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$f\breve{g}$-CLOSED SETS IN A FUZZY SET TOPOLOGY

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Abstract. This paper deals with a new type of fuzzy generalized version of closed sets, called $f\breve{g}$-closed sets, which is already defined in [16]. Again the mutual relationships of this class of sets with other classes defined in [3, 5, 6, 7, 8, 9, 11, 12, 13, 14, 15, 16] are established. Then a new type of closure operator, called $f\breve{g}$-closure operator, is introduced and it is proved that this is an idempotent operator. With the help of this operator, $f\breve{g}$-open, $f\breve{g}$-closed, $f\breve{g}$-continuous and $f\breve{g}$-irresolute functions are introduced and characterized. Afterwards, $f\breve{g}$-regular, $f\breve{g}$-normal, $f\breve{g}$-compact, $f\breve{g}$-T_2-spaces are introduced and characterized. Lastly, applications of the above mentioned functions on these spaces are given.

1. **Introduction**

In [3], the notion of fuzzy generalized closed set has been introduced. Afterwards, dif and only iferent types of generalized versions of fuzzy closed sets are introduced and studied. In this context we have to mention [5, 6, 7, 8, 9, 11, 12, 13, 14, 15, 16]. Here we also introduce a new type of generalized version of fuzzy closed sets, which implies other generalized version of fuzzy closed sets, but not conversely. With the help of this notion a new type of separation axioms are introduced and studied.

Keywords and phrases: $f\breve{g}$-closed set, fuzzy compact space, $f\breve{g}$-closed function, $f\breve{g}$-open q-nbd, $f\breve{g}$-regular space, $f\breve{g}$-continuous function, fuzzy T_2-space, fuzzy semiopen set.
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2. **Preliminaries**

Throughout this paper, by (X, τ) or simply by X we shall mean a fuzzy topological space (fts, for short) in the sense of Chang [20]. A fuzzy set [35] A in an fts X, denoted by $A \in I^X$, is defined to be a mapping from a non-empty set X into the closed interval $I = [0, 1]$. The support [35] of a fuzzy set A, denoted by $suppA$ [35] and is defined by $suppA = \{x \in X : A(x) \neq 0\}$. The fuzzy set with the singleton support $\{x\} \subseteq X$ and the value t $(0 < t \leq 1)$ will be denoted by x_t [35]. 0_X and 1_X are the constant fuzzy sets taking values 0 and 1 respectively in X. The complement [35] of a fuzzy set A in X is denoted by $1_X \setminus A$ and is defined by $(1_X \setminus A)(x) = 1 - A(x)$, for each $x \in X$. For any two fuzzy sets A, B in X, $A \leq B$ means $A(x) \leq B(x)$, for all $x \in X$ [35] while AqB means A is quasi-coincident (q-coincident, for short) [32] with B, i.e., there exists $x \in X$ such that $A(x) + B(x) > 1$. The negation of these two statements will be denoted by $A \not\leq B$ and $A \not{q} B$ respectively. For a fuzzy set A, clA and $intA$ will stand for fuzzy closure [20] and fuzzy interior [20] respectively. A fuzzy set A in an fts X is called fuzzy regular open [2] if $A = intclA$. A fuzzy set A is called a fuzzy neighbourhood (nbd, for short) of a fuzzy point x_t if there exists a fuzzy open set G in X such that $x_t \leq G \leq A$ [32]. If, in addition, A is open, then A is called a fuzzy open nbd [32] of x_t. A fuzzy set A in X is called a q-neighbourhood (q-nbd, for short) [32] of a fuzzy point x_t if there is a fuzzy open set U in X such that $x_t q U \leq A$. If, in addition, A is fuzzy open (resp., fuzzy regular open), then A is called fuzzy open q-nbd [32] (resp., fuzzy regular open q-nbd [2]) of x_t. A fuzzy point x_α is said to be a fuzzy δ-cluster point of a fuzzy set A in an fts X if every fuzzy regular open q-nbd U of x_α is q-coincident with A [24]. The union of all fuzzy δ-cluster points of A is called the fuzzy δ-closure of A, denoted by δclA [24]. A fuzzy set A is called fuzzy δ-closed if $A = \delta clA$ [24] and the complement of a fuzzy δ-closed set is called fuzzy δ-open [24]. The union of all fuzzy δ-open sets contained in a fuzzy set A is called fuzzy δ-interior of A and is denoted by $\delta intA$ [24]. For a fuzzy set A in an fts (X, τ), $\delta cl(1_X \setminus A) = 1_X \setminus \delta intA$ [24]. A fuzzy set A in an fts X is called fuzzy semiopen [2] (respectively, fuzzy β-open [23]) if $A \leq clintA$ (respectively, $A \leq clintclA$). The complement of a fuzzy semiopen (respectively, fuzzy β-open) set is called fuzzy semiclosed [2] (respectively, fuzzy β-closed [23]). The intersection of all fuzzy semiclosed (respectively, fuzzy β-closed) sets containing a fuzzy set A is called fuzzy semiclosure [2] (respectively, fuzzy β-closure [23]) of A, denoted by $sclA$ (respectively, βclA). The collection of all fuzzy semiopen (respectively, fuzzy β-open, fuzzy δ-open) sets in an fts X is denoted by $FSO(X)$ (respectively, $F\beta O(X)$, $F\delta O(X)$) and that of fuzzy semiclosed (respectively, fuzzy β-closed, fuzzy δ-closed) sets is denoted by $FSC(X)$ (respectively, $F\beta C(X)$, $F\delta C(X)$).

3. $f\breve{g}$-Closed Set : Some Properties

In this section we first introduce the class of $f\breve{g}$-closed sets which is strictly larger than the class of fuzzy closed sets. Then some important properties of this class of sets are established here and its mutual relationships with dif and only iferent types of generalized version of fuzzy closed sets, which are defined in [5, 6, 7, 8, 9, 11, 12, 13, 14, 15, 16], are investigated.

Definition 3.1. Let (X, τ) be an fts and $A \in I^X$. Then A is called $f\breve{g}$-closed set in X if $clA \leq U$ whenever $A \leq U \in FSO(X)$.
The complement of $f\breve{g}$-closed set is called $f\breve{g}$-open set.

Remark 3.2. It is clear from definition that every fuzzy closed set is an $f\breve{g}$-closed set. But the converse is not true, in general, as it follows from the following example.

Example 3.3. $f\breve{g}$-closed set $\not\Rightarrow$ fuzzy closed set
Let $X = \{a, b\}$, $\tau = \{0_X, 1_X\}$. Then (X, τ) is an fts. Consider the fuzzy set A, defined by $A(a) = A(b) = 0.5$. Then 1_X is the only fuzzy semiopen set in X containing A and so $clA = 1_X \leq 1_X \Rightarrow A$ is $f\breve{g}$-closed set in X. But $A \notin \tau^c$.

Definition 3.4. Let (X, τ) be an fts and x_α be a fuzzy point in X. Then $A(\in I^X)$ is called an $f\breve{g}$-neighbourhood ($f\breve{g}$-nbd, for short) of x_α if there exists an $f\breve{g}$-open set U in X such that $x_\alpha \leq U \leq A$. If, in addition, A is $f\breve{g}$-open set in X, then A is called an $f\breve{g}$-open nbd of x_α.

Definition 3.5. Let (X, τ) be an fts and x_α be a fuzzy point in X. Then $A(\in I^X)$ is called an $f\breve{g}$-open quasi neighbourhood ($f\breve{g}$ q-nbd, for short) of x_α if there is an $f\breve{g}$-open set U in X such that $x_\alpha q U \leq A$. If, in addition, A is $f\breve{g}$-open set in X, then A is called an $f\breve{g}$-open q-nbd of x_α.

Note 3.6. It is to be noted that arbitrary union of $f\breve{g}$-closed sets is $f\breve{g}$-closed. But intersection of two $f\breve{g}$-closed sets need not be so, as it follows from the following example.

Example 3.7. Let $X = \{a, b\}$, $\tau = \{0_X, 1_X, A\}$ where $A(a) = 0.5, A(b) = 0.4$. Then (X, τ) is an fts. Now $FSO(X) = \{0_X, 1_X, U\}$ where $A \leq U \leq 1_X \setminus A$. Consider the fuzzy sets C and D defined by $C(a) = 0.5, C(b) = 0.6$ and $D(a) = 0.6, D(b) = 0.5$. Then 1_X is the only fuzzy semiopen set in (X, τ) such that $C < 1_X$ and $D < 1_X$ and so $clC = 1_X \leq 1_X$ and $clD = 1_X \leq 1_X$. Consequently, C and D are $f\breve{g}$-closed sets in X. Put $E = C \bigwedge D$. Then $E(a) = E(b) = 0.5$. Now $E \leq E \in FSO(X)$. But $clE = 1_X \setminus A \not\leq E \Rightarrow E$ is not an $f\breve{g}$-closed set in X.

Remark 3.8. From the above discussion we can conclude that the collection of all $f\breve{g}$-open sets does not form a fuzzy topology.

Theorem 3.9. If $A(\in I^X)$ is $f\breve{g}$-closed set in X and $B \in I^X$ is such that $A \leq B \leq clA$, then B is also $f\breve{g}$-closed set in X.

Proof. Let $U \in FSO(X)$ be such that $B \leq U$. Then by hypothesis, $A \leq B \leq U$. As A is $f\breve{g}$-closed set in X, $clA \leq U$ and so $A \leq B \leq clA \leq U \Rightarrow clA \leq clB \leq cl(clA) = clA \leq U \Rightarrow clB \leq U$. Consequently, B is $f\breve{g}$-closed set in X.

Theorem 3.10. Let (X, τ) be an fts and $A, B \in I^X$. If $intA \leq B \leq A$ and A is $f\breve{g}$-open set in X, then B is also $f\breve{g}$-open set in X.

Proof. $intA \leq B \leq A \Rightarrow 1_X \setminus A \leq 1_X \setminus B \leq 1_X \setminus intA = cl(1_X \setminus A)$ where $1_X \setminus A$ is $f\breve{g}$-closed set in X. By Theorem 3.9, $1_X \setminus B$ is $f\breve{g}$-closed set in $X \Rightarrow B$ is $f\breve{g}$-open set in X.

Theorem 3.11. Let (X, τ) be an fts and $A \in I^X$. Then A is $f\breve{g}$-open set in X if and only if $K \leq intA$ whenever $K \leq A$ and $K \in FSC(X)$.

Proof. Let $A(\in I^X)$ be $f\breve{g}$-open set in X and $K \leq A$ where $K \in FSC(X)$. Then $1_X \setminus A \leq 1_X \setminus K$ where $1_X \setminus A$ is $f\breve{g}$-closed set in X and $1_X \setminus K \in FSO(X)$. So $cl(1_X \setminus A) \leq 1_X \setminus K \Rightarrow 1_X \setminus intA \leq 1_X \setminus K \Rightarrow K \leq intA$.

Conversely, let $K \leq intA$ whenever $K \leq A$, $K \in FSC(X)$. Then $1_X \setminus A \leq 1_X \setminus K \in FSO(X)$. Now $1_X \setminus intA \leq 1_X \setminus K \Rightarrow cl(1_X \setminus A) \leq$

$1_X \setminus K \Rightarrow 1_X \setminus A$ is $f\breve{g}$-closed set in $X \Rightarrow A$ is $f\breve{g}$-open set in X.

Theorem 3.12. Let (X, τ) be an fts and $A(\in I^X) \in FSO(X)$. If A is $f\breve{g}$-closed set in X, then A is fuzzy closed set in X.
Proof. Now $A \leq A \in FSO(X)$. By hypothesis, $clA \leq A \Rightarrow A = clA \Rightarrow A$ is a fuzzy closed set in X.

In a similar manner we can state the following theorem easily.

Theorem 3.13. Let (X, τ) be an fts and $A(\in I^X) \in FRO(X)$. If A is a $f\breve{g}$-closed set in X, then A is a fuzzy closed set in X.

Theorem 3.14. Let (X, τ) be an fts and $A(\in I^X)$ be an $f\breve{g}$-closed set and F, a fuzzy closed set in with X $A \not{q} F$. Then $clA \not{q} F$.
Proof. Now $A \not{q} F \Rightarrow A \leq 1_X \setminus F \in \tau$ and hence $1_X \setminus F \in FSO(X)$. By assumption, $clA \leq 1_X \setminus F \Rightarrow clA \not{q} F$.

The converse may not be true, in general, as it seen from the following example.

Example 3.15. Let $X = \{a, b\}$, $\tau = \{0_X, 1_X, A, B\}$ where $A(a) = 0.5, A(b) = 0.6, B(a) = 0.4, B(b) = 0.5$. Then (X, τ) is an fts. Now $FSO(X, \tau) = \{0_X, 1_X, U, V\}$ where $U \geq A$, $B \leq V \leq 1_X \setminus B$. Consider the fuzzy set C defined by $C(a) = C(b) = 0.4$. Then $C \not{q} (1_X \setminus A) \in \tau^c$ ($\in FSC(X)$ also) and $clC = (1_X \setminus A) \not{q} (1_X \setminus A)$. Now $C \leq B \in FSO(X)$ and $clC = 1_X \setminus A \not\leq B \Rightarrow C$ is not $f\breve{g}$-closed set in X.

Definition 3.16. Let (X, τ) be an fts and $A \in I^X$. Then $f\breve{g}$-kernel of A, denoted by $f\breve{g}$-$ker(A)$ is defined by $f\breve{g}$-$ker(A) = \bigwedge\{U \in FSO(X) : A \leq U\}$.

Remark 3.17. The following example shows that the intersection of any two fuzzy semiopen sets may not be fuzzy semiopen, shown in the next example and as a result, $f\breve{g}$-$ker(A)$ is not fuzzy semiopen set in an fts X, in general.

