"Vasile Alecsandri" University of Bacău
Faculty of Sciences
Scientific Studies and Research
Series Mathematics and Informatics
Vol. 28(2018), No. 1, 147-170

PSEUDO CL-SUPERCONTINUOUS FUNCTIONS AND CLOSEDNESS/COMPACTNESS OF THEIR FUNCTION SPACES

D. SINGH, JEETENDRA AGGARWAL AND J. K. KOHLI

Abstract. A new class of functions called 'pseudo cl-supercontinuous' functions is introduced. Basic properties of pseudo cl-supercontinuous functions are studied and their place in the hierarchy of variants of continuity which already exist in the mathematical literature is discussed. The interplay between topological properties and pseudo cl-supercontinuity is investigated. Function spaces of pseudo cl-supercontinuous functions are considered and sufficient conditions for their closedness and compactness in the topology of pointwise convergence are formulated.

1. Introduction

The notion of continuity is of fundamental importance in mathematics and prevades almost all subdisciplines of mathematics. Several variants of continuity occur in the lore of mathematical literature which arise in diverse situations in mathematics and applications of mathematics. Certain of these variants of continuity are stronger than continuity while others are weaker than continuity and yet others are independent of continuity in general.

Keywords and phrases: regular F_σ-set, $D_\delta T_0$-space, ultra Hausdorff space, D_δ- Hausdorff space, locally connected, strongly continuous, topology of pointwise convergence, sum connected space.
(2010) Mathematics Subject Classification: 54C05, 54C10, 54C35, 54G15, 54G20, 54D10

For example, the variants of continuity dealt with in ([23], [32], [33], [35], [38], [40], [50], [57]) are neither weaker nor stronger than continuity. In this paper, we define one such variant of continuity called 'pseudo cl-supercontinuity' and study its basic properties. The class of pseudo cl-supercontinuous functions properly contains the class of quasi cl-supercontinuous functions [23] which in its turn is properly contained in the class of almost cl-supercontinuous functions ([12], [32]) and so includes all cl-supercontinuous ($\equiv$ clopen continuous) functions ([52], [56]). Moreover, the class of pseudo cl-supercontinuous functions properly contains the class of pseudo perfectly continuous functions [35] which in its turn strictly contains the class of quasi perfectly continuous functions [40] and so includes all almost perfectly continuous functions and thus contains all perfectly continuous functions due to Noiri [49]. It is well known that the function space $C(X, Y)$ of all continuous functions from a space X into a space Y is not necessarily closed in Y^X in the topology of pointwise convergence. However, in contrast, Naimpally [47] showed that if X is locally connected and Y is Hausdorff, then the set $S(X, Y)$ of all strongly continuous functions is closed in Y^X in the topology of pointwise convergence. Naimpally's result is extended in ([23], [31], [33], [40]) for larger classes of functions and spaces. The main aim of the present paper is to further strengthen these results to show that if X is sum connected [19] and Y is a $D_\delta T_0$-space [35], then certain classes of functions are identical and closed in Y^X in the topology of pointwise convergence. Moreover, conditions are formulated for these classes of functions to be compact Hausdorff subspaces of Y^X in the topology of pointwise convergence.

Organization of the paper is as follows: Section 2 is devoted to preliminaries and basic definitions. In Section 3 we introduce the notion of 'pseudo cl-supercontinuous function' and elaborate upon its place in the hierarchy of variants of continuity which already exist in the mathematical literature. Basic properties of pseudo cl-supercontinuous functions are studied in Section 4 wherein (i) sufficient conditions are formulated for the composition of two functions to be pseudo cl-supercontinuous; (ii) a sum theorem is proved for a function to be pseudo cl-supercontinuous, whenever it is pseudo cl-supercontinuous on parts; (iii) the graph function is shown to be pseudo cl-supercontinuous if and only if f is pseudo cl-supercontinuous and X is pseudo zero dimensional; (iv) A function into a product space is shown to be pseudo cl-supercontinuous if and only if its composition with each projection is pseudo cl-supercontinuous. Section 5 is devoted to the study of interplay between topological properties and pseudo cl-supercontinuous functions. Section 6 is devoted to function spaces wherein it is shown that if X is sum connected (e.g. connected or locally connected) and Y is a $D_\delta T_0$-space, then the function space $L_p(X,Y)$ of all pseudo cl-supercontinuous functions is closed in Y^X in the topology of pointwise convergence. Moreover, if in addition Y is a compact $D_\delta T_0$-space, then $L_p(X,Y)$ and certain other function spaces are shown to be compact Hausdorff subspaces of Y^X in the topology of pointwise convergence. In Section 7 we consider the reptologization of domain/range of a pseudo cl-upercontinuous function and discuss the change in the topological properties of the function.

2. Basic Definitions and Preliminaries

A subset A of a space X is said to be ***cl-open*** [56] if for each $x \in A$ there exists a clopen set H such that $x \in H \subset A$; or equivalently A is expressible as a union of clopen sets. The complement of a cl-open set is referred to as a ***cl-closed set***. A subset H of a space X is called a **regular $\boldsymbol{G_\delta}$-set** [45] if H is the intersection of a sequence of closed sets whose interiors contain H, i.e. $H = \bigcap\limits_{n=1}^{\infty} F_n = \bigcap\limits_{n=1}^{\infty} F_n^o$, where each F_n is a closed subset of X. The complement of a regular G_δ-set is called a **regular $\boldsymbol{F_\sigma}$-set**. Any intersection of regular G_δ-sets is called **$\boldsymbol{d_\delta}$-closed** [26] and the complement of a d_δ-closed set is called **$\boldsymbol{d_\delta}$-open**. A collection β of subsets of a space X is called an ***open complementary system*** [15] if β consists of open sets such that for every $B \in \beta$, there exist $B_1, B_2, \ldots \in \beta$ with $B = \cup\{X \setminus B_i : i \in N\}$. A subset A of a space X is called a ***strongly open $\boldsymbol{F_\sigma}$-set*** [15] if there exists a countable open complementary system $\beta(A)$ with $A \in \beta(A)$. The complement of a strongly open F_σ-set is called ***strongly closed $\boldsymbol{G_\delta}$-set****. A point $x \in X$ is called a ***$\boldsymbol{\theta}$-adherent point*** [64] of $A \subset X$ if every closed neigbourhood of x intersects A. Let $\boldsymbol{cl_\theta A}$ denote the set of all θ-adherent points of A. The set A is called ***$\boldsymbol{\theta}$-closed*** if $A = \mathrm{cl}_\theta A$. The complement of a θ-closed set is referred to as a ***$\boldsymbol{\theta}$-open set***. A subset A of a space X is said to be ***regular open*** if it is the interior of its closure, i.e., $A = \overline{A}^0$. The complement of a regular open set is referred to as a ***regular closed set***. Any union of regular open sets is called ***$\boldsymbol{\delta}$-open*** [64]. The complement of a δ-open set is referred

*Brandenberg called strongly 'closed G_δ-sets' as 'D-closed' in ([8], [9]).

to as a ***δ-closed set***. An open subset U of a space X is said to be ***r-open*** [34] if for each $x \in U$ there exists a closed set B such that $x \in B \subset U$; or equivalently U is expressible as a union of closed sets.The complement of an r-open set is called ***r-closed set***. An open set G of a space X is said to be ***z*-open** [53] if for each $x \in G$ there exists a cozero set Z in X such that $x \in Z \subset G$; or equivalently G is expressible as a union of cozero sets in X. The complement of a z-open set is called ***z-closed set***.

Next we include definitions of those strong variants of continuity which already exist in the literature and are related to the contents of the present paper.

2.1. **Definitions.** A function $f : X \to Y$ from a topological space X into a topological space Y is said to be

(a) **strongly continuous** [41] if $f(\bar{A}) \subset f(A)$ for each subset A of X.

(b) **perfectly continuous** ([36], [49]) if $f^{-1}(V)$ is clopen in X for every open set $V \subset Y$.

