, τ_1) are fts's. Now $F\delta SC(X, \tau) = \{0_X, 1_X, U\}$ where $A \leq U \leq 1_X \setminus A$. Consider the identity function $i : (X, \tau) \to (X, \tau_1)$. Now $1_X \setminus C \in \tau_1^c$ and $i^{-1}(1_X \setminus C) = 1_X \setminus C < B(\in \tau)$. Then $\delta scl_\tau(1_X \setminus C) = 1_X \setminus C < B$ and so $1_X \setminus C$ is $fg\delta$-semiclosed in (X, τ). Hence i is $fg\delta$-semicontinuous function. But $cl_\tau(1_X \setminus C) = 1_X \setminus A \not\leq B$ implies that $1_X \setminus C$ is not fg-closed in (X, τ) which shows that i is not fg-continuous function.

Example 4.15. Not every fg-continuity implies $fg\delta$-semicontinuity
Let $X = \{a, b\}$, $\tau = \{0_X, 1_X, A, B\}$, $\tau_1 = \{0_X, 1_X, C\}$ where $A(a) = 0.5, A(b) = 0.6, B(a) = 0.5, B(b) = 0.55, C(a) = 0.5, C(b) = 0.6$. Then (X, τ) and (X, τ_1) are fts's. Now $F\delta SC(X, \tau) = \{0_X, 1_X\}$. Consider the identity function $i : (X, \tau) \to (X, \tau_1)$. Now $1_X \setminus C \in \tau_1^c$ and $i^{-1}(1_X \setminus C) = 1_X \setminus C < B(\in \tau)$ and $cl_\tau(1_X \setminus C) = 1_X \setminus A \leq B$. So $1_X \setminus C$ is fg-closed in (X, τ) which shows that i is fg-continuous function. But $\delta scl_\tau(1_X \setminus C) = 1_X \not\leq B$ and so $1_X \setminus C$ is not $fg\delta$-semiclosed in (X, τ). Hence i is not $fg\delta$-semicontinuous function.

Theorem 4.16. Let $f : X \to Y$ be a function. Then the following statements are equivalent:
(i) f is $fg\delta$-semicontinuous,
(ii) for each fuzzy point x_t in X and each fuzzy open set V in Y containing $f(x_t)$, there exists an $fg\delta$-semiopen set U containing x_t such that $f(U) \leq V$,
(iii) $f(g\delta scl(A)) \leq cl(f(A))$, for all $A \in I^X$,
(iv) $g\delta scl(f^{-1}(B)) \leq f^{-1}(clB)$, for all $B \in I^Y$.
Proof (i) $\Rightarrow$ (ii). Let x_t be a fuzzy point in X and V be any fuzzy open set in Y with $f(x_t) \in V$. Then $x_t \in f^{-1}(V)$. Let $U = f^{-1}(V)$. Then U is $fg\delta$-semiopen in X (by (i)) with $x_t \in U$ and $f(U) \leq V$.
(ii) $\Rightarrow$ (i). Let A be any fuzzy open set in Y and x_t be a fuzzy point in X such that $x_t \in f^{-1}(A)$. Then $f(x_t) \in A$. By (ii), there exists an $fg\delta$-semiopen set U in X with $x_t \in U$ such that $f(U) \leq A$. Then $x_t \in U \leq f^{-1}(A)$. Then $x_t \in U = g\delta sint(U) \leq g\delta sint(f^{-1}(A))$. Since x_t is taken arbitrarily and $f^{-1}(A)$ is the union of all fuzzy points in $f^{-1}(A)$, $f^{-1}(A) \leq g\delta sint(f^{-1}(A))$ and so $f^{-1}(A)$ is $fg\delta$-semiopen in X. Hence f is $fg\delta$-semicontinuous function.