Example 3.18. Let $X = \{a, b\}$, $\tau = \{0_X, 1_X, A, B, C, D\}$ where $A(a) = 0.5, A(b) = 0.6, B(a) = 0.6, B(b) = 0.2, C(a) = 0.5, C(b) = 0.2, D(a) = D(b) = 0.6$. Then (X, τ) is an fts.

Now $FSO(X,\tau) = \{0_X, 1_X, U, V, W\}$ where $A \leq U \leq 1_X \setminus C$, $C \leq V \leq 1_X \setminus A$, $W \geq B$. Here the fuzzy sets S, T defined by $S(a) = 0.5, S(b) = 0.7$ and $T(a) = 0.6, T(b) = 0.5$ are fuzzy semiopen sets in X. But $M = S \bigwedge T$, defined by $M(a) = M(b) = 0.5$ is not fuzzy semiopen set in X.

Theorem 3.19. If a fuzzy set A in an fts (X,τ) is $f\breve{g}$-closed, then $clA \leq f\breve{g}\text{-}ker(A)$.

Proof. Let $A(\in I^X)$ be $f\breve{g}$-closed set in X and $x_\alpha \in clA$ be any fuzzy point in X. If possible, let $x_\alpha \notin f\breve{g}\text{-}ker(A)$. Then there exists $V \in FSO(X)$ with $A \leq V$ and $x_\alpha \notin V$. As A is $f\breve{g}$-closed set in X, $clA \leq V \Rightarrow x_\alpha \in V$, a contradiction.

Let us now recall some definitions from [3, 5, 9, 12, 13, 14] for ready references.

Definition 3.20. Let (X,τ) be an fts and $A \in I^X$. Then A is called
(i) fg-closed set [3] if $clA \leq U$ whenever $A \leq U \in \tau$,
(ii) fgs-closed set [5] if $sclA \leq U$ whenever $A \leq U \in \tau$,
(iii) fsg-closed set [5] if $sclA \leq U$ whenever $A \leq U \in FSO(X)$,
(iv) fgp-closed set [5] if $pclA \leq U$ whenever $A \leq U \in \tau$,
(v) fpg-closed set [5] if $pclA \leq U$ whenever $A \leq U \in FPO(X)$,
(vi) $fgpr$-closed set [5] if $pclA \leq U$ whenever $A \leq U \in FRO(X)$,
(vii) $fg\alpha$-closed set [5] if $\alpha clA \leq U$ whenever $A \leq U \in \tau$,
(viii) $f\alpha g$-closed set [5] if $\alpha clA \leq U$ whenever $A \leq U \in F\alpha O(X)$,
(ix) $fg\beta$-closed set [5] if $\beta clA \leq U$ whenever $A \leq U \in \tau$,
(x) $fg\delta_p$-closed set [12] if $\delta pclA \leq U$ whenever $A \leq U \in \tau$,
(xi) $f\delta_p g$-closed set [13] if $\delta pclA \leq U$ whenever $A \leq U \in F\delta PO(X)$,
(xii) $fg\delta$-semiclosed set [14] if $\delta sclA \leq U$ whenever $A \leq U \in \tau$,
(xiii) fgs^*-closed set [9] if $sclA \leq U$ whenever $A \leq U$ where U is fg-open set in X, where the complement of an fg-closed set is called an fg-open set.

Remark 3.21. It is clear from definitions that every $f\breve{g}$-closed set is fg-closed, fgs-closed, fsg-closed, fgp-closed, $fg\alpha$-closed, $f\alpha g$-closed, $fgpr$-closed, $fg\beta$-closed, $fg\delta_p$-closed and $f\delta_p g$-closed. But the converses are not true, in general, as the following examples show.

Example 3.22. fgp-closed set ($fg\beta$-closed set, $fgpr$-closed set, $fg\alpha$-closed set, $f\alpha g$-closed set) $\not\Rightarrow f\breve{g}$-closed set
Let $X = \{a, b\}$, $\tau = \{0_X, 1_X, A\}$ where $A(a) = 0.4, A(b) = 0.7$. Then (X, τ) is an fts. Here $FSO(X) = \{0_X, 1_X, U\}$ where $U \geq A$. Consider the fuzzy set B, defined by $B(a) = B(b) = 0.4$. Then $B \leq A \in FSO(X)$. But $clB = 1_X \not\leq A \Rightarrow B$ is not $f\breve{g}$-closed set in X.
Now $FPC(X) = \{0_X, 1_X, V\}$ where $V \not\geq A$. Now $B \leq A \in \tau$ and $pclB = B \leq A \Rightarrow B$ is fgp-closed set in X.
Again, $B \in F\beta C(X)$ (as $int(cl(intB)) = 0_X < B$) and so $\beta clB = B \leq A \Rightarrow B$ is $fg\beta$-closed set in X.
Now $1_X \in FRO(X)$ only containing B and so $pclB \leq 1_X \Rightarrow B$ is $fgpr$-closed set in X.
Also as $\delta intB = 0_X$, $cl(\delta intB) = 0_X < B$, B is fuzzy δ-preclosed set in X and as a result, B is $fg\delta_p$-closed set in X.
Now consider the fuzzy set C defined by $C(a) = 0.4, C(b) = 0.3$. Then $cl(int(clC)) = cl(int(1_X \setminus A)) = 0_X < C \Rightarrow C$ is fuzzy α-closed set in X and so C is $fg\alpha$-closed set in X. But $C < A \in FSO(X)$ and $clC = 1_X \setminus A \not\leq A \Rightarrow C$ is not $f\breve{g}$-closed set in X.
Now $F\alpha O(X) = \{0_X, 1_X, U\}$ where $U \geq A$ and so $F\alpha C(X) = \{0_X, 1_X, 1_X \setminus U\}$ where $1_X \setminus U \leq 1_X \setminus A$. Now $C \leq A \in F\alpha O(X)$ and $\alpha clC = C \leq A \Rightarrow C$ is $f\alpha g$-closed set in X.

Example 3.23. fgs-closed set, fsg-closed set, fg-closed set $\not\Rightarrow f\breve{g}$-closed set
Let $X = \{a, b\}$, $\tau = \{0_X, 1_X, A\}$ where $A(a) = 0.5, A(b) = 0.4$. Then (X, τ) is an fts. Now $FSO(X) = FSC(X) = \{0_X, 1_X, U\}$ where $A \leq U \leq 1_X \setminus A$. Consider the fuzzy set B defined by $B(a) = B(b) = 0.5$. Then 1_X is the only fuzzy open set in X containing B and so $clB \leq 1_X \Rightarrow B$ is fg-closed set in X. Since $sclB \leq clB$, $sclB \leq 1_X \Rightarrow B$ is fgs-closed set in X. Now $B \in FSO(X) \Rightarrow B \leq B$ and so $sclB = B \leq B \Rightarrow B$ is fsg-closed set in X. But $clB = 1_X \setminus A \not\leq B \Rightarrow B$ is not $f\breve{g}$-closed set in X.

Remark 3.24. (i) $f\breve{g}$-closedness and fpg-closedness are independent concepts,
(ii) $f\breve{g}$-closedness and $fg\delta$-semiclosedness are independent concepts,
(iii) $f\breve{g}$-closedness and fuzzy semiclosedness are independent concepts.

The following examples clarify Remark 3.24.

Example 3.25 $f\breve{g}$-closed set $\not\Rightarrow$ fpg-closed set
Consider Example 3.23 and the fuzzy set C defined by $C(a) = 0.6, C(b) = 0.5$. Then 1_X is the only fuzzy $M \in FSO(X)$ such that $C < M$ and so $clC \leq 1_X$, hence C is an $f\breve{g}$-closed set in X. Now as $int(clC) = 1_X \geq C \Rightarrow C \in FPO(X)$ and so $C \leq C$. But $cl(intC) = clA = 1_X \setminus A \not\leq C \Rightarrow C \notin FPC(X) \Rightarrow pclC \not\leq C \Rightarrow C$ is not an fpg-closed set in X.

Example 3.26. fpg-closed set $\not\Rightarrow$ $f\breve{g}$-closed set
Consider Example 3.22. Here $FPO(X) = \{0_X, 1_X, 1_X \setminus V\}$ where $1_X \setminus V \not\leq 1_X \setminus A$. So $B \in FPO(X)$ with $B \leq B$. Also as $B \not\geq A$, $B \in FPC(X)$. So $pclB = B \leq B \Rightarrow B$ is an fpg-closed set in X. But B is not an $f\breve{g}$-closed set in X.

Example 3.27. $fg\delta$-semiclosed set $\not\Rightarrow$ $f\breve{g}$-closed set
Consider Example 3.23. Consider the fuzzy set D defined by $D(a) = D(b) = 0.5$. Then 1_X is the only fuzzy open set in X containing D and so D is $fg\delta$-semiclosed set in X. Now $D \in FSO(X)$ and so $D \leq D$. But $clD = 1_X \setminus A \not\leq D \Rightarrow D$ is not an $f\breve{g}$-closed set in X.

Example 3.28. $f\breve{g}$-closed set $\not\Rightarrow$ $fg\delta$-semiclosed set
Let $X = \{a, b\}$, $\tau = \{0_X, 1_X, A\}$ where $A(a) = 0.5, A(b) = 0.6$. Then (X, τ) is an fts. Consider the fuzzy set B defined by $B(a) = 0.5, B(b) = 0.4$. Then as $B = 1_X \setminus A \in \tau^c$, B is an $f\breve{g}$-closed set in X. Now $B \leq A \in \tau$. Now $\delta sclB = 1_X \not\leq A \Rightarrow B$ is not an $fg\delta$-semiclosed set in X.

Example 3.29. $f\breve{g}$-closed set $\not\Rightarrow$ fuzzy semiclosed set
Let $X = \{a, b\}$, $\tau = \{0_X, 1_X, A\}$ where $A(a) = A(b) = 0.4$. Then (X, τ) is an fts. Here $FSO(X) = FSC(X) = \{0_X, 1_X, U\}$ where $A \leq U \leq 1_X \setminus A$. Consider the fuzzy set B, defined by $B(a) = 0.6, B(b) = 0.7$. Then 1_X is the only fuzzy semiopen set in X containing B and so $clB \leq 1_X \Rightarrow B$ is $f\breve{g}$-closed set in X. But $B \notin FSC(X)$.

Example 3.30. Fuzzy semiclosed set $\not\Rightarrow$ $f\breve{g}$-closed set
Consider Example 3.23. Here B is not an $f\breve{g}$-closed set. But $B \in FSC(X)$.

Definition 3.31. A fuzzy set A is called an $f\breve{g}$ neighbourhood ($f\breve{g}$-nbd, for short) of a fuzzy point (resp., fuzzy set) x_α (resp., B) in an fts (X, τ) if there exists an $f\breve{g}$-open set U in X such that $x_\alpha \leq U \leq A$ (resp., $B \leq U \leq A$).

Remark 3.32. An $f\breve{g}$-nbd of a fuzzy point in an fts need not be $f\breve{g}$-open set follows from the following example.

Example 3.33. Consider Example 3.23 and the fuzzy point $a_{0.5}$. Here the fuzzy set B is not an $f\breve{g}$-open set in X. But $a_{0.5} \in A \leq B$ where A being a fuzzy open set in X is $f\breve{g}$-open in $X \Rightarrow B$ is an $f\breve{g}$-nbd of $a_{0.5}$.

Note 3.34. Every fuzzy nbd of a fuzzy point is an $f\breve{g}$-nbd of it. But the converse may not be true, as it seen from the following example.

Example 3.35. Consider Example 3.7 and the fuzzy point $a_{0.6}$. Let D be a fuzzy set defined by $D(a) = 0.6, D(b) = 0.3$. Now $(1_X \setminus D)(a) = 0.4, (1_X \setminus D)(b) = 0.7$. Now 1_X is the only fuzzy semiopen set in (X, τ) containing $1_X \setminus D$ and so clearly $1_X \setminus D$ is $f\breve{g}$-closed set in (X, τ). Consequently, D is an $f\breve{g}$-open set in (X, τ). So D is an $f\breve{g}$-nbd of $a_{0.6}$, but D is not a fuzzy nbd of $a_{0.6}$.

Remark 3.36. An $f\breve{g}$-open set is an $f\breve{g}$-nbd of each of its points.

Theorem 3.37. Let (X, τ) be an fts and x_t be a fuzzy point in X. Let $F(\in I^X)$ be an $f\breve{g}$-closed set in X with $x_t \in 1_X \setminus F$. Then there exists an $f\breve{g}$-nbd G of x_t such that $G \not{q} F$.

Proof. Now $x_t \in 1_X \setminus F$ where F is an $f\breve{g}$-closed set is $X \Rightarrow 1_X \setminus F$ being an $f\breve{g}$-open set in an $f\breve{g}$-nbd of x_t. So by definition, there exists an $f\breve{g}$-open set G in X such that $x_t \in G \leq 1_X \setminus F \Rightarrow G \not{q} F$ where G is an $f\breve{g}$-nbd of x_t.

Definition 3.38. The set of all $f\breve{g}$-nbds of a fuzzy point x_t $(0 < t \leq 1)$ in an fts (X, τ) is called the $f\breve{g}$-nbd system at x_t, denoted by $f\breve{g}$-$N(x_t)$.

Theorem 3.39. For a fuzzy point x_t in an fts (X, τ), the following statements hold :

(i) $f\breve{g}$-$N(x_t) \neq \phi$,
(ii) $G \in f\breve{g}$-$N(x_t) \Rightarrow x_t \in G$,
(iii) $G \in f\breve{g}$-$N(x_t)$ and $F \geq G \Rightarrow F \in f\breve{g}$-$N(x_t)$,
(iv) $F, G \in f\breve{g}$-$N(x_t) \Rightarrow F \bigwedge G \in f\breve{g}$-$N(x_t)$,
(v) $G \in f\breve{g}$-$N(x_t) \Rightarrow$ there exists $F \in f\breve{g}$-$N(x_t)$ such that $F \leq G$ and $F \in f\breve{g}$-$N(y_{t'})$ for every $y_{t'} \in F$.