(c) **cl-supercontinuous** [56] ($\equiv$ **clopen continuous** [52])if for each $x \in X$ and each open set V containing $f(x)$ there is a clopen set U containing x such that $f(U) \subset V$.

(d) ***z*-supercontinuous** [24] (respectively $\boldsymbol{D_\delta}$**-supercontinuous** [26], respectively **supercontinuous** [46]) if for each $x \in X$ and for each open set V containing $f(x)$, there exists a cozero set (respectively regular F_σ-set, respectively regular open set) U containing x such that $f(U) \subset V$.

(e) $\mathbf{D}^*$**-supercontinuous** [55] (**D-supercontinuous** [25])if for each $x \in X$ and each open set V containing $f(x)$, there exists a strongly open F_σ-set (open F_σ-set) U containing x such that $f(U) \subset V$.

(f) **R-supercontinuous** [34] if for each $x \in X$ and for each open set V containing $f(x)$, there exists an r-open set U containing x such that $f(U) \subset V$.

(g) **strongly θ-continuous** ([43], [48]) if for each $x \in X$ and for each open set V containing $f(x)$, there exists an open set U containing x such that $f(\overline{U}) \subset V$.

2.2. **Definitions.** A function $f : X \to Y$ from a topological space X into a topological space Y is said to be

(a) $\mathbf{D_\delta}$**-continuous** [27] (respectively **D-continuous** [22], respectively **z-conti-nuous** [53]) if for each $x \in X$ and each regular

F_σ-set (respectively open F_σ-set, respectively cozero set) V containing $f(x)$ there is an open set U containing x such that $f(U) \subset V$.

(b) **almost continuous** [54] (respectively **faintly continuous** [44], respectively **R-continuous** [39]) if for each $x \in X$ and each regular open set (respectively θ-open set, respectively r-open set) V containing $f(x)$ there is an open set U containing x such that $f(U) \subset V$.

(c) $\boldsymbol{d_\delta}$**-map** [28] if for each regular F_σ-set U in Y, $f^{-1}(U)$ is a regular F_σ-set in X.

(d) $\boldsymbol{\theta}$**-continuous** [13] if for each $x \in X$ and each open set V containing $f(x)$ there is an open set U containing x such that $f(\overline{U}) \subset \overline{V}$.

(e) **weakly continuous** [42] if for each $x \in X$ and each open set V containing $f(x)$ there exists an open set U containing x such that $f(U) \subset \overline{V}$.

(f) **quasi** $\boldsymbol{\theta}$**-continuous function** [51] if for each $x \in X$ and each θ-open set V containing $f(x)$ there exists a θ-open set U containing x such that $f(U) \subset V$.

(g) **slightly continuous**† [16] if $f^{-1}(V)$ is open in X for every clopen set $V \subset Y$.

2.3. **Definitions.** A function $f : X \to Y$ from a topological space X into a topological space Y is said to be

(a) $\boldsymbol{\delta}$**-perfectly continuous** [33] if for each δ-open set V in Y, $f^{-1}(V)$ is a clopen set in X.

(b) **almost perfectly continuous** [57] ($\equiv$ **regular set connected** [10]) if $f^{-1}(V)$ is clopen for every regular open set V in Y.

(c) **almost cl-supercontinuous** [32] ($\equiv$ **almost clopen continuous** [12]) if for each $x \in X$ and each regular open set V containing $f(x)$, there is a clopen set U containing x such that $f(U) \subset V$.

(d) **almost** $\boldsymbol{z}$**-supercontinuous** [38] (**almost** $\boldsymbol{D_\delta}$**-supercontinuous**) if for each $x \in X$ and for each regular open set V containing $f(x)$, there exists a cozero set (regular F_σ-set) U containing x such that $f(U) \subset V$.

(e) **almost strongly** $\boldsymbol{\theta}$**-continuous** [50] if for each $x \in X$ and for each regular open set V containing $f(x)$, there exists an open set U containing x such that $f(\overline{U}) \subset V$.

---

†Slightly continuous functions have been referred to as cl-continuous functions in ([27], [39]).

(f) **quasi perfectly continuous** [40] if $f^{-1}(V)$ is clopen in X for every θ-open set V in Y.

(g) **quasi z-supercontinuous** [37] (respectively **quasi cl-supercontinuos** [23], respectively **quasi D_δ-supercontinuos** [38]) if for each $x \in X$ and each θ-open set V containing $f(x)$, there exists a cozero set (respectively clopen set, respectively regular F_σ-set) U containing x such that $f(U) \subset V$.

(h) **pseudo z-supercontinuous** [37] (**pseudo D_δ-supercontinuos** [38]) if for each $x \in X$ and each regular F_σ-set V containing $f(x)$, there exists a cozero set (regular F_σ-set) U containing x such that $f(U) \subset V$.

(i) **δ-continuous** [48] if for each $x \in X$ and for each regular open set V containing $f(x)$, there exists a regular open set U containing x such that $f(U) \subset V$.

2.4. **Definition.** A space X is said to be **D_δ-completely regular** ([27], [30]) if it has a base of regular F_σ-sets.

3. Pseudo cl-supercontinuous functions

3.1. **Definition.** A function $f : X \to Y$ from a topological space X into a topological space Y is said to be **pseudo cl-supercontinuous** if for each $x \in X$ and each regular F_σ-set V containing $f(x)$ there exists a clopen set U containing x such that $f(U) \subset V$.

The nearby diagram (Figure 1) represents an extension of the diagram in [35] and well exhibits the interconnections and interrelations of pseudo cl-supercontinuity with other variants of continuity that already exist in the literature and are related to the theme of the present paper and so it well reflects the place of pseudo cl-supercontinuity in the hierarchy of known variants of continuity.