(i) $\Rightarrow$ (iii). Let $A \in I^X$. Then $cl(f(A))$ is fuzzy closed set in Y. Now $A \leq f^{-1}(f(A)) \leq f^{-1}(cl(f(A)))$ which is $fg\delta$-semiclosed in X (by (i)) and so $g\delta scl(A) \leq g\delta scl(f^{-1}(cl(f(A)))) = f^{-1}(cl(f(A)))$ which implies that $f(g\delta scl(A)) \leq cl(f(A))$.
(iii) $\Rightarrow$ (i). Let V be a fuzzy closed set in Y. Put $U = f^{-1}(V)$. By (iii), $f(g\delta scl(U)) \leq cl(f(U)) = cl(f(f^{-1}(V))) \leq clV = V$ which shows that $g\delta scl(U) \leq f^{-1}(V) = U$. Then U is $fg\delta$-semiclosed in X and hence f is $fg\delta$-semicontinuous function.
(iii) $\Rightarrow$ (iv). Let $B \in I^Y$ and $A = f^{-1}(B)$. Then $A \in I^X$. By (iii), $f(g\delta scl(A)) \leq cl(f(A))$ which implies that $f(g\delta scl(f^{-1}(B))) \leq cl(f(f^{-1}(B))) \leq clB$ and hence $g\delta scl(f^{-1}(B)) \leq f^{-1}(clB)$.
(iv) $\Rightarrow$ (iii). Let $A \in I^X$. Then $f(A) \in I^Y$. By (iv), $g\delta scl(f^{-1}(f(A))) \leq f^{-1}(cl(f(A)))$ and so $g\delta scl(A) \leq g\delta scl(f^{-1}(f(A))) \leq f^{-1}(cl(f(A)))$. Hence $f(g\delta scl(A)) \leq cl(f(A))$.

Remark 4.17. Composition of two $fg\delta$-semicontinuous functions need not be so, as it seen from the following example.

Example 4.18. Let $X = \{a, b\}$, $\tau_1 = \{0_X, 1_X, A, B\}$, $\tau_2 = \{0_X, 1_X\}$, $\tau_3 = \{0_X, 1_X, C\}$ where $A(a) = 0.5, A(b) = 0.6$, $B(a) = 0.5, B(b) = 0.55, C(a) = 0.5, C(b) = 0.6$. Then (X, τ_1), (X, τ_2) and (X, τ_3) are fts's. Consider two identity functions $i_1 : (X, \tau_1) \to (X, \tau_2)$ and $i_2 : (X, \tau_2) \to (X, \tau_3)$. Clearly i_1 and i_2 are $fg\delta$-semicontinuous functions (as every fuzzy set in (X, τ_2) is $fg\delta$-semiclosed set in (X, τ_2)). Let $i_3 = i_2 \circ i_1 : (X, \tau_1) \to (X, \tau_3)$. We claim that i_3 is not $fg\delta$-semicontinuous function. Now $1_X \setminus C \in \tau_3^c$. $i_3^{-1}(1_X \setminus C) = 1_X \setminus C < B (\in \tau_1)$. But $\delta scl_{\tau_1}(1_X \setminus C) = 1_X \not\leq B$. Hence i_3 is not $fg\delta$-semicontinuous function.

Theorem 4.19. If $f : X \to Y$ is $fg\delta$-semicontinuous function and $g : Y \to Z$ is fuzzy continuous function, then $g \circ f : X \to Z$ is $fg\delta$-semicontinuous function.
Proof. Let U be a fuzzy closed set in Z. As g is fuzzy continuous function, $g^{-1}(U)$ is fuzzy closed set in Y. Again f is $fg\delta$-semicontinuous function, $f^{-1}(g^{-1}(U)) = (g \circ f)^{-1}(U)$ is $fg\delta$-semiclosed set in X. Hence $g \circ f$ is $fg\delta$-semicontinuous function.

5. $fg\delta$-Semiregular and $fg\delta$-Seminormal Spaces

Definition 5.1. An fts (X, τ) is said to be $fg\delta$-semiregular space if for any fuzzy point x_t in X and each $fg\delta$-semiclosed set F with $x_t \notin F$, there exist $U, V \in F\delta SO(X)$ such that $x_t \in U, F \leq V$ and $U \not{q} V$.

Definition 5.2. A fuzzy set A in an fts (X, τ) is called an $fg\delta$ q-nbd of a fuzzy point x_α in X if there is an $fg\delta$-semiopen set U in X such that $x_\alpha q U \leq A$. If, in addition, A is $fg\delta$-semiopen in X, then A is called an $fg\delta$-semiopen q-nbd of x_α.

