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A GENERAL FIXED POINT THEOREM FOR A SEQUENCE OF MAPPINGS IN G_p - COMPLETE METRIC SPACES

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Abstract. In this paper, a general fixed point theorem for a sequence of mappings in G_p - complete metric spaces is proved.

1. Introduction

In [8], [9] Dhage introduced a new class of generalized metric spaces, named D - metric space. Mustafa and Sims [16], [17] proved that most of the claims concerning the fundamental topological structures on D - metric spaces are incorrect and introduced an appropriate notion of generalized metric space, named G - metric space. In fact, Mustafa, Sims and other authors [1], [12], [15] - [22], [30] - [35], studied many fixed point results for self mappings in G - metric spaces under certain conditions.

In 1994, Matthews [13] introduced the concept of partial metric spaces as a part of the study of denotional semantics of dataflows and proved the Banach contraction principle in such spaces. Recently, in [2], [3], [7], [10], [11] and in other papers, some fixed point theorems under various contractive conditions in partial metric spaces are proved.

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Quite recently, Zand and Nazhad [37] introduced a generalization and unification of G - metric spaces and partial metric spaces, named G_p - metric spaces. Some fixed point results for mappings in G_p - metric spaces are obtained in [4] - [6] and in other papers.

Several classical fixed point theorems and common fixed point theorems have been unified considering a general condition by an implicit relation in [23], [24] and in other papers.

Recently, the method is used in the study of fixed points in metric spaces, symmetric spaces, quasi - metric spaces, b - metric spaces, ultra - metric spaces, Hilbert metric spaces, reflexive spaces, convex metric spaces, compact metric spaces, paracompact metric spaces, in two and three metric spaces, for single - valued mappings, hybrid pairs of mappings and set - valued mappings. Quite recently, the method is used in the study of fixed points for mappings satisfying a contractive/extensive condition of integral type, in fuzzy metric spaces, probabilistic metric spaces, intuitionistic metric spaces. Also, the method allows the study of local and global properties of fixed point structures.

The study of fixed points for mappings satisfying an implicit relation in G - metric spaces is initiated in [25] - [29] and in other papers. The study of fixed points for mappings satisfying implicit relations in partial metric spaces is initiated in [36].

2. Preliminaries

Definition 2.1 ([37])**.** Let X be a nonempty set. A function $G : X^3 \to \mathbb{R}_+$ is called a G_p - metric on X if the following conditions are satisfied:

(GP_1) : $x = y = z$ if $G_p(x, y, z) = G_p(x, x, x) = G_p(y, y, y) = G_p(z, z, z)$,

(GP_2) : $0 \leq G_p(x, x, x) \leq G_p(x, x, y) \leq G_p(x, y, z)$ for all $x, y, z \in X$,

(GP_3) : $G_p(x, y, z) = G_p(y, z, x) = ...$ (symmetry in all three variables),

(GP_4) : $G_p(x, y, z) \leq G_p(x, a, a) + G_p(a, y, z) - G_p(a, a, a)$ for all $x, y, z, a \in X$.

The pair (X, G_p) is called a G_p - metric space.

Definition 2.2 ([37])**.** Let (X, G_p) be a G_p - metric space and $\{x_n\}$ a sequence in X. A point $x \in X$ is said to be the limit of the sequence $\{x_n\}$ or $x_n \to x$ ($\{x_n\}$ is G_p - convergent to x) if $\lim_{m,n\to\infty} G_p(x, x_n, x_m) = G_p(x, x, x)$.

Theorem 2.3 ([4]). *Let (X, G_p) be a G_p - partial metric space. Then, for any $\{x_n\} \in X$ and $x \in X$, the following conditions are equivalent:*
a) $\{x_n\}$ is G_p - convergent to x,
b) $G_p(x_n, x_n, x) \to G_p(x, x, x)$ as $n \to \infty$,
c) $G_p(x_n, x, x) \to G_p(x, x, x)$ as $n \to \infty$.