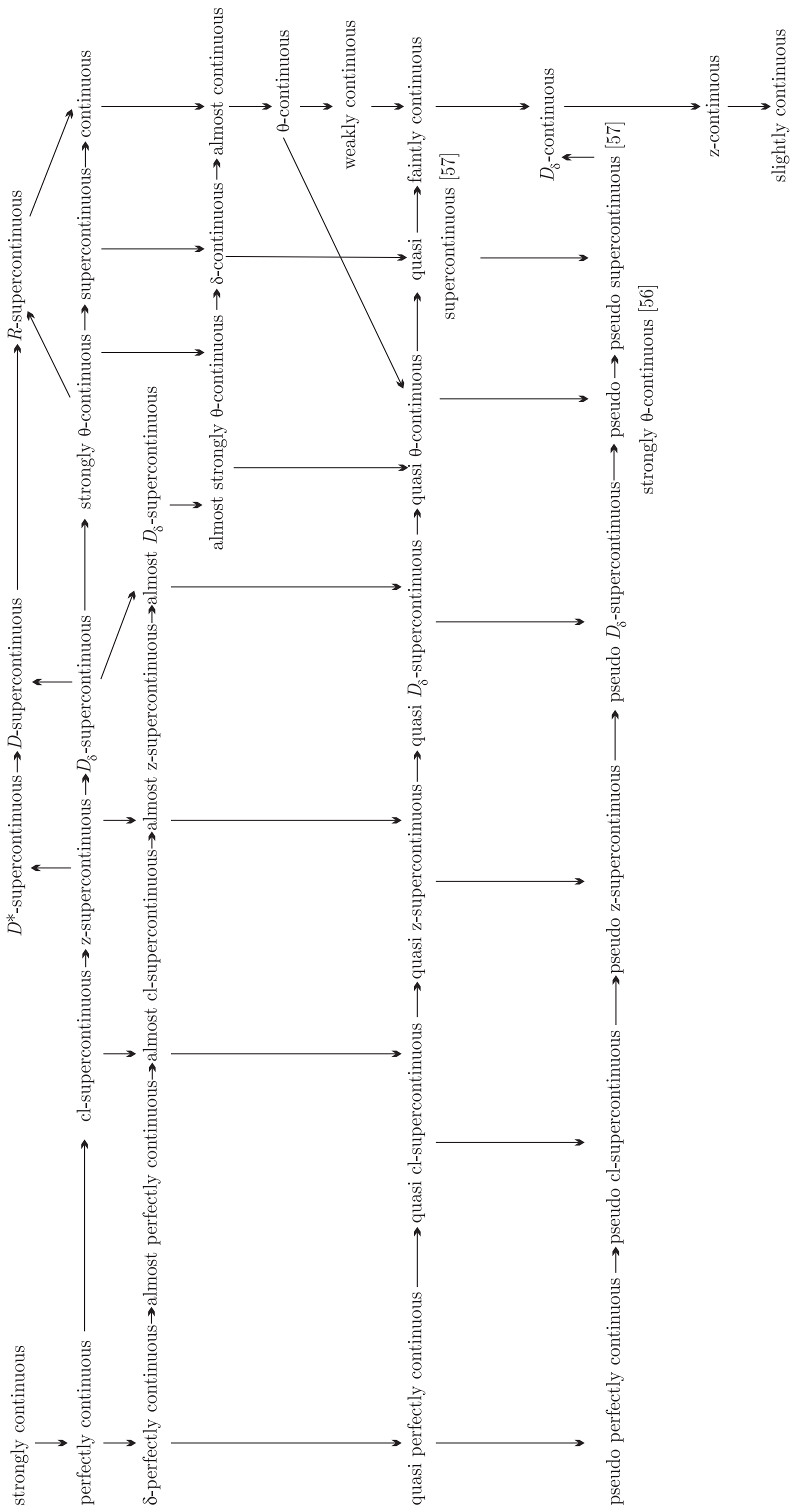
strongly continuous
D^*-supercontinuous
D-supercontinuous
R-supercontinuous
perfectly continuous
cl-supercontinuous
z-supercontinuous
D_δ-supercontinuous
strongly θ-continuous
supercontinuous
continuous
δ-perfectly continuous
almost perfectly continuous
almost cl-supercontinuous
almost z-supercontinuous
almost D_δ-supercontinuous
almost strongly θ-continuous
δ-continuous
almost continuous
θ-continuous
weakly continuous
faintly continuous
quasi perfectly continuous
quasi cl-supercontinuous
quasi z-supercontinuous
quasi D_δ-supercontinuous
quasi θ-continuous
quasi supercontinuous [57]
D_δ-continuous
pseudo perfectly continuous
pseudo cl-supercontinuous
pseudo z-supercontinuous
pseudo D_δ-supercontinuous
pseudo strongly θ-continuous [56]
pseudo supercontinuous [57]
z-continuous
slightly continuous

FIGURE 1

However, none of the implications in Figure 1 is reversible as is shown by examples in ([35],[40], [59]) and the following examples/ observations.

3.2. Let Y be the skyline space due to Heldermann [15] which is a Hausdorff regular space and let X be the same underlying set as that of Y. Suppose X is endowed with indiscrete topology. Let $f : X \to Y$ denote the identity mapping. Since Y is the only regular F_σ-set in the skyline space containing the points p^- and p^+, f is pseudo cl-supercontinuous but it is not quasi cl-supercontinuous.

3.3. If Y is a D_δ-completely regular space, then every pseudo cl-supercontinuous function $f : X \to Y$ from a space X into Y is cl-supercontinuous.

3.4. If X is a zero dimensional space, then every D_δ-continuous function $f : X \to Y$ defined on X is pseudo cl-supercontinuous.

4. Characterizations and Basic Properties

4.1. **Definitions.** A filter base $\mathcal{F}$ in a space X is said to

(i) **cl-converge** [56] to a point $x \in X$, written as $\mathcal{F} \xrightarrow{cl} x$, if every clopen set containing x contains a member of $\mathcal{F}$; and

(ii) $\boldsymbol{d_\delta}$**-converge** [26] to a point $x \in X$, written as $\mathcal{F} \xrightarrow{d_\delta} x$, if every regular F_σ-set containing x contains a member of $\mathcal{F}$.

4.2. **Definitions.** A net (x_λ) in a space X is said to

(i) **cl-converge** [56] to a point $x \in X$, written as $x_\lambda \xrightarrow{cl} x$, if it is eventually in every clopen set containing x; and

(ii) $\boldsymbol{d_\delta}$**-converge** [26] to a point $x \in X$, written as $x_\lambda \xrightarrow{d_\delta} x$, if it is eventually in every regular F_σ-set containing x.

4.3. **Theorem.** *For a function $f : X \to Y$ the following statements are equivalent.*

(a) *f is pseudo cl-supercontinuous*
(b) *$f^{-1}(V)$ is cl-open for each regular F_σ-set $V \subset Y$.*
(c) *$f(\mathcal{F}) \xrightarrow{d_\delta} f(x)$, for every filter base $\mathcal{F}$ in X which cl-converges to x.*
(d) *$f(x_\lambda) \xrightarrow{d_\delta} f(x)$, for every net (x_λ) in X which cl-converges to x.*
(e) *$f([A]_{cl}) \subset [f(A)]_{d_\delta}$ for every set $A \subset X$*
(f) *$[f^{-1}(B)]_{cl} \subset f^{-1}([B]_{d_\delta})$ for every set $B \subset Y$.*
(g) *$f^{-1}(B)$ is cl-closed for each regular G_δ-set $B \subset Y$.*

(h) $f^{-1}(B)$ *is cl-closed for every* d_δ*- closed set* $B \subset Y$.
(i) $f^{-1}(V)$ *is cl-open for every* d_δ*-open set* $V \subset Y$.

We omit the proof of Theorem 4.3.

4.4. **Proposition.** If $f : X \to Y$ is a pseudo cl-supercontinuous function and $g : Y \to Z$ is a D_δ-supercontinuous function, then their composition is cl-supercontinuous.

4.5. **Proposition.** Let $f : X \to Y$ be a slightly continuous function and let $g : Y \to Z$ be pseudo cl-supercontinuous function. Then $g \circ f$ is pseudo cl-supercontinuous.

4.6. **Remark.** The hypothesis of 'slight continuity' on f in Proposition 4.5 can be traded of by any one of the weak variants of continuity in Figure 1 which lie strictly between continuity and slight continuity.

4.7. **Proposition.** If $f : X \to Y$ is a pseudo cl-supercontinuous function and $g : Y \to Z$ is a d_δ-map, then $g \circ f$ is a pseudo cl-supercontinuous function. In particular, composition of two pseudo cl-supercontinuous functions is pseudo cl-supercontinuous.

4.8. **Corollary.** *If* $f : X \to Y$ *is a pseudo cl-supercontinuous function and* $g : Y \to Z$ *is a continuous function, then* $g \circ f$ *is a pseudo cl-supercontinuous function.*

Proof. Every continuous map is a d_δ-map. □

4.9. **Definition** ([56])**.** Let $p : X \to Y$ be a surjection from a topological space X onto a set Y. The collection of all subsets $A \subset Y$ such that $p^{-1}(A)$ is cl-open in X is a topology on Y and is called the **cl-quotient topology**. The map p is called **cl-quotient map**.

4.10. **Proposition.** Let $f : X \to Y$ be a cl-quotient map. Then $g : Y \to Z$ is D_δ-continuous if and only if $g \circ f$ is pseudo cl-supercontinuous.

Proof. Suppose that g is D_δ-continuous and let V be a regular F_σ-set in Z. Then $g^{-1}(V)$ is open in Y. Since f is a cl-quotient map, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is cl-open. Thus $g \circ f$ is pseudo cl-supercontinuous. Conversely, suppose that $g \circ f$ is pseudo cl-supercontinuous and let V be a regular F_σ-set in Z. Then $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is cl-open set in X. Since f is a cl-quotient map, $g^{-1}(V)$ is open in Y and so g is D_δ-continuous. □

4.11. **Proposition.** If $f : X \to Y$ is a surjection which maps clopen sets to open sets and $g : Y \to Z$ is a function such that $g \circ f$ is pseudo cl-supercontinuous, then g is a D_δ-continuous function.