Theorem 5.3. In an fts (X, τ), the following statements are equivalent:
(i) X is $fg\delta$-semiregular,
(ii) for each fuzzy point x_t in X and any $fg\delta$-semiopen q-nbd U of x_t, there exists $V \in F\delta SO(X)$ such that $x_t \in V$ and $\delta sclV \leq U$,
(iii) for each fuzzy point x_t in X and each $fg\delta$-semiclosed set A of X with $x_t \notin A$, there exists $U \in F\delta SO(X)$ with $x_t \in U$ such that $\delta sclU \not{q} A$.
Proof (i) $\Rightarrow$ (ii). Let x_t be a fuzzy point in X and U, any $fg\delta$-semiopen q-nbd of x_t. Then $x_t q U$. Then $U(x) + t > 1$ and so $x_t \notin 1_X \setminus U$ which is $fg\delta$-semiclosed in X. By (i), there exist $V, W \in F\delta SO(X)$ such that $x_t \in V, 1_X \setminus U \leq W$ and $V \not{q} W$. Then $V \leq 1_X \setminus W$ which implies that $\delta sclV \leq \delta scl(1_X \setminus W) = 1_X \setminus W \leq U$.
(ii) $\Rightarrow$ (iii). Let x_t be a fuzzy point in X and A, an $fg\delta$-semiclosed set in X with $x_t \notin A$. Then $A(x) < t \Rightarrow x_t q(1_X \setminus A)$ which is $fg\delta$-semiopen in X. By (ii), there exists $V \in F\delta SO(X)$ such that $x_t \in V$ and $\delta sclV \leq 1_X \setminus A$. Hence $\delta sclV \not{q} A$.
(iii) $\Rightarrow$ (i). Let x_t be a fuzzy point in X and F be any $fg\delta$-semiclosed set in X with $x_t \notin F$. Then by (iii), there exists $U \in F\delta SO(X)$ such that $x_t \in U$ and $\delta sclU \not{q} F$ which implies that $F \leq 1_X \setminus \delta sclU$ (=W, say). Then $W \in F\delta SO(X)$ and $U \not{q} W$ (as $U \not{q}(1_X \setminus \delta sclU)$) and so X is $fg\delta$-semiregular space.

Definition 5.4. An fts (X, τ) is called $fg\delta$-seminormal if for each pair of $fg\delta$-semiclosed sets A, B in X with $A \not{q} B$, there exist $U, V \in F\delta SO(X)$ such that $A \leq U, B \leq V$ and $U \not{q} V$.

Theorem 5.5. An fts (X, τ) is $fg\delta$-seminormal if and only if for every $fg\delta$-semiclosed set F and every $fg\delta$-semiopen set G with

$F \leq G$, there exists $H \in F\delta SO(X)$ such that $F \leq H \leq \delta sclH \leq G$.
Proof. Let X be $fg\delta$-seminormal and let F be $fg\delta$-semiclosed set and G be $fg\delta$-semiopen set with $F \leq G$. Then $F \not{q}(1_X \setminus G)$ where $1_X \setminus G$ is $fg\delta$-semiclosed in X. By hypothesis, there exist $H, T \in F\delta SO(X)$ such that $F \leq H, 1_X \setminus G \leq T$ and $H \not{q}T$. Then $H \leq 1_X \setminus T$ and so $\delta sclH \leq \delta scl(1_X \setminus T) = 1_X \setminus T \leq G$. Hence $F \leq H \leq \delta sclH \leq G$.

Conversely, let A, B be two $fg\delta$-semiclosed sets in X with $A \not{q}B$. Then $A \leq 1_X \setminus B$. By hypothesis, there exists $H \in F\delta SO(X)$ such that $A \leq H \leq \delta sclH \leq 1_X \setminus B$ implies that $A \leq H, B \leq 1_X \setminus \delta sclH = \delta sint(1_X \setminus H) \in F\delta SO(X)$ and $H \not{q}(1_X \setminus \delta sclH)$. Hence X is $fg\delta$-seminormal space.

Definition 5.6. A function $f : X \to Y$ is said to be $fg\delta$-semiirresolute if $f^{-1}(V)$ is $fg\delta$-semiclosed in X for all $fg\delta$-semiclosed set V in Y.