Definition 2.4 ([37]). Let (X, G_p) be a G_p - partial metric space.
1) A sequence $\{x_n\}$ of X is called a G_p - Cauchy sequence if $\lim_{m,n\to\infty} G_p(x_n, x_m, x_m)$ exists and is finite,
2) A G_p - metric space is said to be G_p - complete if and only if every G_p - Cauchy sequence in X converges to $x \in X$ such that $\lim_{n,m\to\infty} G_p(x_n, x_m, x_m) = G_p(x, x, x)$.

Lemma 2.5 ([4]). *Let (X, G_p) be a G_p - metric space. Then:*
1) If $G_p(x, y, z) = 0$ then $x = y = z$,
2) If $x \neq y$ then $G_p(y, x, x) > 0$.

Lemma 2.6 ([8]). *Let (X, G_p) be a G_p - metric space and $\{x_n\}$ a sequence in X which is G_p - convergent to a point $x \in X$ with $G_p(x, x, x) = 0$. Then $\lim_{n\to\infty} G_p(x_n, y, z) = G_p(x, y, z)$ for all $y, z \in X$.*

Proof. By (GP_4)

$$\begin{aligned} G_p(x, y, z) &\leq G_p(x, x_n, x_n) + G_p(x_n, y, z) - G_p(x_n, x_n, x_n) \\ &\leq G_p(x, x_n, x_n) + G_p(x_n, y, z) \end{aligned} \tag{2.1}$$

which implies

$$\begin{aligned} G_p(x, y, z) - G_p(x, x_n, x_n) &\leq G_p(x_n, y, z) \\ &\leq G_p(x_n, x, x) + G_p(x, y, z). \end{aligned}$$

By Theorem 2.3,

$$G_p(x_n, x, x) \to G_p(x, x, x) = 0$$

and

$$G_p(x, x_n, x_n) \to G_p(x, x, x) = 0.$$

Letting n tends to infinity in (2.1) we obtain

$$\lim_{n\to\infty} G_p(x_n, y, z) = G_p(x, y, z).$$

□

Quite recently, Meena and Nema [14] initiated the study of fixed points for sequences of mappings in G - metric spaces.

The purpose of this paper is to obtain some fixed point results for a sequence of mappings satifying an implicit relation in G_p - metric spaces.

3. Implicit relations

Definition 3.1. Let $\mathcal{F}_{pG}$ be the set of all continuous functions $F(t_1,...,t_5):\mathbb{R}_+^5\to\mathbb{R}$ satisfying the following conditions:
(F_1) : F is nonincreasing in variable t_2,t_3,t_4,t_5,
(F_2) : There exists $h\in[0,1)$ such that for all $u,v\geq 0$, $F(u,v,v,u,u)\leq 0$ implies $u\leq hv$,
(F_3) : There exists $k\in[0,1)$ such that for all $t,t'>0$, $F(t,t,t,t,t')\leq 0$ implies $t\leq kt'$.

In the following examples, the proof of property (F_1) is obviously.

Example 3.2. $F(t_1,...,t_5)=t_1-at_2-bt_3-ct_4-dt_5$, *where* $a,d>0,b,c\geq 0$ *and* $a+b+c+d<1$.
(F_2) : *Let* $u,v\geq 0$ *be and* $F(u,v,v,u,u)=u-av-bv-cu-du\leq 0$. *Then* $u\leq hv$, *where* $0\leq h=\dfrac{a+b}{1-(c+d)}<1$.
(F_3) : *Let* $t,t'>0$ *be and* $F(t,t,t,t,t')=t-at-bt-ct-dt'\leq 0$. *Then* $t\leq kt'$, *where* $0<k=\dfrac{d}{1-(a+b+c)}<1$.

Example 3.3. $F(t_1,...,t_5)=t_1-c\max\{t_2,t_3,t_4,t_5\}$, *where* $c\in(0,1)$.
(F_2) : *Let* $u,v\geq 0$ *be and* $F(u,v,v,u,u)=u-c\max\{u,v\}\leq 0$. *If* $u>v$, *then* $u(1-c)\leq 0$, *a contradiction. Hence,* $u\leq v$ *which implies* $u\leq hv$, *where* $0\leq h=c<1$.
(F_3) : *Let* $t,t'>0$ *be and* $F(t,t,t,t,t')=t-c\max\{t,t'\}\leq 0$. *If* $t>t'$, *then* $t(1-c)\leq 0$, *a contradiction. Hence,* $t\leq t'$ *which implies* $t\leq kt'$, *where* $0<k=c<1$.