Proof. Let V be a regular F_σ-set in Z. Since $g \circ f$ is pseudo cl-supercontinuous, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is cl-open set in X. Let $f^{-1}(g^{-1}(V)) = \bigcup\{C_\alpha : \alpha \in \Lambda\}$, where each C_α is a clopen set in X. Again, since f is a surjection which maps clopen sets to open sets, $f(f^{-1}(g^{-1}(V))) = g^{-1}(V) = \bigcup\{f(C_\alpha) : \alpha \in \Lambda\}$ is open being the union of open sets in Y and so g is a D_δ-continuous function. □

4.12. **Corollary.** *If $f : X \to Y$ is a surjection which maps clopen sets to open sets and $g : Y \to Z$ is a function such that $g \circ f$ is pseudo perfectly continuous, then g is a D_δ-continuous function.*

4.13. **Proposition.** If $f : X \to Y$ is a pseudo cl-supercontinuous function and $g : Y \to Z$ is an almost D_δ-supercontinuous function, then their composition $g \circ f$ is almost cl-supercontinuous.

4.14. **Corollary.** ([35, Proposition 4.13]) *If $f : X \to Y$ is a pseudo perfectly continuous function and $g : Y \to Z$ is an almost D_δ-supercontinuous function, then their composition $g \circ f$ is almost cl-supercontinuous.*

4.15. **Proposition.** If $f : X \to Y$ is a pseudo cl-supercontinuous function and $g : Y \to Z$ is a quasi D_δ-supercontinuous function, then their composition $g \circ f$ is quasi cl-supercontinuous.

4.16. **Corollary.** ([35, Proposition 4.14]) *If $f : X \to Y$ is a pseudo perfectly continuous function and $g : Y \to Z$ is quasi D_δ-supercontinuous function, then their composition $g \circ f$ is quasi cl-supercontinuous.*

4.17. **Theorem.** *Let $f : X \to Y$ be a function and let $Q = \{U_\alpha : \alpha \in \Lambda\}$ be a cl-open cover of X. For each $\alpha \in \Lambda$, let $f_\alpha = f_{|U_\alpha} : U_\alpha \to Y$ denote the restriction map. Then f is pseudo cl-supercontinuous if and only if each f_α is pseudo cl-supercontinuous.*

Proof. Necessity is immediate. To prove sufficiency, let V be a regular F_σ-set in Y. Then $f^{-1}(V) = \bigcup\limits_{\alpha \in \Lambda} \{f_\alpha^{-1}(V) : \alpha \in \Lambda\}$. Since each f_α is pseudo cl-supercontinuous, $f_\alpha^{-1}(V)$ is cl-open in U_α and hence in X. Thus $f^{-1}(V)$ is cl-open being the union of cl-open sets and so f is pseudo cl-supercontinuous. □

4.18. **Definition.** A subset S of a space X is said to be **regular G_δ-embedded** [7] in X if every regular G_δ-set in S is the intersection of a regular G_δ-set in X with S; or equivalently every regular F_σ-set in S is the intersection of a regular F_σ-set in X with S.

4.19. **Proposition.** Let $f : X \to Y$ be a pseudo cl-supercontinuous function. If $f(X)$ is regular G_δ-embedded in Y, then $f : X \to f(X)$ is pseudo cl-supercontinuous.

4.20. **Theorem.** *Let $\{f_\alpha : X \to X_\alpha : \alpha \in \Lambda\}$ be a family of functions and let $f : X \to \Pi X_\alpha$ be defined by $f(x) = (f_\alpha(x))_{\alpha\in\Lambda}$ for each $x \in X$. If f is pseudo cl-supercontinuous, then each f_α is pseudo cl-supercontinuous.*

Proof. Let $f : X \to \Pi X_\alpha$ be pseudo cl-supercontinuous. For each $\alpha \in \Lambda$, $f_\alpha = p_\alpha \circ f$, where p_α denotes the projection map $p_\alpha : \Pi X_\alpha \to X_\alpha$. For each $\beta \in \Lambda$, it suffices to prove that f_β is pseudo cl-supercontinuous. To this end, let V_β be a regular F_σ-set in X_β. Then $p_\beta^{-1}(V_\beta) = V_\beta \times \prod\limits_{\alpha\neq\beta} X_\alpha$ is a regular F_σ-set in the product space ΠX_α. Since f is pseudo cl-supercontinuous, in view of Theorem 4.3, $f^{-1}(V_\beta \times \prod\limits_{\alpha\neq\beta} X_\alpha) = f^{-1}(p_\beta^{-1}(V_\beta)) = (p_\beta \circ f)^{-1}(V_\beta) = f_\beta^{-1}(V_\beta)$ is cl-open in X and so f_β is pseudo cl-supercontinuous. □

4.21. **Definition** ([21])**.** A space X is said to be **pseudo zero dimensional** if for each $x \in X$ and each regular F_σ-set U containing x there is a clopen set V such that $x \in V \subset U$.

4.22. **Theorem.** *Let $f : X \to Y$ be a function and let $g : X \to X \times Y$ be the graph function defined by $g(x) = (x, f(x))$ for each $x \in X$. Then g is pseudo cl-supercontinuous, if and only if f is pseudo cl-supercontinuous and X is a pseudo zero dimensional space.*

Proof. Observe that $g = 1_X \times f$, where 1_X denotes the identity function defined on X. Now by Theorem 4.20, g is pseudo cl-supercontinuous if and only if both 1_X and f both are pseudo cl-supercontinuous. Now, pseudo cl-supercontinuity of 1_X implies that every regular F_σ-set in X is cl-open and so X is a pseudo zero dimensional space. □

4.23. **Theorem.** *Let $f : \prod\limits_{\alpha\in\Lambda} X_\alpha \to \prod\limits_{\alpha\in\Lambda} Y_\alpha$ be a mapping defined by $f((x_\alpha)) = (f_\alpha(x_\alpha))$, where $f_\alpha : X_\alpha \to Y_\alpha$ for each $\alpha \in \Lambda$. Then f is pseudo cl-supercontinuous if and only if each f_α is pseudo cl-supercontinuous.*

Proof. To prove necessity, let V_β be a regular F_σ-set in Y_β. Then $\pi_\beta^{-1}(V_\beta) = V_\beta \times \left(\prod_{\alpha\neq\beta} Y_\alpha\right)$ is a subbasic regular F_σ-set in the product space $\prod_{\alpha\in\Lambda} Y_\alpha$. Now, since f is pseudo cl-supercontinuous, $f^{-1}(\pi_\beta^{-1}(V_\beta)) = f_\beta^{-1}(V_\beta)\times\left(\prod_{\alpha\neq\beta} X_\alpha\right)$ is a cl-open set in $\prod_{\alpha\in\Lambda} X_\alpha$. Hence $f_\beta^{-1}(V_\beta)$ is a cl-open set in X_β and so f_β is pseudo cl-supercontinuous.

Conversely, let $V = V_\beta \times \left(\prod_{\alpha\neq\beta} Y_\alpha\right)$ be a subbasic regular F_σ-set in the product space $\prod Y_\alpha$. Then $f^{-1}(V) = f_\beta^{-1}(V_\beta) \times \prod_{\alpha\neq\beta} X_\alpha$. Since each f_β is pseudo cl-supercontinuous, $f_\beta^{-1}(V_\beta)$ is a cl-open subset of the space X_β and so $f^{-1}(V)$ is a cl-open subset of the product space $\prod X_\alpha$. Again, since the arbitrary union and finite intersections of cl-open sets are cl-open, f is pseudo cl-supercontinuous. □

5. Interplay between topological properties and pseudo cl-supercontinuous functions

5.1. **Definitions.** A topological space X is said to be

(a) **weakly cl-normal** if every pair of disjoint cl-closed subsets of X can be separated by disjoint open sets in X.
(b) **weakly $\boldsymbol{\delta}$-normal** [29] if every pair of disjoint regular G_δ-subsets of X can be separated by disjoint open sets in X.