Proof. (i) Since 1_X being an $f\breve{g}$-open set is an $f\breve{g}$-nbd of x_t $(0 < t \leq 1)$, $f\breve{g}$-$N(x_t) \neq \phi$.
(ii) and (iii) are obvious.
(iv) Since every intersection of two $f\breve{g}$-open sets is $f\breve{g}$-open, (iv) is obvious.
(v) Follows from Remark 3.36 and Definition 3.38.

Theorem 3.40. Let x_t be a fuzzy point in an fts (X, τ). Let $f\breve{g}$-$N(x_t)$ be a non-empty collection of fuzzy sets in X satisfying the following conditions :
(1) $G \in f\breve{g}$-$N(x_t) \Rightarrow x_t \in G$,
(2) $F, G \in f\breve{g}$-$N(x_t) \Rightarrow F \bigwedge G \in f\breve{g}$-$N(x_t)$.
Let τ consist of 0_X and all those non-empty fuzzy sets G of X having the property that $x_t \in G \Rightarrow$ there exists an $F \in f\breve{g}$-$N(x_t)$ such that $x_t \in F \leq G$. Then τ is a fuzzy topology on X.

Proof. (i) By hypothesis, $0_X \in \tau$.
(ii) It is clear from the given property of τ that $1_X \in \tau$ as $1_X \in f\breve{g}$-$N(x_t)$ for any fuzzy point x_t $(0 < t \leq 1)$ in an fts X (by (1)).
(iii) Let $G_1, G_2 \in \tau$. If $G_1 \bigwedge G_2 = 0_X$, then by construction of τ, $G_1 \bigwedge G_2 \in \tau$. Suppose $G_1 \bigwedge G_2 \neq 0_X$. Let $x_t \in G_1 \bigwedge G_2$ where $0 < t \leq 1$. Then $G_1(x) \geq t, G_2(x) \geq t$. Since $G_1, G_2 \in \tau$, by definition of τ, there exist $F_1, F_2 \in f\breve{g}$-$N(x_t)$ such that $x_t \in F_1 \leq G_1$, $x_t \in F_2 \leq G_2$. Then $x_t \in F_1 \bigwedge F_2 \leq G_1 \bigwedge G_2$. By (2), $F_1 \bigwedge F_2 \in f\breve{g}$-$N(x_t)$ and so $G_1 \bigwedge G_2 \in \tau$ by construction of τ.
(iv) Let $\mathcal{G} = \{G_\alpha : \alpha \in \Lambda\}$ where $G_\alpha \in \tau$, for each $\alpha \in \Lambda$. Let $x_t \in \bigvee_{\alpha \in \Lambda} G_\alpha$. Then there exists $\beta \in \Lambda$ such that $x_t \in G_\beta$. By definition of τ, there exists $F_\beta \in f\breve{g}$-$N(x_t)$ such that $x_t \in F_\beta \leq G_\beta \leq \bigvee_{\alpha \in \Lambda} G_\alpha \Rightarrow \bigvee_{\alpha \in \Lambda} G_\alpha \in \tau$.

It follows that τ is a fuzzy topology on X.

4. $f\breve{g}$-Closure Operator and $f\breve{g}$-Open and $f\breve{g}$-Closed Functions

In this section we first introduce the notion of $f\breve{g}$-closure operator, which is an idempotent operator. Using this concept as a basic tool, we introduce and characterize $f\breve{g}$-open and $f\breve{g}$-closed functions.

Definition 4.1. Let (X, τ) be an fts and $A \in I^X$. Then $f\breve{g}$-closure and $f\breve{g}$-interior of A, denoted by $f\breve{g}cl(A)$ and $f\breve{g}int(A)$, are defined as follow:
$f\breve{g}cl(A) = \bigwedge\{F : A \leq F, F \text{ is } f\breve{g}\text{-closed set in } X\}$,
$f\breve{g}int(A) = \bigvee\{G : G \leq A, G \text{ is } f\breve{g}\text{-open set in } X\}$.

Remark 4.2. It is obvious that for any $A \in I^X$, $A \leq f\breve{g}cl(A) \leq clA$. If A is $f\breve{g}$-closed set in an fts X, then $A = f\breve{g}cl(A)$. Similarly, $intA \leq f\breve{g}int(A) \leq A$. If A is $f\breve{g}$-open set in an fts X, then $A = f\breve{g}int(A)$. It follows from Example 3.7 that $f\breve{g}cl(A)$ may not be an $f\breve{g}$-closed set in an fts X. Similarly, $f\breve{g}int(A)$ may not by an $f\breve{g}$-open set in an fts X.

Result 4.3. Let (X, τ) be an fts and $A \in I^X$. Then for a fuzzy point x_t in X, $x_t \in f\breve{g}cl(A)$ if and only if every $f\breve{g}$-open q-nbd U of x_t, UqA.

Proof. Let $x_t \in f\breve{g}cl(A)$ for any fuzzy set A in an fts X and F be any $f\breve{g}$-open q-nbd of x_t. Then $x_tqF \Rightarrow x_t \notin 1_X \setminus F$ which is $f\breve{g}$-closed set in X. Then by Definition 4.1, $A \not\leq 1_X \setminus F \Rightarrow$ there exists $y \in X$ such that $A(y) > 1 - F(y) \Rightarrow AqF$.

Conversely, let for every $f\breve{g}$-open q-nbd F of x_t, FqA. Assume that $x_t \notin f\breve{g}cl(A)$. Then by Definition 4.1, there exists an $f\breve{g}$-closed set U in X with $A \leq U$, $x_t \notin U$. Then $x_tq(1_X \setminus U)$ which being an $f\breve{g}$-open set in X is an $f\breve{g}$-open q-nbd of x_t. By assumption, $(1_X \setminus U)qA \Rightarrow (1_X \setminus A)qA$, a contradiction.

Theorem 4.4. Let (X, τ) be an fts and $A, B \in I^X$. Then the following statements are true:
(i) $f\breve{g}cl(0_X) = 0_X$,
(ii) $f\breve{g}cl(1_X) = 1_X$,
(iii) $A \leq B \Rightarrow f\breve{g}cl(A) \leq f\breve{g}cl(B)$,
(iv) $f\breve{g}cl(A \bigvee B) = f\breve{g}cl(A) \bigvee f\breve{g}cl(B)$,
(v) $f\breve{g}cl(A \wedge B) \leq f\breve{g}cl(A) \wedge f\breve{g}cl(B)$, equality does not hold, in general, follows from Example 3.7,

(vi) $f\breve{g}cl(f\breve{g}cl(A)) = f\breve{g}cl(A)$.

Proof. (i), (ii) and (iii) are obvious.

(iv) From (iii), $f\breve{g}cl(A) \bigvee f\breve{g}cl(B) \leq f\breve{g}cl(A \bigvee B)$.

To prove the converse, let $x_\alpha \in f\breve{g}cl(A \bigvee B)$. Then by Result 4.3, for any $f\breve{g}$-open set U in X with $x_\alpha qU$, $Uq(A \bigvee B) \Rightarrow$ there exists $y \in X$ such that $U(y) + max\{A(y), B(y)\} > 1 \Rightarrow$ either $U(y) + A(y) > 1$ or $U(y) + B(y) > 1 \Rightarrow$ either UqA or $UqB \Rightarrow$ either $x_\alpha \in f\breve{g}cl(A)$ or $x_\alpha \in f\breve{g}cl(B) \Rightarrow x_\alpha \in f\breve{g}cl(A) \bigvee f\breve{g}cl(B)$.

(v) Follows from (iii).

(vi) As $A \leq f\breve{g}cl(A)$, for any $A \in I^X$, $f\breve{g}cl(A) \leq f\breve{g}cl(f\breve{g}cl(A))$ (by (iii)).

Conversely, let $x_\alpha \in f\breve{g}cl(f\breve{g}cl(A)) = f\breve{g}cl(B)$ where $B = f\breve{g}cl(A)$. Let U be any $f\breve{g}$-open set in X with $x_\alpha qU$. Then UqB implies that there exists $y \in X$ such that $U(y) + B(y) > 1$. Let $B(y) = t$. Then $y_t qU$ and $y_t \in B = f\breve{g}cl(A) \Rightarrow UqA \Rightarrow x_\alpha \in f\breve{g}cl(A) \Rightarrow f\breve{g}cl(f\breve{g}cl(A)) \leq f\breve{g}cl(A)$. Consequently, $f\breve{g}cl(f\breve{g}cl(A)) = f\breve{g}cl(A)$.

Theorem 4.5. Let (X, τ) be an fts and $A \in I^X$. Then the following statements hold:

(i) $f\breve{g}cl(1_X \setminus A) = 1_X \setminus f\breve{g}int(A)$

(ii) $f\breve{g}int(1_X \setminus A) = 1_X \setminus f\breve{g}cl(A)$.

Proof (i). Let $x_t \in f\breve{g}cl(1_X \setminus A)$ for a fuzzy set A in an fts (X, τ). If possible, let $x_t \notin 1_X \setminus f\breve{g}int(A)$. Then $1 - (f\breve{g}int(A))(x) < t \Rightarrow [f\breve{g}int(A)](x) + t > 1 \Rightarrow f\breve{g}int(A)qx_t \Rightarrow$ there exists at least one $f\breve{g}$-open set $F \leq A$ with $x_t qF \Rightarrow x_t qA$. As $x_t \in f\breve{g}cl(1_X \setminus A), Fq(1_X \setminus A) \Rightarrow Aq(1_X \setminus A)$, a contradiction. Hence

$$f\breve{g}cl(1_X \setminus A) \leq 1_X \setminus f\breve{g}int(A)...(1)$$

Conversely, let $x_t \in 1_X \setminus f\breve{g}int(A)$. Then $1 - [(f\breve{g}int(A)](x) \geq t \Rightarrow x_t \not{q}(f\breve{g}int(A)) \Rightarrow x_t \not{q}F$ for every $f\breve{g}$-open set F contained in A ... (2).

Let U be any $f\breve{g}$-closed set in X such that $1_X \setminus A \leq U$. Then $1_X \setminus U \leq A$. Now $1_X \setminus U$ is $f\breve{g}$-open set in X contained in A. By (2), $x_t \not{q}(1_X \setminus U) \Rightarrow x_t \in U \Rightarrow x_t \in f\breve{g}cl(1_X \setminus A)$ and so

$$1_X \setminus f\breve{g}int(A) \leq f\breve{g}cl(1_X \setminus A)...(3).$$

Combining (1) and (3), (i) follows.

(ii) Putting $1_X \setminus A$ for A in (i), we get $f\breve{g}cl(A) = 1_X \setminus f\breve{g}int(1_X \setminus A) \Rightarrow f\breve{g}int(1_X \setminus A) = 1_X \setminus f\breve{g}cl(A)$.

Let us now recall the following definition from [34] for ready references.

Definition 4.6 [34]. A function $f : X \to Y$ is called fuzzy open (resp., fuzzy closed) if $f(U)$ is fuzzy open (resp., fuzzy closed) set in Y for every fuzzy open (resp., fuzzy closed) set in X.

Let us now introduce the following concept.

Definition 4.7. A function $h : X \to Y$ is called $f\breve{g}$-open function if $f(U)$ is $f\breve{g}$-open set in Y for every fuzzy open set U in X.

Remark 4.8. It is clear that every fuzzy open function is an $f\breve{g}$-open function. But the converse need not be true, as it seen from the following example.

Example 4.9. $f\breve{g}$-open function $\not\Rightarrow$ fuzzy open function
Let $X = \{a, b\}$, $\tau_1 = \{0_X, 1_X, A\}$, $\tau_2 = \{0_X, 1_X\}$ where $A(a) = A(b) = 0.5$. Then (X, τ_1) and (X, τ_2) are fts's. Consider the identity function $i : (X, \tau_1) \to (X, \tau_2)$. Since every fuzzy set in (X, τ_2) is an $f\breve{g}$-open set in (X, τ_2), clearly i is $f\breve{g}$-open function. But $A \in \tau_1$, $i(A) = A \notin \tau_2 \Rightarrow i$ is not a fuzzy open function.

Theorem 4.10. For a bijective function $h : X \to Y$, the following statements are equivalent:
(i) h is $f\breve{g}$-open,
(ii) $h(intA) \leq f\breve{g}int(h(A))$, for all $A \in I^X$,
(iii) for each fuzzy point x_α in X and each fuzzy open set U in X containing x_α, there exists an $f\breve{g}$-open set V containing $h(x_\alpha)$ such that $V \leq h(U)$.

Proof (i) $\Rightarrow$ (ii). Let $A \in I^X$. Then $intA$ is fuzzy open in X. By (i), $h(intA)$ is $f\breve{g}$-open set in Y. Since $h(intA) \leq h(A)$ and $f\breve{g}int(h(A))$ is the union of all $f\breve{g}$-open sets contained in $h(A)$, we have $h(intA) \leq f\breve{g}int(h(A))$.
(ii) $\Rightarrow$ (i). Let U be any fuzzy open set in X. Then $h(U) = h(intU) \leq f\breve{g}int(h(U))$ (by (ii)) $\Rightarrow h(U)$ is $f\breve{g}$-open set in $Y \Rightarrow h$ is $f\breve{g}$-open function.
(ii) $\Rightarrow$ (iii). Let x_α be a fuzzy point in X, and U, a fuzzy open set in X such that $x_\alpha \in U$. Then $h(x_\alpha) \in h(U) = h(intU) \leq f\breve{g}int(h(U))$ (by (ii)). Then $h(U)$ is $f\breve{g}$-open set in Y. Let $V = h(U)$. Then

$h(x_\alpha) \in V$ and $V \leq h(U)$.
(iii) $\Rightarrow$ (i). Let U be an arbitrary fuzzy open set in X and y_α any fuzzy point in $h(U)$, i.e., $y_\alpha \in h(U)$. Then there exists a unique $x \in X$ such that $h(x) = y$ (as h is bijective). Then $[h(U)](y) \geq \alpha \Rightarrow U(h^{-1}(y)) \geq \alpha \Rightarrow U(x) \geq \alpha \Rightarrow x_\alpha \in U$. By (iii), there exists an $f\breve{g}$-open set V in Y such that $h(x_\alpha) \in V$ and $V \leq h(U)$. Then $h(x_\alpha) \in V = f\breve{g}int(V) \leq f\breve{g}int(h(U))$. Since y_α is taken arbitrarily and $h(U)$ is the union of all fuzzy points in $h(U)$, $h(U) \leq f\breve{g}int(f(U)) \Rightarrow h(U)$ is $f\breve{g}$-open set in $Y \Rightarrow h$ is an $f\breve{g}$-open function.