Example 3.4. $F(t_1,...,t_5)=t_1-c\max\left\{t_2,t_3,\dfrac{t_4+t_5}{2}\right\}$, *where* $c\in(0,1)$.

The proof is similar to the proof of Example 3.3.

Example 3.5. $F(t_1,...,t_5)=t_1^2-at_2t_3-bt_3t_4-ct_4t_5$, *where* $c>0,a,b\geq 0$ *and* $a+b+c<1$.
(F_2) : *Let* $u,v\geq 0$ *be and* $F(u,v,v,u,u)=u^2-av^2-buv-cu^2\leq 0$. *If* $u>v$, *then* $u^2[1-(a+b+c)]\leq 0$, *a contradiction. Hence* $u\leq v$, *which implies* $u\leq hv$, *where* $0\leq h=\sqrt{a+b+c}<1$.

(F_3) : *Let* $t, t' > 0$ *be and* $F(t,t,t,t,t') = t^2 - at^2 - bt^2 - ctt' \leq 0$, *which implies* $t\,[1-(a+b)] - ct' \leq 0$, *i.e.* $t \leq kt'$, *where* $0 < k = \dfrac{c}{1-(a+b)} < 1$.

Example 3.6. $F(t_1, ..., t_5) = t_1 - at_2 - b\max\{2t_3, t_4 + t_5\}$, *where* $a \geq 0, b > 0$ *and* $a + 2b < 1$.
(F_2) : *Let* $u, v \geq 0$ *be and* $F(u,v,v,u,u) = u - av - b\max\{2u, 2v\} \leq 0$. *If* $u > v$, *then* $u[1-(a+2b)] \leq 0$, *a contradiction. Hence* $u \leq v$, *which implies* $u \leq hv$, *where* $0 \leq h = a + 2b < 1$.
(F_3) : *Let* $t, t' > 0$ *be and* $F(t,t,t,t,t') = t - at - b\max\{2t, t + t'\} \leq 0$. *If* $t > t'$, *then* $t[1-(a+2b)] \leq 0$, *a contradiction. Hence,* $t \leq t'$ *which implies* $t \leq kt'$, *where* $0 < k = a + 2b < 1$.

Example 3.7. $F(t_1, ..., t_5) = t_1 - at_2 - b\max\{t_3 + t_4, 2t_5\}$, *where* $a, b \geq 0$ *and* $a + 2b < 1$.
The proof is similar to the proof of Example 3.6.

Example 3.8. $F(t_1, ..., t_5) = t_1^2 - at_2^2 - bt_3^2 - ct_4t_5$, *where* $c > 0, a, b \geq 0$ *and* $a + b + c < 1$.
(F_2) : *Let* $u, v \geq 0$ *be and* $F(u,v,v,u,u) = u^2 - av^2 - bv^2 - cu^2 \leq 0$, *which implies* $u \leq hv$, *where* $0 \leq h = \sqrt{\dfrac{a+b}{1-c}} < 1$.
(F_3) : *Let* $t, t' > 0$ *be and* $F(t,t,t,t,t') = t^2 - at^2 - bt^2 - ctt' \leq 0$, *which implies* $t - at - bt - ct' \leq 0$. *Hence* $t \leq kt'$, *where* $0 \leq k = \dfrac{c}{1-(a+b)} < 1$.

Example 3.9. $F(t_1, ..., t_5) = t_1 - c\max\{t_2, t_3, \sqrt{t_4t_5}\}$, *where* $c \in (0,1)$.
(F_2) : *Let* $u, v \geq 0$ *be and* $F(u,v,v,u,u) = u - c\max\{u, v\} \leq 0$. *If* $u > v$, *then* $u\,(1-c) \leq 0$, *a contradiction. Hence* $u \leq v$, *which implies* $u \leq hv$, *where* $0 \leq h = c < 1$.
(F_3) : *Let* $t, t' > 0$ *be and* $F(t,t,t,t,t') = t - c\max\{t, \sqrt{tt'}\} \leq 0$. *If* $t > t'$, *then* $t\,(1-c) \leq 0$, *a contradiction. Hence,* $t \leq t'$ *which implies* $t \leq kt'$, *where* $0 < k = c < 1$.