Theorem 5.7. Let X be an $fg\delta$-seminormal space and $f : X \to Y$ be an $fg\delta$-semiirresolute, fuzzy δ-semiopen bijective function from X onto Y. Then Y is $fg\delta$-seminormal space.
Proof. Let A, B be two $fg\delta$-semiclosed sets in Y with $A \not{q}B$. As f is $fg\delta$-semiirresolute, $f^{-1}(A), f^{-1}(B)$ are $fg\delta$-semiclosed sets in X with $f^{-1}(A) \not{q}f^{-1}(B)$. Since X is $fg\delta$-seminormal space, there exist $U, V \in F\delta SO(X)$ such that $f^{-1}(A) \leq U, f^{-1}(B) \leq V$ and $U \not{q}V$. Since f is bijective, fuzzy δ-semiopen, $A \leq f(U), B \leq f(V)$ and $f(U) \not{q}f(V)$ where $f(U), f(V) \in F\delta SO(Y)$. Hence Y is $fg\delta$-seminormal space.

6. $fg\delta$-Semi T_2-Space

In this section we first introduce a new type of separation axiom, viz., $fg\delta$-semi T_2-space and then it is shown that the inverse image of fuzzy T_2-space [13] under $fg\delta$-semicontinuous function is $fg\delta$-semi T_2 space. Afterwards three different types of fuzzy continuous-like functions are introduced and shown that the inverse image of $fg\delta$-semi T_2 space under these functions are fuzzy T_2-space. Lastly some mutual relationships of these newly defined functions and $fg\delta$-semicontinous functions are established.

We first recall the following definition and theorem from [13] for ready references.

Definition 6.1 [13]. An fts (X, τ) is called fuzzy T_2-space if for any two distinct fuzzy points x_α and y_β : when $x \neq y$, there exist fuzzy open sets U_1, U_2, V_1, V_2 such that $x_\alpha \in U_1$, $y_\beta q V_1, U_1 \not{q} V_1$ and $x_\alpha q U_2, y_\beta \in V_2, U_2 \not{q} V_2$; when $x = y$ and $\alpha < \beta$ (say), there exist fuzzy open sets U and V such that $x_\alpha \in U, y_\beta q V$ and $U \not{q} V$.

Theorem 6.2 [13]. If an fts (X, τ) is fuzzy T_2, then for any two distinct fuzzy points x_α and y_β in X; when $x \neq y$, there exist $U, V \in \tau$ such that $x_\alpha q U, y_\beta q V$ and $U \not{q} V$; when $x = y$ and $\alpha < \beta$ (say), x_α has a fuzzy open nbd U and y_β has a fuzzy open q-nbd V such that $U \not{q} V$.

Definition 6.3. An fts (X, τ) is said to be $fg\delta$-semi T_2-space if for any two distinct fuzzy points x_α and y_β in X; when $x \neq y$, there exist $fg\delta$-semiopen sets U, V in X such that $x_\alpha q U, y_\beta q V, U \not{q} V$; when $x = y$ and $\alpha < \beta$ (say), x_α has an $fg\delta$-semiopen nbd U and y_β has an $fg\delta$-semiopen q-nbd V such that $U \not{q} V$.

Theorem 6.4. If an injective function $f : X \to Y$ is $fg\delta$-semicontinuous from an fts X onto a fuzzy T_2-space Y, then X is $fg\delta$-semi T_2-space.

Proof. Let x_α and y_β be two distinct fuzzy points in X. Then $f(x_\alpha) = z_\alpha$ and $f(y_\beta) = w_\beta$ are two distinct fuzzy points in Y (as f is injective). Let $f(x) = z, f(y) = w$.

Case-1. When $x \neq y$. Then z_α, w_β are two distinct fuzzy points in Y. As Y is fuzzy T_2-space, by Theorem 6.2, there exist fuzzy open sets U, V in Y such that $z_\alpha q U, w_\beta q V$ and $U \not{q} V$. As f is $fg\delta$-semicontinuous, $f^{-1}(U), f^{-1}(V)$ are $fg\delta$-semiopen in X with $x_\alpha q f^{-1}(U), y_\beta q f^{-1}(V)$ and $f^{-1}(U) \not{q} f^{-1}(V)$ [Indeed, $z_\alpha q U$ implies that $U(z) + \alpha > 1$ and so $U(f(x)) + \alpha > 1$. Then $[f^{-1}(U)](x) + \alpha > 1$. Hence $x_\alpha q f^{-1}(U)$].