Theorem 4.11. If $h : X \to Y$ is $f\breve{g}$-open, bijective function, then the following statements are true:
(i) for each fuzzy point x_α in X and each fuzzy open q-nbd U of x_α, there exists an $f\breve{g}$-open q-nbd V of $h(x_\alpha)$ in Y such that $V \leq h(U)$,
(ii) $h^{-1}(f\breve{g}cl(B)) \leq cl(h^{-1}(B))$, for all $B \in I^Y$.
Proof (i). Let x_α be a fuzzy point in X and U be any fuzzy open q-nbd of x_α in X. Then $x_\alpha q U = intU \Rightarrow h(x_\alpha) q h(intU) \leq f\breve{g}int(h(U))$ (by Theorem 4.10 (i)$\Rightarrow$(ii)) implies that there exists at least one $f\breve{g}$-open q-nbd V of $h(x_\alpha)$ in Y with $V \leq h(U)$..
(ii) Let x_α be any fuzzy point in X such that $x_\alpha \notin cl(h^{-1}(B))$ for any $B \in I^Y$. Then there exists a fuzzy open q-nbd U of x_α in X such that $U \not{q} h^{-1}(B)$. Now

$$h(x_\alpha) q h(U) ...(1)$$

where $h(U)$ is $f\breve{g}$-open set in Y. Now $h^{-1}(B) \leq 1_X \setminus U$ which is a fuzzy closed set in $X \Rightarrow B \leq h(1_X \setminus U)$ (as h is injective) and $h(1_X \setminus U) \leq 1_Y \setminus h(U) \Rightarrow B \not{q} h(U)$. Let $V = 1_Y \setminus h(U)$. Then $B \leq V$ which is $f\breve{g}$-closed set in Y. We claim that $h(x_\alpha) \notin V$. If possible, let $h(x_\alpha) \in V = 1_Y \setminus h(U)$. Then $1 - [h(U)](h(x)) \geq \alpha \Rightarrow h(U) \not{q} h(x_\alpha)$, contradicting (1). So $h(x_\alpha) \notin V \Rightarrow h(x_\alpha) \notin f\breve{g}cl(B) \Rightarrow x_\alpha \notin h^{-1}(f\breve{g}cl(B)) \Rightarrow h^{-1}(f\breve{g}cl(B)) \leq cl(h^{-1}(B))$.

Theorem 4.12. An injective function $h : X \to Y$ is $f\breve{g}$-open if and only if for each $B \in I^Y$ and F, a fuzzy closed set in X with $h^{-1}(B) \leq F$, there exists an $f\breve{g}$-closed set V in Y such that $B \leq V$ and $h^{-1}(V) \leq F$.
Proof. Let $B \in I^Y$ and F, a fuzzy closed set in X with $h^{-1}(B) \leq F$. Then $1_X \setminus h^{-1}(B) \geq 1_X \setminus F$ where $1_X \setminus F$ is a fuzzy open set in $X \Rightarrow h(1_X \setminus F) \leq h(1_X \setminus h^{-1}(B)) \leq 1_Y \setminus B$ (as h is injective) where

$h(1_X \setminus F)$ is an $f\breve{g}$-open set in Y. Let $V = 1_Y \setminus h(1_X \setminus F)$. Then V is $f\breve{g}$-closed set in Y such that $B \leq V$. Now $h^{-1}(V) = h^{-1}(1_Y \setminus h(1_X \setminus F)) = 1_X \setminus h^{-1}(h(1_X \setminus F)) \leq F$.

Conversely, let F be a fuzzy open set in X. Then $1_X \setminus F$ is a fuzzy closed set in X. We have to show that $h(F)$ is an $f\breve{g}$-open set in Y. Now $h^{-1}(1_Y \setminus h(F)) \leq 1_X \setminus F$. By assumption, there exists an $f\breve{g}$-closed set V in Y such that

$$1_Y \setminus h(F) \leq V ...(1)$$

and $h^{-1}(V) \leq 1_X \setminus F$. Therefore, $F \leq 1_X \setminus h^{-1}(V)$ implies that

$$h(F) \leq h(1_X \setminus h^{-1}(V)) \leq 1_Y \setminus V ...(2)$$

(as h is injective). Combining (1) and (2), $h(F) = 1_Y \setminus V$ which is an $f\breve{g}$-open set in Y.

Definition 4.13. A function $h : X \to Y$ is called an $f\breve{g}$-closed function if $h(A)$ is $f\breve{g}$-closed set in Y for each fuzzy closed set A in X.

Remark 4.14. It is obvious that every fuzzy closed function is $f\breve{g}$-closed function, but the converse may not be true as it follows from Example 4.9. Here every fuzzy set in (X, τ_2) is $f\breve{g}$-closed set in (X, τ_2) and so clearly i is an $f\breve{g}$-closed function. But $1_X \setminus A \in \tau_1^c$, $i(1_X \setminus A) = 1_X \setminus A \notin \tau_2^c \Rightarrow i$ is not a fuzzy closed function.

Theorem 4.15. A bijective function $h : X \to Y$ is $f\breve{g}$-closed if and only if $f\breve{g}cl(h(A)) \leq h(clA)$, for all $A \in I^X$.

Proof. Let us suppose that $h : X \to Y$ is an $f\breve{g}$-closed function and $A \in I^X$. Then $h(cl(A))$ is $f\breve{g}$-closed set in Y. Since $h(A) \leq h(clA)$ and $f\breve{g}cl(h(A))$ is the intersection of all $f\breve{g}$-closed sets in Y containing $h(A)$, we have $f\breve{g}cl(h(A)) \leq h(clA)$.

Conversely, let for any $A \in I^X$, $f\breve{g}cl(h(A)) \leq h(clA)$. Let U be any fuzzy closed set in X. Then $h(U) = h(clU) \geq f\breve{g}cl(h(U)) \Rightarrow h(U)$ is an $f\breve{g}$-closed set in $Y \Rightarrow h$ is an $f\breve{g}$-closed function.

Theorem 4.16. If $h : X \to Y$ is an $f\breve{g}$-closed bijective function, then the following statements hold:

(i) for each fuzzy point x_α in X and each fuzzy closed set U in X with $x_\alpha \not{q} U$, there exists an $f\breve{g}$-closed set V in Y with $h(x_\alpha) \not{q} V$ such that $V \geq h(U)$,

(ii) $h^{-1}(f\breve{g}int(B)) \geq int(h^{-1}(B))$, for all $B \in I^Y$.

Proof (i). Let x_α be a fuzzy point in X and U be any fuzzy closed set in X with $x_\alpha \not{q} U = clU \Rightarrow h(x_\alpha) \not{q} h(clU) \geq f\breve{g}cl(h(U))$ (by Theorem 4.15) $\Rightarrow h(x_\alpha) \not{q} V$ for some $f\breve{g}$-closed set V in Y with $V \geq h(U)$.

(ii). Let $B \in I^Y$ and x_α be any fuzzy point in X such that $x_\alpha \in int(h^{-1}(B))$. Then there exists a fuzzy open set U in X with $U \leq h^{-1}(B)$ such that $x_\alpha \in U$. Then $1_X \setminus U \geq 1_X \setminus h^{-1}(B) \Rightarrow h(1_X \setminus U) \geq h(1_X \setminus h^{-1}(B))$ where $h(1_X \setminus U)$ is an $f\breve{g}$-closed set in Y. Let $V = 1_Y \setminus h(1_X \setminus U)$. Then V is an $f\breve{g}$-open set in Y and $V = 1_Y \setminus h(1_X \setminus U) \leq 1_Y \setminus h(1_X \setminus h^{-1}(B)) \leq 1_Y \setminus (1_Y \setminus B) = B$ (as h is injective). Now $U(x) \geq \alpha \Rightarrow x_\alpha \not{q} (1_X \setminus U) \Rightarrow h(x_\alpha) \not{q} h(1_X \setminus U)$ $\Rightarrow h(x_\alpha) \leq 1_Y \setminus h(1_X \setminus U) = V \Rightarrow h(x_\alpha) \in V = f\breve{g}int(V) \leq f\breve{g}int(B) \Rightarrow x_\alpha \in h^{-1}(f\breve{g}int(B))$. Since x_α is taken arbitrarily, $int(h^{-1}(B)) \leq h^{-1}(f\breve{g}int(B))$, for all $B \in I^Y$.

Remark 4.17. A composition of two $f\breve{g}$-closed (resp., $f\breve{g}$-open) functions need not be so, as it is seen from the following example.

Example 4.18. Let $X = \{a, b\}$, $\tau_1 = \{0_X, 1_X, A\}$, $\tau_2 = \{0_X, 1_X\}$, $\tau_3 = \{0_X, 1_X, B\}$ where $A(a) = A(b) = 0.5$, $B(a) = 0.5, B(b) = 0.4$. Then (X, τ_1), (X, τ_2) and (X, τ_3) are fts's. Consider two identity functions $i_1 : (X, \tau_1) \to (X, \tau_2)$ and $i_2 : (X, \tau_2) \to (X, \tau_3)$. Clearly i_1 and i_2 are $f\breve{g}$-open and $f\breve{g}$-closed functions. Let $i_3 = i_2 \circ i_1 : (X, \tau_1) \to (X, \tau_3)$. Now $A = 1_X \setminus A$. So $A \in \tau_1$ as well as $A \in \tau_1^c$. Then $i_3(A) = A \leq A \in FSO(X, \tau_3)$ and $cl_{\tau_3}(A) = 1_X \setminus B \not\leq A \Rightarrow A$ is not an $f\breve{g}$-open and A is not an $f\breve{g}$-closed set in $(X, \tau_3) \Rightarrow i_3$ is not nor an $f\breve{g}$-open, neither an $f\breve{g}$-closed function.

Theorem 4.19. If $h_1 : X \to Y$ is a fuzzy closed (resp., fuzzy open) function and $h_2 : Y \to Z$ is an $f\breve{g}$-closed (resp., $f\breve{g}$-open) function, then $h_2 \circ h_1 : X \to Z$ is $f\breve{g}$-closed (resp., $f\breve{g}$-open) function.

5. $f\breve{g}$-Regular, $f\breve{g}$-Normal and $f\breve{g}$-Compact Spaces

In this section two new types of separation axioms are introduced and studied. Afterwards, a new type of compact space is introduced.

Definition 5.1. An fts (X, τ) is said to be an $f\breve{g}$-regular space if for any fuzzy point x_t in X and each $f\breve{g}$-closed set F in X with $x_t \notin F$, there exist $U, V \in FSO(X)$ such that $x_t \in U$, $F \leq V$ and

$U \not q V$.

Theorem 5.2. In an fts (X, τ), the following statements are equivalent:
(i) X is $f\breve{g}$-regular,
(ii) for each fuzzy point x_t in X and any $f\breve{g}$-open q-nbd U of x_t, there exists $V \in FSO(X)$ such that $x_t \in V$ and $sclV \leq U$,
(iii) for each fuzzy point x_t in X and each $f\breve{g}$-closed set A of X with $x_t \notin A$, there exists $U \in FSO(X)$ with $x_t \in U$ such that $sclU \not q A$.

Proof (i) $\Rightarrow$ (ii). Let x_t be a fuzzy point in X and U, any $f\breve{g}$-open q-nbd of x_t. Then $x_t q U \Rightarrow U(x) + t > 1 \Rightarrow x_t \notin 1_X \setminus U$ which is an $f\breve{g}$-closed set in X. By (i), there exist $V, W \in FSO(X)$ such that $x_t \in V, 1_X \setminus U \leq W$ and $V \not q W$. Then $V \leq 1_X \setminus W \Rightarrow sclV \leq scl(1_X \setminus W) = 1_X \setminus W \leq U$.
(ii) $\Rightarrow$ (iii). Let x_t be a fuzzy point in X and A an $f\breve{g}$-closed set in X with $x_t \notin A$. Then $A(x) < t \Rightarrow x_t q(1_X \setminus A)$, where $(1_X \setminus A)$, being an $f\breve{g}$-open set in X, is $f\breve{g}$-open q-nbd of x_α. So by (ii), there exists $V \in FSO(X)$ such that $x_t \in V$ and $sclV \leq 1_X \setminus A$. Then $sclV \not q A$.
(iii) $\Rightarrow$ (i). Let x_t be a fuzzy point in X and F be any $f\breve{g}$-closed set in X with $x_t \notin F$. Then by (iii), there exists $U \in FSO(X)$ such that $x_t \in U$ and $sclU \not q F$. Then $F \leq 1_X \setminus sclU$ $(=: V)$. So $V \in FSO(X)$ and $V \not q U$ as $U \not q (1_X \setminus sclU)$. Consequently, X is an $f\breve{g}$-regular space.