5.2. **Theorem.** *Let $f : X \to Y$ be a pseudo cl-supercontinuous function from a weakly cl-normal space X into a space Y. If (i) f is an open bijection; or (ii) f is a closed surjection, then Y is a weakly δ-normal space.*

Proof. Let A and B be disjoint regular G_δ-closed subsets of Y. Since f is pseudo cl-supercontinuous, $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint cl-closed subsets of X. Since X is a weakly cl-normal space, there exist disjoint open sets U and V containing $f^{-1}(A)$ and $f^{-1}(B)$, respectively.

(i) In case f is an open bijection, $f(U)$ and $f(V)$ are disjoint open sets containing A and B respectively.
(ii) In case f is a closed surjection, the sets $W_1 = Y \setminus f(X\setminus U)$ and $W_2 = Y \setminus f(X \setminus V)$ are open in Y. It is easily verified that W_1 and W_2 are disjoint and contain A and B respectively.

□

5.3. **Definition.** A space X is said to be $\boldsymbol{D_\delta}$**-compact** [28] (**mildly compact**‡[62]) if every cover of X by regular F_σ-sets (clopen sets) has a finite subcover.

5.4. **Proposition.** Let $f : X \to Y$ be a pseudo cl-supercontinuous function from a mildly compact space X onto a space Y. Then Y is a D_δ-compact space.

Proof. Let $\{G_\alpha : \alpha \in \Lambda\}$ be a cover of Y by regular F_σ-sets. Since f is pseudo cl-supercontinuous, the collection $\Omega = \{f^{-1}(G_\alpha) : \alpha \in \Lambda\}$ is a cl-open cover of X. Since each cl-open set is a union of clopen sets, let $\Upsilon = \{U_\beta : \beta \in \Gamma\}$ be the natural refinement of Ω into clopen sets. Since X is a mildly compact space, let $\{U_{\beta_1}, \ldots, U_{\beta_n}\}$ be a finite subcollection of Υ which covers X. Then $\{f(U_{\beta_1}), \ldots, f(U_{\beta_n})\}$ is a cover of Y. Since each U_β is contained in some $f^{-1}(G_\alpha)$, each $f(U_\beta)$ is contained in some G_α. Thus finitely many G_α's cover Y and so Y is D_δ-compact. □

5.5. **Corollary.** ([35, Proposition 5.7]). *Let $f : X \to Y$ be a pseudo perfectly continuous function from a mildly compact space X onto Y. Then Y is a D_δ-compact space.*

5.6. **Definitions.** A space is said to be a

(i) $\boldsymbol{D_\delta T_0}$**-space** [35] if for each pair of distinct points x and y in X there exists a regular F_σ-set containing one of the points x and y but not the other.

‡Sostak [61] calls mildly compact spaces as clustered spaces.

(ii) ***D_δ*-Hausdorff** [27] (**ultra Hausdorff** [62]) if every pair of distinct points in X are contained in disjoint regular F_σ-sets(clopen sets).

The following diagram illustrates the relationships that exist among $D_\delta T_0$-spaces and other strong variants of Hausdorffness.

$$
\begin{tikzcd}
& \text{ultra Hausdorff} \arrow[d] & \\
& \text{functionally Hausdorff} \arrow[d] & \\
D_\delta T_0\text{-space} \arrow[ddr] & D_\delta\text{-Hausdorff} \arrow[l] \arrow[d] & \\
& \theta\text{-Hausdorff} \arrow[r] & \text{Urysohn} \arrow[dl] \\
& \text{Hausdorff} &
\end{tikzcd}
$$

FIGURE 2

Every $D_\delta T_0$-space is Hausdorff. However, even a Hausdorff regular space need not be a $D_\delta T_0$-space (see [35, Example 5.3])

5.7. **Proposition.** Let $f : X \to Y$ be a pseudo cl-supercontinuous injection into a $D_\delta T_0$-space Y. Then X is an ultra Hausdorff space.

Proof. Let $x, y \in X$, $x \neq y$. Then $f(x) \neq f(y)$. Since Y is a $D_\delta T_0$-space, there exists a regular F_σ-set V containing one of the points $f(x)$ and $f(y)$ but not both. To be precise, suppose that $f(x) \in V$. Since f is pseudo cl-supercontinuous, $f^{-1}(V)$ is a cl-open set containing x but not y. Let U be a clopen set containing x such that $U \subset f^{-1}(V)$. Then U and $X \setminus U$ are disjoint clopen sets containing x and y respectively. Hence X is an ultra Hausdorff space. ∎

5.8. **Corollary.** ([35, Proposition 5.4]) *Let $f : X \to Y$ be a pseudo perfectly continuous injection into a $D_\delta T_0$-space Y. Then X is an ultra Hausdorff space.*

Proof. Every pseudo perfectly continuous function is pseudo cl-supercontinuous. ∎

5.9. **Definition.** A space X is said to be a ***δ*-completely regular space** ([27], [30]) if for each regular G_δ-set F and a point $x \notin F$, there exists a continuous function $f : X \to [0, 1]$ such that $f(x) = 0$ and $f(F) = 1$.

5.10. **Theorem.** *Let $f : X \to Y$ be an open closed pseudo cl-supercontinuous surjection. Then Y is a δ-completely regular space.*

Proof. Let $K \subset Y$ be a regular G_δ-set and let $y \notin K$. Since f is pseudo cl-supercontinuous, $f^{-1}(K)$ is cl-closed and so $f^{-1}(K) = \bigcap_{\alpha \in \Lambda} F_\alpha$, where each F_α is a clopen set. Let $x_0 \in f^{-1}(y)$. Then $x_0 \notin f^{-1}(K)$ and hence $x_0 \notin F_{\alpha_0}$ for some $\alpha_0 \in \Lambda$. Since F_{α_0} is clopen, its characteristic function $\phi : X \to [0,1]$ is continuous and $\phi(x_0) = 0$ and $\phi(f^{-1}(K)) = 1$. Define $\hat{\varphi} : Y \to [0,1]$ by taking $\hat{\varphi}(y) = \sup\{\phi(x) : x \in f^{-1}(y)\}$. Then $\hat{\varphi}(z) = 0$, $\hat{\varphi}(K) = 1$ and by [11, Exercise 16] $\hat{\varphi}$ is continuous. Hence Y is a δ-completely regular space. □

5.11. **Corollary.** ([35, Theorem 5.10]) *Let $f : X \to Y$ be an open closed pseudo perfectly continuous surjection. Then Y is a δ-completely regular space.*

5.12. **Remark.** There exists no open closed pseudo cl-supercontinuous surjection from a space onto a non δ-completely regular space.

5.13. **Theorem.** *Let $f, g : X \to Y$ be pseudo cl-supercontinuous functions from a space X into a D_δ-Hausdorff space Y. Then the equalizer $E = \{x \in X : f(x) = g(x)\}$ of the functions f and g is a cl-closed subset of X.*

Proof. To show that the set E is cl-closed in X, we shall show that its complement $X \setminus E$ is cl-open. To this end, let $x \in X \setminus E$. Then $f(x) \neq g(x)$. Since Y is D_δ-Hausdorff, there exist disjoint regular F_σ-sets U and V containing $f(x)$ and $g(x)$, respectively. Since f and g are pseudo cl-supercontinuous, there exist clopen sets G_1 and G_2 containing x such that $f(G_1) \subset U$ and $g(G_2) \subset V$. Then $G = G_1 \cap G_2$ is a clopen set containing x. Since U and V are disjoint, $G \subset X \setminus E$ and so $X \setminus E$ being the union of clopen sets is cl-open. □

5.14. **Corollary.** ([35, Proposition 5.12]) *Let $f, g : X \to Y$ be pseudo perfectly functions from a space X into a D_δ-Hausdorff space Y. Then the equalizer $E = \{x \in X : f(x) = g(x)\}$ of the functions f and g is a cl-closed subset of X.*

5.15. **Proposition.** Let $f : X \to Y$ be a pseudo cl-supercontinuous function from a space X into a D_δ-Hausdorff space Y. Then the set $A = \{(x_1, x_2) \in X \times X : f(x_1) = f(x_2)\}$ is cl-closed in $X \times X$.