Example 3.10. $F(t_1, ..., t_5) = t_1 - c\max\left\{t_2, t_3, \dfrac{2t_4 + t_5}{3}, \dfrac{2t_5 + t_4}{3}\right\}$, *where* $c \in (0,1)$.
(F_2) : *Let* $u, v \geq 0$ *be and* $F(u,v,v,u,u) = u - c\max\{u, v\} \leq 0$. *If* $u > v$, *then* $u\,(1-c) \leq 0$, *a contradiction. Hence* $u \leq v$, *which implies* $u \leq hv$, *where* $0 \leq h = c < 1$.

(F_3) : *Let* $t, t' > 0$ *be and* $F(t,t,t,t,t') = t - c\max\left\{t, \frac{2t+t'}{3}, \frac{2t'+t}{3}\right\} \leq 0$. *If* $t > t'$, *then* $t(1-c) \leq 0$, *a contradiction. Hence,* $t \leq t'$ *which implies* $t \leq kt'$, *where* $0 < k = c < 1$.

Example 3.11. $F(t_1, ..., t_5) = t_1 - a\max\{t_2, t_3\} - b\max\{t_4, t_5\}$, *where* $a, b \geq 0$ *and* $a + b < 1$.
(F_2) : *Let* $u, v \geq 0$ *be and* $F(u,v,v,u,u) = u - av - bu \leq 0$, *which implies* $u \leq hv$, *where* $0 \leq h = \frac{a}{1-b} < 1$.
(F_3) : *Let* $t, t' > 0$ *be and* $F(t,t,t,t,t') = t - at - b\max\{t, t'\} \leq 0$. *If* $t > t'$, *then* $t[1-(a+b)] \leq 0$, *a contradiction. Hence,* $t \leq t'$ *which implies* $t \leq kt'$, *where* $0 < k = a + b < 1$.

4. Main results

Theorem 4.1. *Let* (X, G_p) *be a* G_p *- metric space and* $\{T_n\}_{n\in\mathbb{N}}$ *be a sequence of mappings such that for all* $x, y, z \in X$ *and* $i, j, k \in \mathbb{N}$

$$\begin{gathered} F(G_p(T_ix, T_jy, T_kz), G_p(x,y,z), G_p(x, T_ix, T_jy), \\ G_p(y, T_jy, T_kz), G_p(z, T_kz, T_ix)) \leq 0, \end{gathered} \tag{4.1}$$

where $F \in \mathcal{F}_{pG}$. *Then,* $\{T_n\}_{n\in\mathbb{N}}$ *has a unique common fixed point.*

Proof. Let $x_0 \in X$ be any arbitrary point. We define a sequence $\{x_n\}$ in X with

$$x_{n+1} = T_{n+1}x_n, \ n = 0, 1, 2, ...$$

By (4.1) we have successively

$$\begin{gathered} F(G_p(T_nx_{n-1}, T_{n+1}x_n, T_{n+2}x_{n+1}), G_p(x_{n-1}, x_n, x_{n+1}), \\ G_p(x_{n-1}, T_nx_{n-1}, T_{n+1}x_n), G_p(x_n, T_{n+1}x_n, T_{n+2}x_{n+1}), \\ G_p(x_{n+1}, T_{n+2}x_{n+1}, T_nx_{n-1})) \leq 0, \end{gathered}$$

$$\begin{gathered} F(G_p(x_n, x_{n+1}, x_{n+2}), G_p(x_{n-1}, x_n, x_{n+1}), G_p(x_{n-1}, x_n, x_{n+1}), \\ G_p(x_n, x_{n+1}, x_{n+2}), G_p(x_{n+1}, x_{n+2}, x_n)) \leq 0. \end{gathered}$$