Definition 5.3. An fts (X, τ) is called $f\breve{g}$-normal space if for each pair of $f\breve{g}$-closed sets A, B in X with $A \ /qB$, there exist $U, V \in FSO(X)$ such that $A \leq U, B \leq V$ and $U \ \not q V$.

Theorem 5.4. An fts (X, τ) is an $f\breve{g}$-normal space if and only if for every $f\breve{g}$-closed set F and $f\breve{g}$-open set G with $F \leq G$, there exists $H \in FSO(X)$ such that $F \leq H \leq sclH \leq G$.

Proof. Let X be an $f\breve{g}$-normal space and let F be $f\breve{g}$-closed set and G be $f\breve{g}$-open set in X with $F \leq G$. Then $F \not q (1_X \setminus G)$ where $1_X \setminus G$ is $f\breve{g}$-closed set in X. By hypothesis, there exist $H, T \in FSO(X)$ such that $F \leq H, 1_X \setminus G \leq T$ and $H \not q T$. Then $H \leq 1_X \setminus T \leq G$. Therefore, $F \leq H \leq sclH \leq scl(1_X \setminus T) = 1_X \setminus T \leq G$.

Conversely, let A, B be two $f\breve{g}$-closed sets in X with $A \not q B$. Then $A \leq 1_X \setminus B$. By hypothesis, there exists $H \in FSO(X)$ such that $A \leq H \leq sclH \leq 1_X \setminus B \Rightarrow A \leq H, B \leq 1_X \setminus sclH$ $(=: V)$. Then $V \in FSO(X)$ and so $B \leq V$. Also as $H \not q (1_X \setminus sclH)$, $H \not q V$.

Consequently, X is an $f\breve{g}$-normal space.

Definition 5.5. Let (X,τ) be an fts and $A \in I^X$. A collection $\mathcal{U}$ of fuzzy sets in X is called a fuzzy cover of A if $\bigcup \mathcal{U} \geq A$ [25]. If each member of $\mathcal{U}$ is fuzzy open (resp., fuzzy regular open, fuzzy semiopen) in X, then $\mathcal{U}$ is called a fuzzy open [25] (resp., fuzzy regular open [2], fuzzy semiopen [28]) cover of A. If, in particular, $A = 1_X$, we get the definition of fuzzy cover of X as $\bigcup \mathcal{U} = 1_X$ [20].

Definition 5.6. Let (X,τ) be an fts and $A \in I^X$. Then a fuzzy cover $\mathcal{U}$ of A (resp., of X) is said to have a finite subcover $\mathcal{U}_0$ if $\mathcal{U}_0$ is a finite subcollection of $\mathcal{U}$ such that $\bigcup \mathcal{U}_0 \geq A$ [25]. If, in particular $A = 1_X$, we get $\bigcup \mathcal{U}_0 = 1_X$ [20].

Definition 5.7. Let (X,τ) be an fts and $A \in I^X$. Then A is called fuzzy compact [20] (resp., fuzzy almost compact [21], fuzzy nearly compact [29], fuzzy semicompact [19]) set if every fuzzy open (resp., fuzzy open, fuzzy regular open, fuzzy semiopen) cover $\mathcal{U}$ of A has a finite subcollection $\mathcal{U}_0$ such that $\bigcup \mathcal{U}_0 \geq A$ (resp., $\bigcup\limits_{U \in \mathcal{U}_0} clU \geq A$, $\bigcup \mathcal{U}_0 \geq A$, $\bigcup \mathcal{U}_0 \geq A$). If, in particular, $A = 1_X$, we get the definition of fuzzy compact [20] (resp., fuzzy almost compact [21], fuzzy nearly compact [22], fuzzy semicompact [28]) space as $\bigcup \mathcal{U}_0 = 1_X$ (resp., $\bigcup\limits_{U \in \mathcal{U}_0} clU = 1_X$, $\bigcup \mathcal{U}_0 = 1_X$, $\bigcup \mathcal{U}_0 = 1_X$).

Let us now introduce the following concept.

Definition 5.8. Let (X,τ) be an fts and $A \in I^X$. Then A is called $f\breve{g}$-compact if every fuzzy cover $\mathcal{U}$ of A by $f\breve{g}$-open sets of X has a finite subcover. If, in particular, $A = 1_X$, we get the definition of $f\breve{g}$-compact space X.

Theorem 5.9. Every $f\breve{g}$-closed set in an $f\breve{g}$-compact space X is $f\breve{g}$-compact.

Proof. Let $A (\in I^X)$ be an $f\breve{g}$-closed set in an $f\breve{g}$-compact space X. Let $\mathcal{U}$ be a fuzzy cover of A by $f\breve{g}$-open sets of X. Then $\mathcal{V} = \mathcal{U} \bigcup (1_X \setminus A)$ is a fuzzy cover of X by $f\breve{g}$-open sets of X. As X is $f\breve{g}$-compact space, $\mathcal{V}$ has a finite subcollection $\mathcal{V}_0$ which also covers X. If $\mathcal{V}_0$ contains $1_X \setminus A$, we omit it and get a finite subcover of A.

Hence A is $f\breve{g}$-compact set.

Remark 5.10. It is clear from definitions that every $f\breve{g}$-compact space is fuzzy compact (resp., fuzzy almost compact, fuzzy nearly compact).

6. $f\breve{g}$-Continuous and $f\breve{g}$-Irresolute Functions

In this section we first introduce and study $f\breve{g}$-continuous function. The collection of all $f\breve{g}$-continuous functions is strictly larger than the collection of all fuzzy continuous functions defined between two fts's. It is shown that the image of an $f\breve{g}$-continuous function from an $f\breve{g}$-regular (resp., $f\breve{g}$-normal, $f\breve{g}$-compact) space is fuzzy regular [27] (resp., fuzzy normal [26], fuzzy compact [20], fuzzy nearly compact [22], fuzzy almost compact [21]). Afterwards, the notion of $f\breve{g}$-irresolute function is introduced and it is shown that the family of all $f\breve{g}$-irresolute functions is strictly smaller than that of $f\breve{g}$-continuous function, and also that $f\breve{g}$-irresolute function and fuzzy continuous function are independent concepts. Lastly, we establish that an $f\breve{g}$-regular (resp., $f\breve{g}$-normal, $f\breve{g}$-compact) space remains invariant under $f\breve{g}$-irresolute functions.

We first recall the following definition for ready references.

Definition 6.1 [20]. A function $h : X \to Y$ is said to be a fuzzy continuous function if $h^{-1}(V)$ is fuzzy open set in X for every fuzzy open set V in Y.

Definition 6.2. A function $h : X \to Y$ is said to be an $f\breve{g}$-continuous function if $h^{-1}(V)$ is $f\breve{g}$-open set in X for every fuzzy open set V in Y.

Remark 6.3. Since every fuzzy open set in an fts X is $f\breve{g}$-open, we can conclude that every fuzzy continuous function is $f\breve{g}$-continuous function. But the converse need not be true, as it seen from the following example.

Example 6.4. $f\breve{g}$-continuous function $\not\Rightarrow$ fuzzy continuous function

Let $X = \{a, b\}$, $\tau_1 = \{0_X, 1_X\}$, $\tau_2 = \{0_X, 1_X, A\}$ where

$A(a) = A(b) = 0.5$. Then (X, τ_1) and (X, τ_2) are fts's. Consider the identity function $i : (X, \tau_1) \to (X, \tau_2)$. Since every fuzzy set in (X, τ_1) is $f\breve{g}$-open set in (X, τ_1), so clearly i is $f\breve{g}$-continuous function. But $A \in \tau_2$, $i^{-1}(A) = A \notin \tau_1 \Rightarrow i$ is not a fuzzy continuous function.

Theorem 6.5. Let $h : (X, \tau) \to (Y, \sigma)$ be a function. Then the following statements are equivalent:
(i) h is $f\breve{g}$-continuous function,
(ii) for each fuzzy point x_α in X and each fuzzy open nbd V of $h(x_\alpha)$ in Y , there exists an $f\breve{g}$-open nbd U of x_α in X such that $h(U) \leq V$,
(iii) $h(f\breve{g}cl(A)) \leq cl(h(A))$, for all $A \in I^X$,
(iv) $f\breve{g}cl(h^{-1}(B)) \leq h^{-1}(clB)$, for all $B \in I^Y$.

Proof (i) $\Rightarrow$ (ii). Let x_α be a fuzzy point in X and V any fuzzy open nbd of $h(x_\alpha)$ in Y. Then $x_\alpha \in h^{-1}(V)$ which is $f\breve{g}$-open in X (by (i)). Let $U = h^{-1}(V)$. Then $h(U) = h(h^{-1}(V)) \leq V$.
(ii) $\Rightarrow$ (i). Let A be any fuzzy open set in Y and let x_α be a fuzzy point in X such that $x_\alpha \in h^{-1}(A)$. Then $h(x_\alpha) \in A$ where A is a fuzzy open nbd of $h(x_\alpha)$ in Y. By (ii), there exists an $f\breve{g}$-open nbd U of x_α in X such that $h(U) \leq A$. Then $x_\alpha \in U \leq h^{-1}(A) \Rightarrow x_\alpha \in U = f\breve{g}int(U) \leq f\breve{g}int(h^{-1}(A))$. Since x_α is taken arbitrarily and $h^{-1}(A)$ is the union of all fuzzy points in $h^{-1}(A)$, $h^{-1}(A) \leq f\breve{g}int(h^{-1}(A)) \Rightarrow h^{-1}(A)$ is an $f\breve{g}$-open set in $X \Rightarrow h$ is an $f\breve{g}$-continuous function.
(i) $\Rightarrow$ (iii). Let $A \in I^X$. Then $cl(h(A))$ is a fuzzy closed set in Y. By (i), $h^{-1}(cl(h(A)))$ is an $f\breve{g}$-closed set in X. Now $A \leq h^{-1}(h(A)) \leq h^{-1}(cl(h(A)))$ and so $f\breve{g}cl(A) \leq f\breve{g}cl(h^{-1}(cl(h(A)))) = h^{-1}(cl(h(A))) \Rightarrow h(f\breve{g}cl(A)) \leq cl(h(A))$.
(iii) $\Rightarrow$ (i). Let V be a fuzzy closed set in Y. Put $U = h^{-1}(V)$. Then $U \in I^X$. By (iii), $h(f\breve{g}cl(U)) \leq cl(h(U)) = cl(h(h^{-1}(V))) \leq clV = V \Rightarrow f\breve{g}cl(U) \leq h^{-1}(V) = U \Rightarrow U$ is an $f\breve{g}$-closed set in $X \Rightarrow f$ is an $f\breve{g}$-continuous function.
(iii) $\Rightarrow$ (iv). Let $B \in I^Y$ and $A = h^{-1}(B)$. Then $A \in I^X$. By (iii), $h(f\breve{g}cl(A)) \leq cl(h(A)) \Rightarrow h(f\breve{g}cl(h^{-1}(B))) \leq cl(h(h^{-1}(B))) \leq clB \Rightarrow f\breve{g}cl(h^{-1}(B)) \leq h^{-1}(clB)$.
(iv) $\Rightarrow$ (iii). Let $A \in I^X$. Then $h(A) \in I^Y$. By (iv), $f\breve{g}cl(h^{-1}(h(A))) \leq h^{-1}(cl(h(A))) \Rightarrow f\breve{g}cl(A) \leq f\breve{g}cl(h^{-1}(h(A))) \leq h^{-1}(cl(h(A))) \Rightarrow h(f\breve{g}cl(A)) \leq cl(h(A))$.

Remark 6.6. The composition of two $f\breve{g}$-continuous functions need not be so, as it is seen from the following example.

Example 6.7. Let $X = \{a, b\}$, $\tau_1 = \{0_X, 1_X, A\}$, $\tau_2 = \{0_X, 1_X\}$, $\tau_3 = \{0_X, 1_X, B\}$ where $A(a) = 0.5, A(b) = 0.4, B(a) = 0.5, B(b) = 0.6$. Then (X, τ_1), (X, τ_2) and (X, τ_3) are fts's. Consider two identity functions $i_1 : (X, \tau_1) \to (X, \tau_2)$ and $i_2 : (X, \tau_2) \to (X, \tau_3)$. Then clearly i_1 and i_2 are $f\breve{g}$-continuous functions. Now $1_X \setminus B \in \tau_3^c$. So $(i_2 \circ i_1)^{-1}(1_X \setminus B) = 1_X \setminus B \leq A \in FSO(X, \tau_1)$. But $cl_{\tau_1}(1_X \setminus B) = 1_X \setminus A \not\leq A \Rightarrow 1_X \setminus B$ is not an $f\breve{g}$-closed set in $(X, \tau_1) \Rightarrow i_2 \circ i$ is not an $f\breve{g}$-continuous function.

Theorem 6.8. If $h_1 : X \to Y$ is an $f\breve{g}$-continuous function and $h_2 : Y \to Z$ is a fuzzy continuous function, then $h_2 \circ h_1 : X \to Z$ is an $f\breve{g}$-continuous function.
Proof. Obvious.

Let us now recall some definitions from [27, 26, 5, 33] for ready references.

Definition 6.9 [27]. An fts (X, τ) is called fuzzy regular space if for any fuzzy point x_α in X and any fuzzy closed set F in X with $x_\alpha \notin F$, there exist fuzzy open sets U, V in X such that $x_\alpha \in U$, $F \leq V$ and $U \not{q} V$.

Definition 6.10 [26]. An fts (X, τ) is called fuzzy normal space if for each pair of fuzzy closed sets A, B in X with $A \not{q} B$, there exist fuzzy open sets U, V in X such that $A \leq U, B \leq V$ and $U \not{q} V$.

Definition 6.11 [5]. An fts (X, τ) is called fT_s-space if every fuzzy semiopen set in X is fuzzy open set in X.