Proof. To prove that A is a cl-closed subset of $X \times X$, we shall show that $X \times X \setminus A$ is cl-open. So let $(x, y) \notin A$. Then $f(x) \neq f(y)$. Since Y is a D_δ-Hausdorff space, there exist disjoint regular F_σ-sets U and V containing $f(x)$ and $f(y)$ respectively. Since f is pseudo cl-supercontinuous there exist clopen sets G_1 and G_2 in X containing x and y respectively such that $f(G_1) \subset U$ and $f(G_2) \subset V$. Then $G_1 \times G_2$ is a clopen subset of $X \times X$ containing (x, y) and $(G_1 \times G_2) \cap A = \emptyset$. Hence $G_1 \times G_2 \subset X \times X \setminus A$ and so $X \times X \setminus A$ is cl-open. □

5.16. **Corollary.** ([35, Proposition 5.13]) *Let $f : X \to Y$ be a pseudo perfectly function from a space X into a D_δ-Hausdorff space Y. Then the set $A = \{(x_1, x_2) \in X \times X : f(x_1) = f(x_2)\}$ is a cl-closed in $X \times X$.*

The following gives sufficient conditions for the direct and inverse preservation of pseudo zero dimensionality under mappings.

5.17. **Theorem.** ([21]) *Let $f : X \to Y$ be a pseudo cl-supercontinuous bijection which maps regular F_σ-sets to regular F_σ-sets. Then X is a pseudo zero dimensional space. Further, if in addition f maps clopen sets to clopen sets, then Y is also a pseudo zero dimensional space.*

5.18. **Definitions.** A space X is said to be

(i) **pseudo hyperconnected** if there exists no nonempty proper regular G_δ-set in X or equivalently there exists no nonempty proper regular F_σ-set in X, i.e. X is the only nonempty regular F_σ-set in X.

(ii) **hyperconnected** ([1], [63]) if every nonempty open subset of X is dense in X, i.e. X is the only nonempty regular open set in X.

5.19. **Proposition.** Let $f : X \to Y$ be a pseudo cl-supercontinuous surjection from a connected space X onto Y. Then Y is pseudo hyperconnected.

Proof. Suppose Y is not pseudo hyperconnected and let V be a nonempty proper regular F_σ-set in Y. Since f is pseudo cl-supercontinuous, $f^{-1}(V)$ is a nonempty proper cl-open subset of X contradicting the fact that X is connected. □

5.20. **Remark.** There exists no pseudo cl-supercontinuous surjection from a connected space onto a non pseudo hyperconnected space.

6. Function Spaces and Pseudo Perfectly Continuous Functions

It is of fundamental importance in topology, analysis and other branches of mathematics to know whether a given function space is closed/compact in Y^X in the topology of pointwise convergence. So it is of considerable significance both from intrinsic interest as well as from applications viewpoint to formulate conditions on the spaces X, Y so that certain subsets of $C(X,Y)$ or Y^X are closed/compact in the topology of pointwise convergence. Results of this type and Ascoli type theorems abound in the literature (see [2], [17]). Naimpally's result [47] that in contrast to continuous functions, the set $S(X,Y)$ of strongly continuous functions is closed in Y^X in the topology of pointwise convergence if X is locally connected and Y is Hausdorff; is extended to a larger framework by Kohli and Singh [31], wherein it is shown that if X is sum connected and Y is Hausdorff, then the function space $P(X,Y)$ of all perfectly continuous functions as well as the function space $L(X,Y)$ of all cl-supercontinuous functions is closed in Y^X in the topology of pointwise convergence. This result is further extended in ([33], [40], [57]) for the set $P_\Delta(X,Y)$ of all δ-perfectly continuous functions as well as for the set $P_\delta(X,Y)$ of all almost perfectly continuous ($\equiv$ regular set connected) functions and the set $P_q(X,Y)$ of all quasi perfectly continuous functions under the same hypotheses on X and Y. Moreover, in [35] the same result is shown to be true for the set $P_p(X,Y)$ of all pseudo perfectly continuous functions if the hypothesis on Y is strengthened to be a $D_\delta T_0$-space. Herein we further strengthen these results for the set $L_p(X,Y)$ of all pseudo cl-supercontinuous functions to show that if X is a sum connected space and Y is a $D_\delta T_0$-space, then all the ten classes of functions are identical, i.e. $S(X,Y) = P(X,Y) = L(X,Y) = P_\Delta(X,Y) = P_\delta(X,Y) = P_q(X,Y) = P_p(X,Y) = L_\delta(X,Y) = L_q(X,Y) = L_p(X,Y)$ and are closed in Y^X in the topology of pointwise convergence. Note that $L_\delta(X,Y)$ denotes the set of all almost cl-supercontinuous functions and $L_q(X,Y)$ denotes the set of all quasi cl-supercontinuous functions from X into Y.

6.1. **Proposition.** Let $f : X \to Y$ be a pseudo cl-supercontinuous function into a $D_\delta T_0$-space Y. Then f is constant on each connected subset of X. In particular, if X is connected, then f is constant on X and hence strongly continuous.

Proof. Assume contrapositive and let C be the connected subset of X such that $f(C)$ is not a singleton. Let $f(x)$, $f(y) \in f(C)$, $f(x) \neq f(y)$. Since Y is a $D_\delta T_0$-space, there exists a regular F_σ-set V containing one of the points $f(x)$ and $f(y)$ but not other. For definiteness assume that $f(x) \in V$. Since f is pseudo cl-supercontinuous, $f^{-1}(V) \cap C$ is a nonempty proper cl-open subset of

C and so C contains a nonempty proper clopen set, contradicting the fact that C is connected. The last part of the theorem is immediate, since every constant function is strongly continuous. □

6.2. **Corollary.** *Let $f : X \to Y$ be a pseudo cl-supercontinuous function from a sum connected space X into a $D_\delta T_0$-space Y. Then f is constant on each component of X and hence strongly continuous.*

Proof. Since X is a sum connected space, each component of X is clopen in X. Hence it follows that any union of components of X and the complement of this union are complementary clopen sets in X. By above propositon f is constant on each component on X. Therefore, for every subset H of Y, $f^{-1}(H)$ and $X \setminus f^{-1}(H)$ are complementary clopen sets in X being the union of components of X. So f is strongly continuous. □

Next, we quote the following results from ([23], [35], [57]).

6.3. **Theorem** ([57, Theorem 4.5])**.** *Let $f : X \to Y$ be a function from a sum connected space X into a δT_0-space Y. Then the following statements are equivalent.*

(a) *f is strongly continuous.*
(b) *f is perfectly continuous.*
(c) *f is cl-supercontinuous.*
(d) *f is δ-perfectly continuous.*
(e) *f is almost perfectly continuous.*

6.4. **Theorem** ([23, Theorem 9.8])**.** *Let $f : X \to Y$ be a function from a sum connected space X into a Hausdorff space Y. Then the following statements are equivalent.*

(a) *f is strongly continuous.*
(b) *f is perfectly continuous.*
(c) *f is cl-supercontinuous.*
(d) *f is δ-perfectly continuous.*
(e) *f is almost perfectly continuous.*
(f) *f is almost cl-supercontinuous.*
(g) *f is quasi cl-supercontinuous.*

6.5. **Theorem** ([35, Theorem 6.7])**.** *Let $f : X \to Y$ be a function from a sum connected space X into a $D_\delta T_0$-space Y. Then the following statements are equivalent.*

(a) *f is strongly continuous.*
(b) *f is perfectly continuous.*

(c) *f is cl-supercontinuous.*
(d) *f is δ-perfectly continuous.*
(e) *f is almost perfectly continuous.*
(f) *f is quasi perfectly continuous.*
(g) *f is pseudo perfectly continuous.*
(h) *f is almost cl-supercontinuous.*

6.6. **Remark.** In the original version of ([35, Theorem 6.5]) the equivalence (h) does not appear. However, it is immediate in view of Theorem 6.4 and the fact that the class of almost cl-supercontinuous functions lies between the classes of cl-supercontinuous functions and quasi cl-supercontinuous functions.