By (F_2) we obtain

$$G_p(x_n, x_{n+1}, x_{n+2}) \leq hG_p(x_{n-1}, x_n, x_{n+1})$$

which implies

$$G_p(x_n, x_{n+1}, x_{n+2}) \leq h^nG_p(x_0, x_1, x_2). \tag{4.2}$$

Now, for any positive integers $k \geq m \geq n \geq 1$ we obtain by (GP_4)

$$\begin{aligned} G_p(x_n, x_m, x_k) &\leq G_p(x_n, x_{n+1}, x_{n+2}) + G_p(x_{n+1}, x_{n+2}, x_{n+2}) + ... + \\ &\quad + G_p(x_{k-2}, x_{k-1}, x_k) \\ &\leq h^n \left(1 + h + ... + h^{k-n}\right) G_p(x_0, x_1, x_2) \\ &\leq \frac{h^n}{1-h} G_p(x_0, x_1, x_2) \to 0 \text{ as } n \to \infty. \end{aligned}$$

Since by (GP_2), $G_p(x_n, x_m, x_m) \leq G_p(x_n, x_m, x_k)$ it follows that $G_p(x_n, x_m, x_m) \to 0$ as $n, m \to \infty$ and thus $\{x_n\}$ is a G_p - Cauchy sequence, since (X, G_p) is a G_p - complete metric space.

By Theorem 2.3, (4.2) and Definition 2.2, there exists $u \in X$ such that

$$\begin{aligned} \lim_{m,n \to \infty} G_p(x_n, x_m, x_m) &= \lim_{n \to \infty} G_p(u, x_n, x_n) \\ &= G(u, u, u) = 0. \end{aligned}$$

Now we prove that u is a common fixed point of $\{T_n\}_{n \in \mathbb{N}}$. By (4.1) we have successively

$$\begin{gathered} F(G_p(T_n x_{n-1}, T_j u, T_k u), G_p(x_{n-1}, u, u), G_p(x_{n-1}, T_n x_{n-1}, T_j u), \\ G_p(u, T_j u, T_k u), G_p(u, T_k u, T_n x_{n-1})) \leq 0, \end{gathered}$$

$$\begin{gathered} F(G_p(x_n, T_j u, T_k u), G_p(x_{n-1}, u, u), G_p(x_{n-1}, x_n, T_j u), \\ G_p(u, T_j u, T_k u), G_p(u, T_k u, x_n)) \leq 0. \end{gathered} \tag{4.3}$$

By Lemma 2.6 we obtain

$$\lim_{n \to \infty} G_p(x_{n-1}, x, x) = G_p(u, u, u) = 0.$$

On the other hand, by (GP_4) and (GP_2)

$$\begin{aligned} G_p(x_{n-1}, x_n, T_j u) &\leq G_p(x_{n-1}, x_n, x_n) + G_p(x_n, u, T_j u) \\ &\leq G_p(x_{n-1}, x_n, x_{n+1}) + G_p(x_n, u, T_j u). \end{aligned}$$

Letting n tends to infinity, by (4.2) and Lemma 2.6 we obtain

$$\lim_{n \to \infty} G_p(x_{n-1}, x_n, T_j u) \leq \lim_{n \to \infty} G_p(x_n, u, T_j u).$$

Similarly, by Lemma 2.6 we have

$$\lim_{n \to \infty} G_p(u, x_n, T_k u) = G_p(u, u, T_k u).$$

Letting n tends to infinity in (4.3), using (F_1), we obtain

$$\begin{gathered} F(G_p(u, T_j u, T_k u), 0, G_p(u, u, T_j u), \\ G_p(u, T_j u, T_k u), G_p(u, u, T_k u)) \leq 0. \end{gathered} \tag{4.4}$$

By (GP_2),

$$G_p(u,u,T_ju) \leq G_p(u,T_ju,T_ku),$$

$$G_p(u,u,T_ku) \leq G_p(u,T_ju,T_ku).$$

By (4.4) and (F_1) we obtain

$$F(G_p(u,T_ju,T_ku), G_p(u,T_ju,T_ku), G_p(u,T_ju,T_ku), \\ G_p(u,T_ju,T_ku), G_p(u,T_ju,T_ku)) \leq 0.$$

By (F_2) we have $G_p(u,T_ju,T_ku) \leq kG_p(u,T_ju,T_ku)$, which implies $G_p(u,T_ju,T_ku) = 0$. By Lemma 2.5 (1), $u = T_ju = T_ku$. Hence u is a common fixed point of $\{T_n\}_{n\in\mathbb{N}}$.