Definition 6.12 [33]. A function $f : X \to Y$ is called fuzzy presemiopen if $h(U) \in FSO(Y)$ for every $U \in FSO(X)$.

Theorem 6.13. If a bijective function $h : X \to Y$ is $f\breve{g}$-continuous, fuzzy open function from an $f\breve{g}$-regular, fT_s-space X onto an fts Y, then Y is fuzzy regular space.
Proof. Let y_α be a fuzzy point in Y and F, a fuzzy closed set in Y with $y_\alpha \notin F$. As h is bijective, there exists unique

$x \in X$ such that $h(x) = y$. So $h(x_\alpha) \notin F \Rightarrow x_\alpha \notin h^{-1}(F)$ where $h^{-1}(F)$ is $f\breve{g}$-closed set in X (as h is an $f\breve{g}$-continuous function). By hypothesis, there exist $U, V \in FSO(X)$ such that $x_\alpha \in U, h^{-1}(F) \leq V$ and $U \not{q} V$. Then $h(x_\alpha) \in h(U)$, $F = h(h^{-1}(F))$ (as h is bijective)$\leq h(V)$ and $h(U) \not{q} h(V)$. Since X is fT_s-space, U, V are fuzzy open sets in X. Now as h is a fuzzy open function, $h(U), h(V)$ are fuzzy open sets in Y with $y_\alpha \in h(U), F \leq h(V)$ and $h(U) \not{q} h(V)$ (Indeed, $h(U)qh(V) \Rightarrow$ there exists $z \in Y$ such that $[h(U)](z) + [h(V)](z) > 1 \Rightarrow U(h^{-1}(z)) + V(h^{-1}(z)) > 1$ as h is bijective $\Rightarrow UqV$, a contradiction). Hence Y is a fuzzy regular space.

In a similar manner we can state the following theorems easily.

Theorem 6.14. If a bijective function $h : X \to Y$ is $f\breve{g}$-continuous, fuzzy presemiopen function from an $f\breve{g}$-regular (resp., $f\breve{g}$-normal) space X onto an fT_s-space Y, then Y is fuzzy regular (resp., fuzzy normal) space.

Theorem 6.15. If a bijective function $h : X \to Y$ is $f\breve{g}$-continuous, fuzzy open function from an $f\breve{g}$-normal, fT_s-space X onto an fts Y, then Y is fuzzy normal space.

Definition 6.16. A function $h : X \to Y$ is called $f\breve{g}$-irresolute function if $h^{-1}(U)$ is an $f\breve{g}$-open set in X for every $f\breve{g}$-open set U in Y.

We can state the following theorems easily. Their proofs are similar to that of Theorem 6.13.

Theorem 6.17. If a bijective function $h : X \to Y$ is an $f\breve{g}$-irresolute, fuzzy presemiopen function from an $f\breve{g}$-regular (resp., $f\breve{g}$-normal) space X onto an fts Y, then Y is an $f\breve{g}$-regular (resp., $f\breve{g}$-normal) space.

Theorem 6.18. If a bijective function $h : X \to Y$ is an $f\breve{g}$-irresolute, fuzzy open function from an $f\breve{g}$-regular (resp., $f\breve{g}$-normal), fT_s-space X onto an fts Y, then Y is an $f\breve{g}$-regular (resp., $f\breve{g}$-normal) space.

Theorem 6.19. A function $h : X \to Y$ is $f\breve{g}$-irresolute function if and only if for each fuzzy point x_α in X and each $f\breve{g}$-open nbd V

in Y of $h(x_\alpha)$, there exists an $f\breve{g}$-open nbd U in X of x_α such that $h(U) \leq V$.

Proof. Let $h : X \to Y$ be an $f\breve{g}$-irresolute function. Let x_α be a fuzzy point in X and V be any $f\breve{g}$-open nbd of $h(x_\alpha)$ in Y. Then $h(x_\alpha) \in V \Rightarrow x_\alpha \in h^{-1}(V)$ which being an $f\breve{g}$-open set in X is an $f\breve{g}$-open nbd of x_α in X. Put $U = h^{-1}(V)$. Then U is an $f\breve{g}$-open nbd of x_α in X and $h(U) = h(h^{-1}(V)) \leq V$.

Conversely, let A be an $f\breve{g}$-open set in Y and x_α be any fuzzy point in X such that $x_\alpha \in h^{-1}(A)$. Then $h(x_\alpha) \in A$. By hypothesis, there exists an $f\breve{g}$-open nbd U of x_α in X such that $h(U) \leq A \Rightarrow x_\alpha \in U = f\breve{g}int(U) \leq f\breve{g}int(h^{-1}(A))$. Since x_α is taken arbitrarily and $h^{-1}(A)$ is the union of all fuzzy points in $h^{-1}(A)$, $h^{-1}(A) \leq f\breve{g}int(h^{-1}(A)) \Rightarrow h^{-1}(A) = f\breve{g}int(h^{-1}(A)) \Rightarrow h^{-1}(A)$ is $f\breve{g}$-open set in $X \Rightarrow h$ is an $f\breve{g}$-irresolute function.

Theorem 6.20. Let $h : X \to Y$ be an $f\breve{g}$-continuous function from X onto an fts Y and $A(\in I^X)$be an $f\breve{g}$-compact set in X. Then $h(A)$ is a fuzzy compact (resp., fuzzy almost compact, fuzzy nearly compact) set in Y.

Proof. Let $\mathcal{U} = \{U_\alpha : \alpha \in \Lambda\}$ be a fuzzy cover of $h(A)$ by fuzzy open (resp., fuzzy open, fuzzy regular open) sets of Y. Then $h(A) \leq \bigcup\limits_{\alpha\in\Lambda} U_\alpha \Rightarrow A \leq h^{-1}(\bigcup\limits_{\alpha\in\Lambda} U_\alpha) = \bigcup\limits_{\alpha\in\Lambda} h^{-1}(U_\alpha)$. Then $\mathcal{V} = \{h^{-1}(U_\alpha) : \alpha \in \Lambda\}$ is a fuzzy cover of A by $f\breve{g}$-open sets of X as h is an $f\breve{g}$-continuous function. As A is $f\breve{g}$-compact set in X, there exists a finite subcollection Λ_0 of Λ such that $A \leq \bigcup\limits_{\alpha\in\Lambda_0} h^{-1}(U_\alpha) \Rightarrow h(A) \leq h(\bigcup\limits_{\alpha\in\Lambda_0} h^{-1}(U_\alpha) \leq \bigcup\limits_{\alpha\in\Lambda_0} U_\alpha \Rightarrow h(A)$ is fuzzy compact (resp., fuzzy almost compact, fuzzy nearly compact) set in Y.

Now we can state the following theorems easily the proofs of which are same as that of Theorem 6.20.

Theorem 6.21. Let $h : X \to Y$ be an $f\breve{g}$-irresolute function from X onto an fts Y and $A(\in I^X)$ be an $f\breve{g}$-compact set in X. Then $h(A)$ is $f\breve{g}$-compact (resp., fuzzy compact, fuzzy almost compact, fuzzy nearly compact) set in Y.

Theorem 6.22. Let $h : X \to Y$ be an $f\breve{g}$-continuous function from an $f\breve{g}$-compact space X onto an fts Y . Then Y is a fuzzy compact (resp., fuzzy almost compact, fuzzy nearly compact) space.

Theorem 6.23. Let $h : X \to Y$ be an $f\breve{g}$-irresolute function from an $f\breve{g}$-compact space X onto an fts Y . Then Y is $f\breve{g}$-compact (resp., fuzzy compact, fuzzy almost compact, fuzzy nearly compact) space.

Remark 6.24. It is clear from definitions that $f\breve{g}$-irresolute function is $f\breve{g}$-continuous. But the converse may not be true, as it seen from the following example.

Example 6.25. $f\breve{g}$-continuity $\not\Rightarrow$ $f\breve{g}$-irresoluteness
Let $X = \{a, b\}$, $\tau_1 = \{0_X, 1_X, A\}$, $\tau_2 = \{0_X, 1_X\}$ where $A(a) = 0.5, A(b) = 0.4$. Then $(X.\tau_1)$ and (X, τ_2) are fts's. Consider the identity function $i : (X, \tau_1) \to (X, \tau_2)$. Clearly i is an $f\breve{g}$-continuous function. Consider the fuzzy set B, defined by $B(a) = B(b) = 0.5$. Then B is $f\breve{g}$-closed set in (X, τ_2) as every fuzzy set in (X, τ_2) is $f\breve{g}$-closed set in (X, τ_2). Now $i^{-1}(B) = B \leq B \in FSO(X, \tau_1)$. But $cl_{\tau_1} B = 1_X \setminus A \not\leq B \Rightarrow B$ is not $f\breve{g}$-closed set in $(X, \tau_1) \Rightarrow i$ is not $f\breve{g}$-irresolute function.

Remark 6.26. Fuzzy continuity and $f\breve{g}$-irresoluteness are independent concepts follows from the following examples.

Example 6.27. Fuzzy continuity $\not\Rightarrow$ $f\breve{g}$-irresoluteness
Consider Example 6.21. Here i is clearly fuzzy continuous function. But it is shown that i is not an $f\breve{g}$-irresolute function.

Example 6.28. $f\breve{g}$-irresoluteness $\not\Rightarrow$ fuzzy continuity
Let $X = \{a, b\}$, $\tau_1 = \{0_X, 1_X\}$, $\tau_2 = \{0_X, 1_X, A\}$ where $A(a) = 0.5, A(b) = 0.4$. Then (X, τ_1) and (X, τ_2) are fts's. Consider the identity function $i : (X, \tau_1) \to (X, \tau_2)$. Since every fuzzy set in (X, τ_1) is $f\breve{g}$-closed set in (X, τ_1), i is clearly $f\breve{g}$-irresolute function. But $A \in \tau_2$, $i^{-1}(A) = A \notin \tau_1 \Rightarrow i$ is not a fuzzy continuous function.

7. $f\breve{g}$-T_2-Space

In this section a new type of fuzzy separation axiom, viz., $f\breve{g}$-T_2-space is introduced and it is shown that the inverse image of fuzzy T_2-space [27] under $f\breve{g}$-continuous function is $f\breve{g}$-T_2-space. Afterwards, one strong and one weak form of $f\breve{g}$-continuous function are introduced. It is shown that the image of fuzzy regular (resp., fuzzy normal, fuzzy compact, fuzzy semicompact) space under these functions is $f\breve{g}$-regular (resp., $f\breve{g}$-normal, $f\breve{g}$-compact) space.

We first recall the following definition and theorem from [27, 29] for ready references.

Definition 7.1 [27]. An fts (X, τ) is called fuzzy T_2-space if for any two distinct fuzzy points x_α and y_β; when $x \neq y$, there exist fuzzy open sets U_1, U_2, V_1, V_2 such that $x_\alpha \in U_1, y_\beta q V_1, U_1 \not{q} V_1$ and $x_\alpha q U_2, y_\beta \in V_2, U_2 \not{q} V_2$; when $x = y$ and $\alpha < \beta$ (say), there exist fuzzy open sets U and V in X such that $x_\alpha \in U, y_\beta q V$ and $U \not{q} V$.

Theorem 7.2 [29]. An fts (X, τ) is fuzzy T_2-space if and only if for any two distinct fuzzy points x_α and y_β in X; when $x \neq y$, there exist fuzzy open sets U, V in X such that $x_\alpha q U$, $y_\beta q V$ and $U \not{q} V$; when $x = y$ and $\alpha < \beta$ (say), x_α has a fuzzy open nbd U and y_β has a fuzzy open q-nbd V such that $U \not{q} V$.

Definition 7.3. An fts (X, τ) is called $f\breve{g}$-T_2-space, if for any two distinct fuzzy points x_α and y_β in X; when $x \neq y$, there exist $f\breve{g}$-open sets U, V in X such that $x_\alpha q U$, $y_\beta q V$ and $U \not{q} V$; when $x = y$ and $\alpha < \beta$ (say), x_α has an $f\breve{g}$-open nbd U and y_β has an $f\breve{g}$-open q-nbd V such that $U \not{q} V$.

Theorem 7.4. If an injective function $h : X \to Y$ is $f\breve{g}$-continuous function from an fts X onto a fuzzy T_2-space Y, then X is $f\breve{g}$-T_2 space.

Proof. Let x_α and y_β be two distinct fuzzy points in X. Then $h(x_\alpha)$ $(= z_\alpha$, say) and $h(y_\beta)(= w_\beta$, say) are two distinct fuzzy points in Y.

Case I. Suppose $x \neq y$. Then $z \neq w$. Since Y is fuzzy T_2-space, there exist fuzzy open sets U, V in Y such that $z_\alpha q U, w_\beta q V$ and $U \not{q} V$. As h is $f\breve{g}$-continuous function, $h^{-1}(U)$ and $h^{-1}(V)$ are $f\breve{g}$-open sets in X with $x_\alpha q h^{-1}(U), y_\beta q h^{-1}(V)$ and $h^{-1}(U) \not{q} h^{-1}(V)$ [Indeed, $z_\alpha q U \Rightarrow U(z) + \alpha > 1 \Rightarrow U(h(x)) + \alpha > 1 \Rightarrow [h^{-1}(U)](x) + \alpha > 1 \Rightarrow x_\alpha q h^{-1}(U)$. Again, $h^{-1}(U) q h^{-1}(V) \Rightarrow$ there exists $t \in X$ such that $[h^{-1}(U)](t) + [h^{-1}(V)](t) > 1 \Rightarrow U(h(t)) + V(h(t)) > 1 \Rightarrow U q V$, a contradiction].