6.7. **Theorem** ([35, Theorem 6.8])**.** *Let X be a sum connected space and let Y be a $D_\delta T_0$-space. Then $S(X,Y) = P(X,Y) = L(X,Y) = P_\Delta(X,Y) = P_\delta(X,Y) = P_q(X,Y) = P_p(X,Y)$ is closed in Y^X in the topology of pointwise convergence.*

6.8. **Theorem.** *Let $f : X \to Y$ be a function from a sum connected space X into a $D_\delta T_0$-space Y. Then the following statements are equivalent.*

(a) *f is strongly continuous.*
(b) *f is perfectly continuous.*
(c) *f is cl-supercontinuous.*
(d) *f is δ-perfectly continuous.*
(e) *f is almost perfectly continuous.*
(f) *f is almost cl-supercontinuous.*
(g) *f is quasi cl-supercontinuous.*
(h) *f is quasi perfectly continuous.*
(i) *f is pseudo perfectly continuous.*
(j) *f is pseudo cl-supercontinuous.*

6.9. **Theorem.** *Let X be a sum connected space and let Y be a $D_\delta T_0$-space. Then $S(X,Y) = P(X,Y) = L(X,Y) = P_\Delta(X,Y) = P_\delta(X,Y) = P_q(X,Y) = P_p(X,Y) = L_\delta(X,Y) = L_q(X,Y) = L_p(X,Y)$ is closed in Y^X in the topology of pointwise convergence.*

6.10. **Theorem** ([35, Theorem 6.9])**.** *If X is a sum connected space and Y is a compact $D_\delta T_0$-space, then the spaces $S(X,Y) = P(X,Y) = L(X,Y) = P_\Delta(X,Y) = P_\delta(X,Y) = P_q(X,Y) = P_p(X,Y)$ are compact Hausdorff subspaces of Y^X in the topology of pointwise convergence.*

6.11. **Theorem.** *If X is a sum connected space and Y is a compact $D_\delta T_0$-space, then the spaces $S(X,Y) = P(X,Y) = L(X,Y) = P_\Delta(X,Y) = P_\delta(X,Y) = P_q(X,Y) = P_p(X,Y) = L_\delta(X,Y) = L_q(X,Y) = L_p(X,Y)$ are compact Hausdorff subspaces of Y^X in the topology of pointwise convergence.*

7. Change of Topology and Pseudo cl-supercontinuous functions

The technique of change of topology of a space is of considerable significance and prevalant throughout the mathematics. It is widely used in topology, functional analysis and several other branches of mathematics. For example, weak and weak* topologies of a Banach space, weak and strong operator topologies on $\mathfrak{B}(H)$ the space of operators on a Hilbert space, the hull kernel topology and the multitude of other topologies on $Id(A)$ the space of all closed two sided ideals of a Banach algebra A ([4], [5], [6], [60]). Furthermore, to taste the flavour of applications of the technique in topology see ([14], [18], [20], [34], [56], [65]).

Here we restrict ourselves to the retopologization of the domain/range or both domain and range of a pseudo cl-supercontinuous function and reflect upon the change in the nature of pseudo cl-supercontinuous function. This change suggests alternative proofs of certain results of the preceding sections.

Let (X,τ) be a topological space and let

(i) β_{d_δ} denote the collection of all regular F_σ-subsets of the space (X,τ). Since the intersection of two regular F_σ-sets is a regular F_σ-set, the collection β_{d_δ} constitutes the base for a topology on X which we denote by τ_{d_δ}. The topology τ_{d_δ} has been used in the literature. For example(see [26],[34]).

(ii) β_{cl} denote the collection of all clopen subsets of (X,τ). Since the intersection of two clopen sets is a clopen set, the collection β_{cl} is a base for a topology on X which we denote by τ_{cl}. The topology τ_{cl} has been extensively referred to in the mathematical literature. For example (see [12], [34],[56], [62]).

7.1. **Proposition.** A topological space (X,τ) is D_δ-completely regular if and only if $\tau = \tau_{d_\delta}$.

7.2. **Proposition.** A topological space (X,τ) is zero dimensional space if and only if $\tau = \tau_{cl}$.

7.3. **Theorem.** *For a function $f : (X, \tau) \to (Y, \nu)$, the following statements are equivalent.*

(a) $f : (X, \tau) \to (Y, \nu)$ *is pseudo cl-supercontinuous.*
(b) $f : (X, \tau) \to (Y, \nu_{d_\delta})$ *is cl-supercontinuous.*
(c) $f : (X, \tau_{cl}) \to (Y, \nu)$ *is* D_δ*-continuous.*
(d) $f : (X, \tau_{cl}) \to (Y, \nu_{d_\delta})$ *is continuous.*

References

[1] N. Ajmal and J.K. Kohli, *Properties of hyperconnected spaces, their mappings into Hausdorff spaces and embeddings into hyperconnected spaces*, Acta Math. Hungar. 60(1-2) (1992), 41-49.

[2] A.V. Arhangel´skii, *General Topology III*, Springer Verlag, Berlin Heidelberg, 1995.

[3] C.E. Aull, *Functionally regular spaces*, Indag. Math. 38 (1976), 281-288.

[4] F. Beckhoff, *Topologies on the spaces of ideals of a Banach algebra*, Stud. Math. 115(1995), 189-205.

[5] F. Beckhoff, *Topologies of compact families on the ideal space of a Banach algebra*, Stud. Math. 118(1996), 63-75.

[6] F. Beckhoff, *Topologies on the ideal space of a Banach algebra and spectral synthesis*, Proc. Amer. Math. Soc. 125(1997), 2859-2866.

[7] R.N. Bhaumik, *Role of regular* G_δ*-subsets in set-theoretic topology*, The Mathematics Student, 70(1-4), (2001), 99-104.

[8] H. Brandenburg, *On a class of nearness spaces and the epireflective hull of developable topological spaces*, Proc. Internat. Symposium on Topology and its Applications, (Beograd, 1977).

[9] H. Brandenburg, *On spaces with a* G_δ*-basis*, Arch. Math. (Basel) 35(1980), 544-547.

[10] J. Dontchev, M. Ganster and I. Reilly, *More on almost s-continuity*, Indian J. Math. 41(1999), 139-146.

[11] J. Dugundji, *Topology*, Allyn and Bacon, Boston, 1966.

[12] E. Ekici, *Generalization of perfectly continuous, regular set-connected and clopen functions*, Acta Math Hungar. 107(3) (2005), 193-206.

[13] S. Fomin, *Extensions of topological spaces*, Ann. of Math. 44(1943), 471-480.

[14] A.M. Gleason, *Universal locally connected refinements*, Illinois J.Math. 7(1963), 521-531.

[15] N.C. Heldermann, *Developability and some new regularity axioms*, Canadian J. Math. 33(3) (1981), 641-663.

[16] R.C. Jain, *The role of regularly open sets in general topology*, Ph.D. Thesis, Meerut University, Institute of Advanced Studies, Meerut, India (1980).

[17] J.L. Kelley, General Topology, Van Nostrand, New York, 1955.