Suppose that $\{T_n\}_{n\in\mathbb{N}}$ have an other fixed point v. By (4.1) we obtain

$$F(G_p(T_iu,T_ju,T_kv), G_p(u,u,v), G_p(u,T_iu,T_ju), \\ G_p(u,T_ju,T_kv), G_p(v,T_kv,T_iu)) \leq 0,$$

$$F(G_p(u,u,v), G_p(u,u,v), G_p(u,u,u), \\ G_p(u,u,v), G_p(v,v,u)) \leq 0.$$

By (GP_3), $G_p(u,u,u) \leq G_p(u,u,v)$. Hence, by (F_1) we obtain

$$F(G_p(u,u,v), G_p(u,u,v), G_p(u,u,v), \\ G_p(u,u,v), G_p(v,v,u)) \leq 0,$$

which implies by (F_3) that

$$G_p(u,u,v) \leq kG_p(u,v,v).$$

Similarly we obtain

$$G_p(u,v,v) \leq kG_p(u,u,v).$$

Hence

$$G_p(u,u,v) \leq kG_p(u,v,v) \leq k^2 G_p(u,u,v),$$

which implies

$$G_p(u,u,v)\left(1-k^2\right) \leq 0,$$

i.e.

$$G_p(u,u,v) = 0.$$

By Lemma 2.5 (1), $u = v$. Therefore, u is the unique common fixed point of $\{T_n\}_{n\in\mathbb{N}}$. $\square$

Theorem 4.2. *Let (X, G_p) be a G_p - complete metric space and $\{T_n\}_{n\in\mathbb{N}}$ be a sequence of mappings such that for all $x, y, z \in X$ and $i, j, k \in \mathbb{N}$, one of the following inequalities hold:*

$$\begin{aligned} F(&G_p(T_ix, T_jy, T_kz), G_p(x, y, z), G_p(x, T_ix, T_ix), \\ &G_p(y, T_jy, T_jy), G_p(z, T_kz, T_kz)) \leq 0, \end{aligned} \tag{4.5}$$

$$\begin{aligned} F(&G_p(T_ix, T_jy, T_kz), G_p(x, y, z), G_p(x, T_jy, T_jy), \\ &G_p(y, T_kz, T_kz), G_p(z, T_ix, T_ix)) \leq 0, \end{aligned} \tag{4.6}$$

$$\begin{aligned} F(&G_p(T_ix, T_jy, T_kz), G_p(x, y, z), G_p(x, x, T_ix), \\ &G_p(y, y, T_jy), G_p(z, z, T_kz)) \leq 0, \end{aligned} \tag{4.7}$$

$$\begin{aligned} F(&G_p(T_ix, T_jy, T_kz), G_p(x, y, z), G_p(x, x, T_jy), \\ &G_p(y, y, T_kz), G_p(z, z, T_ix)) \leq 0, \end{aligned} \tag{4.8}$$

where $F \in \mathcal{F}_{pG}$. Then, $\{T_n\}_{n\in\mathbb{N}}$ has a unique common fixed point.

Proof. We prove this theorem in the case of inequality (4.5).

By (GP_3) we have

$$\begin{aligned} G_p(x, T_ix, T_ix) &\leq G_p(x, T_ix, T_jy), \\ G_p(y, T_jy, T_jy) &\leq G_p(z, T_jy, T_kz), \\ G_p(z, T_kz, T_kz) &\leq G_p(z, T_kz, T_ix). \end{aligned}$$

By (4.5) and (F_1) we obtain

$$\begin{aligned} F(&G_p(T_ix, T_jy, T_kz), G_p(x, y, z), G_p(x, T_ix, T_jy), \\ &G_p(z, T_jy, T_kz), G_p(z, T_kz, T_ix)) \leq 0, \end{aligned}$$

which is inequality (4.1). By Theorem 4.1, $\{T_n\}_{n\in\mathbb{N}}$ has a unique common fixed point.

In cases (4.6), (4.7), (4.8), the proof is similar. □

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