Case-2. When $x = y$ and $\alpha < \beta$ (say). As Y is fuzzy T_2-space, by Theorem 6.2, there exist fuzzy open sets U, V in Y such that $z_\alpha \in U, w_\beta q V$ and $U \not{q} V$. Then $U(z) \geq \alpha$ implies that $U(f(x)) \geq \alpha$ and so $[f^{-1}(U)](x) \geq \alpha$. Then $x_\alpha \in f^{-1}(U), y_\beta q f^{-1}(V)$ and $f^{-1}(U) \not{q} f^{-1}(V)$, where $f^{-1}(U)$ and $f^{-1}(V)$ are $fg\delta$-semiopen in X. Consequently, X is $fg\delta$-semi T_2-space.

In a similar manner we can easily state the following theorem.

Theorem 6.5. If an injective function $f : X \to Y$ is $fg\delta$-semiirresolute from an fts X into an $fg\delta$-semi T_2-space Y, then X is $fg\delta$-semi T_2-space.

Definition 6.6. An fts (X, τ) is said to be fuzzy δ-semi T_2-space if for any two distinct fuzzy points x_α and y_β in X; when $x \neq y$, there exist $U, V \in F\delta SO(X)$ such that $x_\alpha qU, y_\beta qV$ and $U \not{q} V$; when $x = y$ and $\alpha < \beta$ (say), x_α has a fuzzy δ-semiopen nbd U and y_β has a fuzzy δ-semiopen q-nbd V such that $U \not{q} V$.

Definition 6.7. A function $f : X \to Y$ is called
(i) strongly $fg\delta$-semicontinuous if $f^{-1}(V)$ is fuzzy open in X for every $fg\delta$-semiopen set V of Y,
(ii) weakly $fg\delta$-semicontinuous if $f^{-1}(V) \in F\delta SO(X)$ for every $fg\delta$-semiopen set V of Y,
(iii)$fg\delta^*$-semicontinuous if $f^{-1}(V)$ is $fg\delta$-semiopen in X for every $V \in F\delta SO(Y)$.

Now we can easily state the following theorems the proof of which are similar as that of Theorem 6.4.

Theorem 6.8. If an injective function $f : X \to Y$ is strongly $fg\delta$-semicontinuous from an fts X into an $fg\delta$-semi T_2-space Y, then X is fuzzy T_2-space.

Theorem 6.9. If an injective function $f : X \to Y$ is weakly $fg\delta$-semicontinuous from an fts X into an $fg\delta$-semi T_2-space Y, then X is fuzzy δ-semi T_2-space.

Theorem 6.10. If an injective function $f : X \to Y$ is $fg\delta^*$-semicontinuous from an fts X into a fuzzy δ-semi T_2-space Y, then X is $fg\delta$-semi T_2-space.

Remark 6.11. Strongly $fg\delta$-semicontinuity and weakly $fg\delta$-semicontinuity are independent notions follows from the next two examples.

Example 6.12. Not every Strongly $fg\delta$-semicontinuity implies weakly $fg\delta$-semicontinuity
Let $X = \{a\}$, $\tau_1 = \{0_X, 1_X, A\}$, $\tau_2 = \{0_X, 1_X, B\}$ where $A(a) \leq 0.4, B(a) = 0.6$. Then (X, τ_1) and (X, τ_2) are fts's. Consider the identity function $i : (X, \tau_1) \to (X, \tau_2)$. Now $F\delta SO(X, \tau_1) = \{0_X, 1_X, M\}$ where $M(a) = 0.4$ and $F\delta SC(X, \tau_1) = \{0_X, 1_X, 1_X \setminus M\}$ where $(1_X \setminus M)(a) = 0.6$. Clearly any fuzzy set $C > B$ is $fg\delta$-semiclosed in (X, τ_2). Then

$i^{-1}(C) = C \in \tau_1^c$ which shows that$i$ is strongly $fg\delta$-semicontinuous. Let D be a fuzzy set in X defined by $D(a) = 0.7$. Now $i^{-1}(D) = D$. Then $D \notin F\delta SC(X, \tau_1)$ and so i is not weakly $fg\delta$-semicontinuous.