[18] J.K. Kohli, *A class of mappings containing all continuous and all semiconnected mappings*, Proc. Amer. Math. Soc. 72(1) (1978), 175-181.

[19] J.K. Kohli, *A class of spaces containing all connected and all locally connected spaces*, Math. Nachricten 82(1978), 121-129.

[20] J.K. Kohli, *Change of topology, characterizations and product theorems for semilocally P-spaces*, Houston J. Math. 17(3) (1991), 335-350.

[21] J.K. Kohli, *Localization, generalizations and factorization of zero dimensionality*, (preprint).

[22] J.K. Kohli, *D-continuous functions, D-regular spaces and D-Hausdorff spaces*, Bull. Cal. Math. Soc. 84(1992), 39-46.

[23] J.K. Kohli and J. Aggarwal, *Quasi cl-supercontinuous functions and their function spaces*, Demonstratio Math. 45(3) (2012), 677-697.

[24] J.K. Kohli and R. Kumar, *z-supercontinuous functions*, Indian J. Pure Appl. Math. 33(7), (2002), 1097-1108.

[25] J.K. Kohli and D. Singh, *D-supercontinuous functions*, Indian J. Pure Appl. Math. 32(2) (2001), 227-235.

[26] J.K. Kohli and D. Singh, *D_δ-supercontinuous functions*, Indian J. Pure Appl. Math. 34(7) (2003), 1089-1100.

[27] J.K. Kohli and D. Singh, *Between weak continuity and set connectedness*, Studii Si Cercetari Stiintifice Ser. Matem. Univ. Bacau Nr. 15 (2005), 55-65.

[28] J.K. Kohli and D. Singh, *Between compactness and quasicompactness*, Acta, Math. Hungar. 106(4) (2005), 317-329.

[29] J.K. Kohli and D. Singh, *Weak normality properties and factorizations of normality*, Acta Math. Hungar. 110(1-2) (2006), 67-80.

[30] J.K. Kohli and D. Singh, *Between regularity and complete regularity and a factorization of complete regularity*, Studii Si Cercetari Stiintifice Seria Matematica 17 (2007), 125-134.

[31] J.K. Kohli and D. Singh, *Function spaces and strong variants of continuity*, Applied General Topology 9(1) (2008), 33-38.

[32] J.K. Kohli and D. Singh,*Almost cl-supercontinuous functions*, Applied General Topology 10(1) (2009), 1-12.

[33] J.K. Kohli and D. Singh, *δ-perfectly continuous functions*, Demonstratio Mathematica 42(1) (2009), 221-231.

[34] J.K. Kohli, D. Singh and J. Aggarwal, *R-supercontinuous functions*, Demonstratio Mathematica 43(3) (2010), 703-723.

[35] J.K. Kohli, D. Singh, J. Aggarwal and M. Rana, *Pseudo perfectly continuous functions and closedness/compactness of their function spaces*, Applied General Topology 14(1) (2013), 115-134.

[36] J.K. Kohli, D. Singh and C.P. Arya, *Perfectly continuous functions*, Stud. Cerc. St. Ser. Mat. Nr. 18, (2008), 99-110.

[37] J.K. Kohli, D. Singh and R. Kumar, *Quasi z-supercontinuous functions and pseudo z-supercontinuous functions*, Studii Si Cercetari Stiintifice Ser. Matem. Univ. Bacau Nr. 14(2004), 43-56.

[38] J.K. Kohli, D. Singh and R. Kumar, *Generalizations of z-supercontinuous functions and D_δ-supercontinuous functions*, Applied General Topology 9(2) (2008), 239-251.

[39] J.K. Kohli, D. Singh, R. Kumar and J. Aggarwal, *Between continuity and set conectedness*, Applied General Topology 11(1) (2010), 43-55.

[40] J.K. Kohli, D. Singh and B.K. Tyagi, *Quasi perfectly continuous functions and their function spaces*, Scientific Studies and Research Series Mathematics and Informatics, 21(2) (2011), 23-40

[41] N. Levine, *Strong continuity in topological spaces*, Amer. Math. Monthly, 67(1960), 269.

[42] N. Levine, *A decomposition of continuity in topological space*, Amer. Math. Monthly, 68 (1961), 44-46.

[43] P.E. Long and L. Herrington, *Strongly θ-continuous functions*, J. Korean Math. Soc. 8(1981), 21-28.

[44] P.E. Long and L. Herrington, *T_θ-topology and faintly continuous functions*, Kyungpook Math. J. 22 (1982), 7-14.

[45] J. Mack, *Countable paracompactness and weak normality properties*, Trans. Amer. Math. Soc. 148(1970), 265-272.

[46] B.M. Munshi and D.S. Bassan, *Super-continuous mappings*, Indian J. Pure Appl. Math. 13(1982), 229-236.

[47] S.A. Naimpally, *On strongly continuous functions*, Amer. Math. Monthly 74(1967), 166-168.

[48] T. Noiri, *On δ-continuous functions*, J. Korean Math. Soc. 16(1980), 161-166.

[49] T. Noiri, *Supercontinuity and some strong forms of continuity*, Indian J. Pure. Appl. Math. 15(3) (1984), 241-250.

[50] T. Noiri and Sin Min Kang, *On almost strongly θ-continuous functions*, Indian J. Pure Appl. Math. 15(1) (1984), 1-8.

[51] T. Noiri and V. Popa, *Weak forms of faint continuity*, Bull. Math. de la Soc. Sci. Math. de la Roumanie 34(82) (1990), 263-270.

[52] I.L. Reilly and M. K. Vamanamurthy, *On super-continuous mappings*, Indian J. Pure. Appl. Math. 14(6) (1983), 767-772.

[53] M.K. Singal and S.B. Nimse, *z-continuous mappings*, The Mathematics Student 66(1-4) (1997), 193-210.

[54] M.K. Singal and A.R. Singal, *Almost continuous mappings*, Yokohama Math. J. 16 (1968), 63-73.

[55] D. Singh, *D^*-supercontinuous functions*, Bull. Cal. Math. Soc. 94(2) (2002), 67-76.

[56] D. Singh, *cl-supercontinuous functions*, Applied General Topology, 8(2), (2007), 293-300.

[57] D. Singh, *Almost perfectly continuous functions*, Quaestiones Mathematicae 33(2) (2010), 211-221.

[58] D. Singh, *Pseudo strongly θ-continuous functions*, Scientific Studies and Research Series Mathematics and Informatics 23(2) (2013), 127-140.

[59] D. Singh and J.K. Kohli, *On certain generalizations of supercontinuity/δ-continuity*, Scientific Studies and Research Series Mathematics and Informatics 12(2) (2012), 119-136.

[60] D.W.B. Somerset, *Ideal spaces of Banach algebras*, Proc. London Math. Soc. 78(3) (1999), 369-400.

[61] A. Sostak, *A class of topological spaces containing all bicompact and all connected spaces*, General Topology and its Relations to Modern Analysis and Algebra IV, Proceedings of the 4th Prague Topological Symposium (1976), Part B, 445-451.

[62] R. Staum, *The Algebra of bounded continuous functions into a nonarchimedean field*, Pac. J. Math. 50(1) (1974), 169-185.

[63] L.A. Steen and J.A. Seeback, Jr., *Counter Examples in Topology*, Springer Verlag, New York, 1978.

[64] N.V. Veličko, *H-Closed topological spaces*, Amer. Math. Soc. Transl. 78 (1968), 103-118.

[65] G.S. Young, *Introduction of local connectivity by change of topology*, Amer. J. Math. 68 (1946), 479-494.

Department of Mathematics, Sri Aurobindo College, University of Delhi, New Delhi 110017, India dstopology@rediffmail.com

Department of Mathematics, Shivaji College, University of Delhi, New Delhi 110027, India jitenaggarwal@gmail.com

Department of Mathematics, Hindu College, University of Delhi, Delhi 110007, India. jk_kohli@yahoo.co.in