Case II. Suppose $x = y$ and $\alpha < \beta$ (say). Then $z = w$ and $\alpha < \beta$. Since Y is fuzzy T_2-space, there exist a fuzzy open nbd

U of x_α and a fuzzy open q-nbd V of w_β such that $U \not{q} V$. Then $U(z) \geq \alpha \Rightarrow [h^{-1}(U)](x) \geq \alpha \Rightarrow x_\alpha \in h^{-1}(U), y_\beta q h^{-1}(V)$ and $h^{-1}(U) \not{q} h^{-1}(V)$ where $h^{-1}(U)$ and $h^{-1}(V)$ are $f\breve{g}$-open sets in X as h is $f\breve{g}$-continuous function. Consequently, X is $f\breve{g}$-T_2-space.

In a similar manner, we can prove the following theorems.

Theorem 7.5. If a bijective function $h : X \to Y$ is $f\breve{g}$-irresolute function from an fts X onto an $f\breve{g}$-T_2-space Y, then X is $f\breve{g}$-T_2-space.

Theorem 7.6. If a bijective function $h : X \to Y$ is $f\breve{g}$-open function from a fuzzy T_2-space X onto an fts Y, then Y is $f\breve{g}$-T_2-space.

Definition 7.7. A function $h : X \to Y$ is called

(i) $f\breve{g}$-strongly continuous if $h^{-1}(V)$ is fuzzy open set in X for every $f\breve{g}$-open set V in Y,

(ii) $f\breve{g}$-weakly continuous if $h^{-1}(V) \in FSO(X)$ for every $f\breve{g}$-open set V in Y.

Remark 7.8. It is clear from definitions that $f\breve{g}$-strongly continuous function is $f\breve{g}$-weakly continuous function as well as $f\breve{g}$-continuous function and $f\breve{g}$-irresolute function. But the converses are not true, in general, as it follow from the following examples.

Example 7.9. $f\breve{g}$-weakly continuity $\not\Rightarrow$ $f\breve{g}$-strongly continuity
Let $X = \{a\}$, $\tau_1 = \{0_X, 1_X, A\}$, $\tau_2 = \{0_X, 1_X, B\}$ where $A(a) \leq 0.3, B(a) \leq 0.4$. Then (X, τ_1) and (X, τ_2) are fts's. Consider the identity function $i : (X, \tau_1) \to (X, \tau_2)$. Now $FSO(X, \tau_1) = \{0_X, 1_X, V\}$ where $0 < V(a) \leq 0.7$ and $FSO(X, \tau_2) = \{0_X, 1_X, U\}$ where $0 < U(a) \leq 0.6$. The collection of all $f\breve{g}$-closed sets in (X, τ_2) is $\{0_X, 1_X, 1_X \setminus B\}$ and that of $f\breve{g}$-open sets is $\{0_X, 1_X, B\}$. Now consider any fuzzy set W defined by $W \leq B$, i.e., $W(a) \leq 0.4$. Now $i^{-1}(W) = W \in FSO(X, \tau_1) \Rightarrow i$ is $f\breve{g}$-weakly continuous function. But if we consider the fuzzy set C defined by $C(a) = 0.4$, then C is $f\breve{g}$-open set in (X, τ_2). But $i^{-1}(C) = C \notin \tau_1 \Rightarrow i$ is not $f\breve{g}$-strongly continuous function.

Example 7.10. $f\breve{g}$-continuity and $f\breve{g}$-irresoluteness $\not\Rightarrow$ $f\breve{g}$-strongly continuity
Let $X = \{a\}$, $\tau_1 = \{0_X, 1_X\}$, $\tau_2 = \{0_X, 1_X, A\}$ where $A(a) = 0.6$.

Then (X, τ_1) and (X, τ_2) are fts's. Consider the identity function $i : (X, \tau_1) \to (X, \tau_2)$. Since every fuzzy set in (X, τ_1) is $f\breve{g}$-closed as well as $f\breve{g}$-open set in (X, τ_1), i is $f\breve{g}$-continuous as well as $f\breve{g}$-irresolute function. Now $A \in \tau_2$ is $f\breve{g}$-open set in (X, τ_2). But $i^{-1}(A) = A \notin \tau_1 \Rightarrow i$ is not $f\breve{g}$-strongly continuous function.

Remark 7.11. $f\breve{g}$-weakly continuity and $f\breve{g}$-irresoluteness are independent concepts follow from the following examples.

Example 7.12. $f\breve{g}$-irresoluteness $\not\Rightarrow$ $f\breve{g}$-weakly continuity
Consider Example 7.10. Since every fuzzy set in (X, τ_1) is $f\breve{g}$-open set in (X, τ_1), i is $f\breve{g}$-irresolute function. But $i^{-1}(A) = A \notin FSO(X, \tau_1) \Rightarrow i$ is not $f\breve{g}$-weakly continuous function.

Example 7.13. $f\breve{g}$-weakly continuity $\not\Rightarrow$ $f\breve{g}$-irresoluteness
Consider Example 7.9. Here i is $f\breve{g}$-weakly continuous function. Consider the fuzzy set V defined by $V(a) = 0.6$. Then $V \in \tau_2^c \Rightarrow V$ is $f\breve{g}$-closed set in (X, τ_2). Now $i^{-1}(V) = V \leq V \in FSO(X, \tau_1)$. But $clV = D$ where D is defined by $D(a) = 0.7$ and so $clV \not\leq V \Rightarrow V$ is not $f\breve{g}$-closed set in $(X, \tau_1) \Rightarrow i$ is not $f\breve{g}$-irresolute function.

Let us now recall following two definitions from [6, 10] for ready references.

Definition 7.14 [6]. An fts (X, τ) is called fuzzy s-regular if for any fuzzy point x_α in X and any fuzzy semiclosed set F in X with $x_\alpha \notin F$, there exist fuzzy semiopen sets U, V in X such that $x_\alpha \in U, F \leq V$ and $U \not{q} V$.

Definition 7.15 [6]. An fts (X, τ) is called fuzzy s-normal if for each pair of fuzzy semiclosed sets A, B in X with $A \not{q} B$, there exist fuzzy semiopen sets U, V in X such that $A \leq U, B \leq V$ and $U \not{q} V$.

Definition 7.16 [10]. An fts (X, τ) is called fuzzy semi-T_2-space if for any two distinct fuzzy points x_α and y_β; when $x \neq y$, there exist fuzzy semiopen sets U, V in X such that $x_\alpha q U$, $y_\beta q V$ and $U \not{q} V$; when $x = y$ and $\alpha < \beta$ (say), x_α has a fuzzy semi-nbd U and y_β has a fuzzy semi-q-nbd V in X such that $U \not{q} V$.

Now we can state the following theorems easily the proofs of which are similar to that of Theorem 6.13, Theorem 6.14, Theorem 6.15, Theorem 6.17, Theorem 6.18, Theorem 6.20, Theorem 6.21 and Theorem 7.4.

Theorem 7.17. If a bijective function $h : X \to Y$ is an $f\breve{g}$-strongly continuous, fuzzy open function from a fuzzy regular (resp., fuzzy normal) space X onto an fts Y, then Y is an $f\breve{g}$-regular (resp., $f\breve{g}$-normal) space.

Theorem 7.18. If a bijective function $h : X \to Y$ is an $f\breve{g}$-weakly continuous, fuzzy presemiopn function from a fuzzy s-regular (resp., fuzzy s-normal) space X onto an fts Y, then Y is an $f\breve{g}$-regular (resp., $f\breve{g}$-normal) space.

Theorem 7.19. If a bijective function $h : X \to Y$ is an $f\breve{g}$-strongly continuous (resp., $f\breve{g}$-weakly continuous) function from X onto an $f\breve{g}$-T_2-space Y, then X is a fuzzy T_2-space (resp., fuzzy semi-T_2-space).

Theorem 7.20. If a bijective function $h : X \to Y$ is an $f\breve{g}$-strongly (resp., $f\breve{g}$-weakly) continuous function from a fuzzy compact (resp., fuzzy semicompact) space X onto an fts Y, then Y is an $f\breve{g}$-compact space.

8. **Mutual Relationship Between Functions**

In this section we first establish the mutual relation between $f\breve{g}$-closed function with the functions defined in [5, 9, 11, 12, 13, 14] and then find the mutual relationship of $f\breve{g}$-continuous function with the functions defined in [5, 7, 9, 11, 12, 13, 14, 15].

We first recall the following definitions from [5, 9, 11, 12, 13, 14] for ready references.

Definition 8.1. Let (X, τ_1) and (Y, τ_2) are fts's and $h : (X, \tau_1) \to (Y, \tau_2)$ be a function. Then h is called an
(i) fg-closed function [5] if $h(A)$ is fg-closed set in Y for every $A \in \tau_1^c$,
(ii) fgp-closed function [5] if $h(A)$ is fgp-closed set in Y for every $A \in \tau_1^c$,
(iii) $fg\beta$-closed function [11] if $h(A)$ is $fg\beta$-closed set in Y for every $A \in \tau_1^c$,
(iv) $fg\alpha$-closed function [5] if $h(A)$ is $fg\alpha$-closed set in Y for every

$A \in \tau_1^c$,
(v) $fgpr$-closed function [15] if $h(A)$ is $fgpr$-closed set in Y for every $A \in \tau_1^c$,
(vi) $fg\delta_p$-closed function [12] if $h(A)$ is $fg\delta_p$-closed set in Y for every $A \in \tau_1^c$,
(vii) $f\delta_p g$-closed function [13] if $h(A)$ is $f\delta_p g$-closed set in Y for every $A \in \tau_1^c$,
(viii) $fg\delta$-semiclosed function [14] if $h(A)$ is $fg\delta$-semiclosed set in Y for every $A \in \tau_1^c$,
(ix) fgs^*-closed function [9] if $h(A)$ is fgs^*-closed set in Y for every $A \in \tau_1^c$.

Note 8.2. It is clear from definitions that $f\breve{g}$-closed function is fg-closed, fgp-closed, $fg\beta$-closed, $fg\alpha$-closed, $fgpr$-closed, $fg\delta_p$-closed function. But the converses are not true, in general, follows from the following example.

Example 8.3. fg-closed, fgp-closed, $fg\beta$-closed, $fg\alpha$-closed, $fgpr$-closed, $fg\delta_p$-closed function $\not\Rightarrow$ $f\breve{g}$-closed function
Let $X = \{a, b\}$, $\tau_1 = \{0_X, 1_X, A\}$, $\tau_2 = \{0_X, 1_X, B\}$ where $A(a) = A(b) = 0.5, B(a) = 0.5, B(b) = 0.4$. Then (X, τ_1) and (X, τ_2) are fts's. Consider the identity function $i : (X, \tau_1) \to (X, \tau_2)$. Here $A \in \tau_1^c$. Now $i(A) = A \leq A \in FSO(X, \tau_2)$. But $cl_{\tau_2} A = 1_X \setminus B \not\leq A \Rightarrow A$ is not $f\breve{g}$-closed set in $(X, \tau_2) \Rightarrow i$ is not $f\breve{g}$-closed function. Since 1_X is the only fuzzy open set in (X, τ_2) containing A, we say that A is fg-closed set, fgp-closed set, $fg\beta$-closed set, $fg\alpha$-closed set, $fgpr$-closed set, $fg\delta_p$-closed set in (X, τ_2). Consequently, i is fg-closed, fgp-closed, $fg\beta$-closed, $fg\alpha$-closed, $fgpr$-closed, $fg\delta_p$-closed function.

Note 8.4. $f\breve{g}$-closed function is independent concept of $f\delta_p g$-closed function, $fg\delta$-semiclosed function and fgs^*-closed function, as it seen from the following examples.

Example 8.5. $f\delta_p g$-closed function, $fg\delta$-semiclosed function $\not\Rightarrow$ $f\breve{g}$-closed function
Consider Example 8.3. Here i is not $f\breve{g}$-closed function. Since only fuzzy δ-preopen set in (X, τ_2) containing the fuzzy set A is 1_X, so clearly i is $f\delta_p g$-closed function. Again 1_X is the only fuzzy open set

in (X,τ_2) containing A and so clearly i is $fg\delta$-semiclosed function.

Example 8.6. $f\check{g}$-closed function $\not\Rightarrow$ $fg\delta$-semiclosed function
Let $X = \{a,b\}$, $\tau_1 = \{0_X, 1_X, A\}$, $\tau_2 = \{0_X, 1_X, A, B\}$ where $A(a) = 0.5, A(b) = 0.6, B(a) = 0.5, B(b) = 0.55$. Then (X,τ_1) and (X,τ_2) are fts's. Consider the identity function $i : (X,\tau_1) \to (X,\tau_2)$. Now $1_X \setminus A \in \tau_1^c$. So $i(1_X \setminus A) = 1_X \setminus A$. Now $FSO(X,\tau_2) = \{0_X, 1_X, U\}$ where $U \geq B$. So $1_X \setminus A \leq B \in FSO(X,\tau_2)$. Now $cl_{\tau_2}(1_X \setminus A) = 1_X \setminus A \leq B \Rightarrow 1_X \setminus A$ is $f\check{g}$-closed set in (X,τ_2) $\Rightarrow$ i is $f\check{g}$-closed function. But $\delta scl_{\tau_2}(1_X \setminus A) = 1_X \not\leq B$, but $1_X \setminus A \leq B \in \tau_2 \Rightarrow 1_X \setminus A$ is not $fg\delta$-semiclosed set in $(X,\tau_2) \Rightarrow i$ is not $fg\delta$-semiclosed function.