Example 6.13. Not every Weakly $fg\delta$-semicontinuity implies strongly $fg\delta$-semicontinuity
Let $X = \{a\}$, $\tau_1 = \{0_X, 1_X, A\}$, $\tau_2 = \{0_X, 1_X, B\}$ where $A(a) = 0.3, B(a) = 0.6$. Then (X, τ_1) and (X, τ_2) are fts's. Consider the identity function $i : (X, \tau_1) \to (X, \tau_2)$. Now $F\delta SO(X, \tau_1) = \{0_X, 1_X, U\}$ where $A \le U \le 1_X \setminus A$ and $F\delta SC(X, \tau_1) = \{0_X, 1_X, 1_X \setminus U\}$ where $A \le 1_X \setminus U \le 1_X \setminus A$. Again, $F\delta SO(X, \tau_2) = F\delta SC(X, \tau_2) = \{0_X, 1_X\}$. Now any fuzzy set $C > B$ is $fg\delta$-semiclosed in (X, τ_2). Let D be a fuzzy set in X defined by $D(a) = 0.65$. Then D is $fg\delta$-semiclosed in (X, τ_2). Then $i^{-1}(D) = D$. Now $int_{\tau_1}(\delta cl_{\tau_1} D) = int_{\tau_1}(1_X \setminus A) = A < D$ which shows that i is weakly $fg\delta$-semicontinuous function. But $D \notin \tau_1^c$ and hence i is not strongly $fg\delta$-semicontinuous function.

Remark 6.14. Weakly $fg\delta$-semicontinuous function is $fg\delta^*$-semicontinuous, but not conversely follows from the next example.

Example 6.15. $fg\delta^*$-semicontinuity may not imply weakly $fg\delta$-semicontinuity
Consider Example 6.12. Here i is not weakly $fg\delta$-semicontinuous. Now $F\delta SO(X, \tau_2) = F\delta SC(X, \tau_2) = \{0_X, 1_X\}$ and so obviously i is $fg\delta^*$-semicontinuous.

Remark 6.16. In Example 3.29, it is shown that fuzzy closed set need not be $fg\delta$-semiclosed and so we can conclude that strongly $fg\delta$-semicontinuity does not imply $fg\delta^*$-semicontinuity. The next example shows that $fg\delta^*$-semicontinuity does not imply strongly $fg\delta$-semicontinuity, i.e., strongly $fg\delta$-semicontinuity and $fg\delta^*$-semicontinuity are independent concepts.

Example 6.17. $fg\delta^*$-semicontinuity may not imply strongly $fg\delta$-semicontinuity
Let $X = \{a\}$, $\tau_1 = \{0_X, 1_X\}$, $\tau_2 = \{0_X, 1_X, B\}$ where $B(a) = 0.6$. Then (X, τ_1) and (X, τ_2) are fts's. Consider the identity function $i : (X, \tau_1) \to (X, \tau_2)$. Now $F\delta SO(X, \tau_2) = F\delta SC(X, \tau_2) = \{0_X, 1_X\}$ and so obviously i is $fg\delta^*$-semicontinuous function. Now any fuzzy

set $C > B$ is $fg\delta$-semiclosed in (X, τ_2). Let D be a fuzzy set in X defined by $D(a) = 0.7$. Then D is $fg\delta$-semiclosed in (X, τ_2). Then $i^{-1}(D) = D \notin \tau_1^c$ and hence i is not strongly $fg\delta$-semicontinuous function.

Remark 6.18. The next examples establish the mutual relationships between $fg\delta$-semicontinuity with the functions defined in Definition 6.7.

Example 6.19. $fg\delta$-semicontinuity may not imply $fg\delta^*$-semicontinuity
Let $X = \{a\}$, $\tau_1 = \{0_X, 1_X, B\}$, $\tau_2 = \{0_X, 1_X, A\}$ where $A(a) = 0.3, B(a) = 0.1$. Then (X, τ_1) and (X, τ_2) are fts's. Consider the identity function $i : (X, \tau_1) \to (X, \tau_2)$. Now $F\delta SO(X, \tau_1) = F\delta SC(X, \tau_1) = \{0_X, 1_X\}$ and $F\delta SO(X, \tau_2) = F\delta SC(X, \tau_2) = \{0_X, 1_X, U\}$ where $A \leq U \leq 1_X \setminus U$. Now $1_X \setminus A \in \tau_2^c$. Then $i^{-1}(1_X \setminus A) = 1_X \setminus A < 1_X$ only in (X, τ_1) and so $\delta scl_{\tau_1}(1_X \setminus A) \leq 1_X$. Then i is $fg\delta$-semicontinuous function. Now $V \in F\delta SC(X, \tau_2)$ where $V(a) = 0.5$. Then $i^{-1}(V) = V < B \in \tau_1$. But $\delta scl_{\tau_1} V = 1_X \not\leq B$ which shows that i is not $fg\delta^*$-semicontinuous function.

Example 6.20. $fg\delta^*$-semicontinuity may not imply $fg\delta$-semicontinuity
Let $X = \{a\}$, $\tau_1 = \{0_X, 1_X, A\}$, $\tau_2 = \{0_X, 1_X, B\}$ where $A(a) \leq 0.4, B(a) = 0.62$. Then (X, τ_1) and (X, τ_2) are fts's. Consider the identity function $i : (X, \tau_1) \to (X, \tau_2)$. Now $F\delta SO(X, \tau_1) = \{0_X, 1_X, U\}$ where $A \leq U \leq 1_X \setminus A$ and $F\delta SC(X, \tau_1) = \{0_X, 1_X, 1_X \setminus U\}$ where $A \leq 1_X \setminus U \leq 1_X \setminus A$. Also $F\delta SO(X, \tau_2) = F\delta SC(X, \tau_2) = \{0_X, 1_X\}$. Then clearly i is $fg\delta^*$-emicontinuous. Now $1_X \setminus B \in \tau_2^c$. Then $i^{-1}(1_X \setminus B) = 1_X \setminus B \leq 1_X \setminus B \in \tau_1$. But $\delta scl_{\tau_1}(1_X \setminus B) = M \not\leq 1_X \setminus B$ where $M(a) = 0.4$ which shows that $1_X \setminus B$ is not $fg\delta$-semiclosed in (X, τ_1) and hence i is not $fg\delta$-semicontinuous.

Example 6.21. Strongly $fg\delta$-semicontinuity may not imply $fg\delta$-semicontinuity
Consider Example 6.20. Here i is not $fg\delta$-semicontinuous. Now any fuzzy set $C > B$ is $fg\delta$-semiclosed in (X, τ_2). Here $i^{-1}(C) = C \in \tau_1^c$

which implies that i is strongly $fg\delta$-semicontinuous.

Example 6.22. $fg\delta$-semicontinuity may not imply strongly $fg\delta$-semicontinuity, weakly $fg\delta$-semicontinuity
Let $X = \{a\}$, $\tau_1 = \{0_X, 1_X, A\}$, $\tau_2 = \{0_X, 1_X\}$ where $A(a) = 0.5$. Then (X, τ_1) and (X, τ_2) are fts's. Consider the identity function $i : (X, \tau_1) \to (X, \tau_2)$. Clearly i is $fg\delta$-semicontinuous function. Now every fuzzy set in (X, τ_2) is $fg\delta$-semiclosed. Consider the fuzzy set B, defined by $B(a) = 0.2$. Then B is $fg\delta$-semiclosed in (X, τ_2). But $i^{-1}(B) = B \notin \tau_1^c$ which shows that i is not strongly $fg\delta$-semicontinuous function. Again $B \notin F\delta SC(X, \tau_1)$. Indeed, $int_{\tau_1}(\delta cl_{\tau_1} B) = int_{\tau_1} A = A \not\leq B$ and so i is not weakly $fg\delta$-semicontinuous function.

Example 6.23. Weakly $fg\delta$-semicontinuity may not imply $fg\delta$-semicontinuity
Let $X = \{a\}$, $\tau_1 = \{0_X, 1_X, A\}$, $\tau_2 = \{0_X, 1_X, B\}$ where $A(a) = 0.55, B(a) \geq 0.6$. Then (X, τ_1) and (X, τ_2) are fts's. Consider the identity function $i : (X, \tau_1) \to (X, \tau_2)$. The collection of all $fg\delta$-semiclosed sets in $(X, \tau_2) = \{0_X, 1_X\}$ as $F\delta SO(X, \tau_2) = F\delta SC(X, \tau_2) = \{0_X, 1_X\}$ and so i is clearly weakly $fg\delta$-semicontinuous function. Now $F\delta SO(X, \tau_1) = F\delta SC(X, \tau_1) = \{0_X, 1_X\}$. Consider the fuzzy set C, defined by $C(a) = 0.4$. Then $C \in \tau_2^c$. Now $i^{-1}(C) = C < A \in \tau_1$. But $\delta scl_{\tau_1} C = 1_X \not\leq A$ and so i is not $fg\delta$-semicontinuous function.

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