Example 8.7. $f\check{g}$-closed function $\not\Rightarrow$ $f\delta_p g$-closed function, fgs^*-closed function
Let $X = \{a,b\}$, $\tau_1 = \{0_X, 1_X, B\}$, $\tau_2 = \{0_X, 1_X, A\}$ where $A(a) = 0.5, A(b) = 0.2, B(a) = 0.4, B(b) = 0.7$. Then (X,τ_1) and (X,τ_2) are fts's. Consider the identity function $i : (X,\tau_1) \to (X,\tau_2)$. Now $FSO(X,\tau_2) = \{0_X, 1_X, C\}$ where $A \leq C \leq 1_X \setminus A$, $F\delta PO(X,\tau_2) = \{0_X, 1_X, U, V\}$ where $U \leq A, V \not\leq 1_X \setminus A$ and $F\delta PC(X,\tau_2) = \{0_X, 1_X, 1_X \setminus U, 1_X \setminus V\}$ where $1_X \setminus U \geq 1_X \setminus A, 1_X \setminus V \not\geq A$. Now $1_X \setminus B \in \tau_1^c$. Then $i(1_X \setminus B) = 1_X \setminus B < 1_X (\in FSO(X,\tau_2))$ only and so $cl_{\tau_2}(1_X \setminus B) \leq 1_X \Rightarrow 1_X \setminus B$ is an $f\check{g}$-closed set in $(X,\tau_2) \Rightarrow i$ is $f\check{g}$-closed function. Again $1_X \setminus B \in F\delta PO(X,\tau_2)$. So $1_X \setminus B \leq 1_X \setminus B$. But $\delta pcl_{\tau_2}(1_X \setminus B) = C$ where C is defined by $C(a) = 0.6, C(b) = 0.8 \Rightarrow C \not\leq 1_X \setminus B \Rightarrow 1_X \setminus B$ is not an $f\delta_p g$-closed set in $(X,\tau_2) \Rightarrow i$ is not $f\delta_p g$-closed function. Also the collection of all fg-open sets in (X,τ_2) is $\{0_X, 1_X, V\}$ where $V \not\geq 1_X \setminus A$. Now $i(1_X \setminus B) = 1_X \setminus B \leq 1_X \setminus B$ which is fg-open set in (X,τ_2). But $scl_{\tau_2}(1_X \setminus B) = 1_X \not\leq 1_X \setminus B \Rightarrow 1_X \setminus B$ is not fgs^*-closed set in $(X,\tau_2) \Rightarrow i$ is not fgs^*-closed function.

Example 8.8. fgs^*-closed function $\not\Rightarrow$ $f\check{g}$-closed function
Let $X = \{a,b\}$, $\tau_1 = \{0_X, 1_X, A, B\}$, $\tau_2 = \{0_X, 1_X, F\}$ where $A(a) = 0.4, A(b) = 0.55, B(a) = 0.5, B(b) = 0.6, F(a) = 0.4, F(b) = 0.5$. Then (X,τ_1) and (X,τ_2) are fts's. Consider the identity function $i : (X,\tau_1) \to (X,\tau_2)$. Here $FSO(X,\tau_2) = FSC(X,\tau_2) = \{0_X, 1_X, U\}$ where $F \leq U \leq 1_X \setminus F$. Now as $i(1_X \setminus A) = 1_X \setminus A \in FSC(X,\tau_2)$ and $i(1_X \setminus B) = 1_X \setminus B \in FSC(X,\tau_2)$, so i is clearly fgs^*-closed

function. Now $1_X \setminus B \leq C \in FSO(X, \tau_2)$ where C is defined by $C(a) = C(b) = 0.5$. But $cl_{\tau_2}(1_X \setminus B) = 1_X \setminus F \not\leq C \Rightarrow 1_X \setminus B$ is not $f\breve{g}$-closed set in $(X, \tau_2) \Rightarrow i$ is not $f\breve{g}$-closed function.

Definition 8.9. Let (X, τ_1) and (Y, τ_2) are fts's and $h : (X, \tau_1) \to (Y, \tau_2)$ be a function. Then h is called
(i) fg-continuous function [5] if $h^{-1}(V)$ is fg-closed set in X for every $V \in \tau_2^c$,
(ii) fgp-continuous function [7] if $h^{-1}(V)$ is fgp-closed set in X for every $V \in \tau_2^c$,
(iii) fpg-continuous function [7] if $h^{-1}(V)$ is fpg-closed set in X for every $V \in \tau_2^c$,
(iv) $fg\alpha$-continuous function [5] if $h^{-1}(V)$ is $fg\alpha$-closed set in X for every $V \in \tau_2^c$,
(v) $f\alpha g$-continuous function [5] if $h^{-1}(V)$ is $f\alpha g$-closed set in X for every $V \in \tau_2^c$,
(vi) $fg\beta$-continuous function [11] if $h^{-1}(V)$ is $fg\beta$-closed set in X for every $V \in \tau_2^c$,
(vii) $fgpr$-continuous function [15] if $h^{-1}(V)$ is $fgpr$-closed set in X for every $V \in \tau_2^c$,
(viii) $fg\delta_p$-continuous function [13] if $h^{-1}(V)$ is $fg\delta_p$-closed set in X for every $V \in \tau_2^c$,
(ix) $f\delta_p g$-continuous function [13] if $h^{-1}(V)$ is $f\delta_p g$-closed set in X for every $V \in \tau_2^c$,
(x) $fg\delta$-semiclosed function [14] if $h^{-1}(V)$ is $fg\delta$-semiclosed set in X for every $V \in \tau_2^c$,
(xi) fgs^*-continuous function [9] if $h^{-1}(V)$ is fs^*g-closed set in X for every $V \in \tau_2^c$.

Remark 8.10. It is clear from Remark 3.21 that $f\breve{g}$-continuous function is fg-continuous, fgp-continuous, $fg\alpha$-continuous, $f\alpha g$-continuous, $fg\beta$-continuous, $fgpr$-continuous, $fg\delta_p$-continuous, $f\delta_p g$-continuous function. But the converses are not true, in general, follow from the following example.

Example 8.11. fg-continuous, fgp-continuous, $fg\alpha$-continuous, $f\alpha g$-continuous, $fg\beta$-continuous, $fgpr$-continuous, $fg\delta_p$-continuous, $f\delta_p g$-continuous function $\not\Rightarrow f\breve{g}$-continuous function
Let $X = \{a, b\}$, $\tau_1 = \{0_X, 1_X, A\}$, $\tau_2 = \{0_X, 1_X, B\}$ where $A(a) = 0.5, A(b) = 0.4, B(a) = B(b) = 0.5$. Then (X, τ_1) and (X, τ_2)

are fts's. Consider the identity function $i : (X, \tau_1) \to (X, \tau_2)$. Now $1_X \setminus B \in \tau_2^c$. So $i^{-1}(1_X \setminus B) = 1_X \setminus B \leq 1_X \setminus B \in FSO(X, \tau_1)$. But $cl_{\tau_1}(1_X \setminus B) = 1_X \setminus A \not\leq 1_X \setminus B \Rightarrow 1_X \setminus B$ is not $f\breve{g}$-closed set in $(X, \tau_1) \Rightarrow i$ is not $f\breve{g}$-continuous function. But 1_X is the only fuzzy open as well as fuzzy regular open, fuzzy α-open, fuzzy δ-preopen set in (X, τ_1) containing $1_X \setminus B$ and so i is fg-continuous, fgp-continuous, $fg\alpha$-continuous, $f\alpha g$-continuous, $fg\beta$-continuous, $fgpr$-continuous, $fg\delta_p$-continuous, $f\delta_p g$-continuous function.

Remark 8.12. The concept of $f\breve{g}$-continuity is independent of the concepts of fpg-continuity and $fg\delta$-semicontinuity and fgs^*-continuity follow from the following examples.

Example 8.13. fpg-continuity $\not\Rightarrow f\breve{g}$-continuity
Consider Example 8.10. Here 1_X is the only fuzzy preopen set in (X, τ_1) containing $1_X \setminus B$, so clearly i is fpg-continuous function, though i is not $f\breve{g}$-continuous function.

Example 8.14. $f\breve{g}$-continuity $\not\Rightarrow fpg$-continuity, fgs^*-continuity
Let $X = \{a, b\}$, $\tau_1 = \{0_X, 1_X, A\}$, $\tau_2 = \{0_X, 1_X, B\}$ where $A(a) = 0.5, A(b) = 0.4, B(a) = 0.4, B(b) = 0.5$. Then (X, τ_1) and (X, τ_2) are fts's. Consider the identity function $i : (X, \tau_1) \to (X, \tau_2)$. Here $1_X \setminus B \in \tau_2^c$. So $i^{-1}(1_X \setminus B) = 1_X \setminus B < 1_X(\in FSO(X, \tau_1))$ only and so $cl_{\tau_1}(1_X \setminus B) \leq 1_X \Rightarrow 1_X \setminus B$ is $f\breve{g}$-closed set in $(X, \tau_1) \Rightarrow i$ is $f\breve{g}$-continuous function. Now $1_X \setminus B \in FPO(X, \tau_1)$ as $int_{\tau_1}(cl_{\tau_1}(1_X \setminus B)) = 1_X > 1_X \setminus B$. So $1_X \setminus B \leq 1_X \setminus B \in FPO(X, \tau_1)$. But $pcl_{\tau_1}(1_X \setminus B) = 1_X \not\leq 1_X \setminus B \Rightarrow 1_X \setminus B$ is not an fpg-closed set in $(X, \tau_1) \Rightarrow i$ is not fpg-continuous function. Now the collection of all fg-open sets in (X, τ_1) is $\{0_X, 1_X, V\}$ where $V \not\geq 1_X \setminus A$. Now $i^{-1}(1_X \setminus B) = 1_X \setminus B \leq 1_X \setminus B$ which is fg-open set in (X, τ_1). But $scl_{\tau_1}(1_X \setminus B) = 1_X \not\leq 1_X \setminus B \Rightarrow 1_X \setminus B$ is not fgs^*-closed set in $(X, \tau_1) \Rightarrow i$ is not fgs^*-continuous function.

Example 8.15. $f\breve{g}$-continuity $\not\Rightarrow fg\delta$-semicontinuity
Let $X = \{a, b\}$, $\tau_1 = \{0_X, 1_X, A\}$, $\tau_2 = \{0_X, 1_X, B\}$ where $A(a) = 0.5, A(b) = 0.6, B(a) = 0.5, B(b) = 0.7$. Then (X, τ_1) and (X, τ_2) are fts's. Consider the identity function $i : (X, \tau_1) \to (X, \tau_2)$. Now $1_X \setminus B \in \tau_2^c$, $i^{-1}(1_X \setminus B) = 1_X \setminus B < A \in FSO(X, \tau_1)$. $cl_{\tau_1}(1_X \setminus B) = 1_X \setminus A < A \Rightarrow 1_X \setminus B$ is $f\breve{g}$-closed set in $(X, \tau_1) \Rightarrow i$ is $f\breve{g}$-continuous function. Now $1_X \setminus B < A \in \tau_1$.

But $\delta scl_{\tau_1}(1_X \setminus B) = 1_X \not\leq A \Rightarrow 1_X \setminus B$ is not $fg\delta$-semiclosed set in $(X, \tau_1) \Rightarrow i$ is not $fg\delta$-semicontinuous function.

Example 8.16. $fg\delta$-semicontinuity $\not\Rightarrow f\breve{g}$-continuity
Consider Example 8.10. Here i is not $f\breve{g}$-continuous function. Since 1_X is the only fuzzy open set in (X, τ_1) containing $1_X \setminus B$, so $1_X \setminus B$ is $fg\delta$-semiclosed set in $(X, \tau_1) \Rightarrow i$ is $fg\delta$-semicontinuous function.

Example 8.17. fgs^*-continuity $\not\Rightarrow f\breve{g}$-continuity
Consider Example 8.8. Here $1_X \setminus F \in \tau_2^c$. Then $i^{-1}(1_X \setminus F) = 1_X \setminus F$. Here the collection of all fg-open sets in (X, τ_1) is $\{0_X, 1_X, V, W\}$ where $0.5 \leq V(a) < 0.6, V(b) \geq 0.5, W \not\geq 1_X \setminus B$. So 1_X is the only fg-open set in (X, τ_1) containing $1_X \setminus F$ and so clearly i is fgs^*-continuous function. Again $FSO(X, \tau_1) = \{0_X, 1_X, U\}$ where $U \geq A$. So $1_X \setminus F \leq D \in FSO(X, \tau_1)$ where $D(a) = 0.6, D(b) = 0.55$. But $cl_{\tau_1}(1_X \setminus F) = 1_X \not\leq D \Rightarrow 1_X \setminus F$ is not $f\breve{g}$-closed set in $(X, \tau_1) \Rightarrow i$ is not $f\breve{g}$-continuous function.

References

[1] Arya, S.P. and Singal, N.; Some forms of fuzzy continuity, *Jour. Fuzzy Math.*, 2 (1994), 723-731.

[2] Azad, K.K.; On fuzzy semi-continuity, fuzzy almost continuity and fuzzy weakly continuity, *J.Math. Anal. Appl.*, 82 (1981), 14-32.

[3] Balasubramanian, G. and Sundaram, P.; On some generalizations of fuzzy continuous functions, *Fuzzy Sets and Systems*, 86 (1997), 93-100.

[4] Bhattacharyya, Anjana and M.N. Mukherjee; On fuzzy δ-almost continuous and δ^*-almost continuous functions, *J. Tripura Math. Soc.*, 2 (2000), 45-57.

[5] Bhattacharyya, Anjana; $fg^*\alpha$-continuous functions in fuzzy topological spaces, *International Journal of Scientific and Engineering Research*, Vol. 4, Issue 8 (2013), 973-979.

[6] Bhattacharyya, Anjana; Unified version of fuzzy generalized open sets on fuzzy topological spaces, *Analele Universităţii Oradea Fasc. Matematica*, Tom XXII, Issue No. 1 (2015), 69-74.

[7] Bhattacharyya, Anjana; Fuzzy generalized continuity, *Annals of Fuzzy Mathematics and Informatics*, Vol. 11, No. 4 (April 2016), 645-659.

[8] Bhattacharyya, Anjana; Several concepts of fuzzy generalized continuity in fuzzy topological spaces, *Analele Universităţii Oradea Fasc. Matematica*, TOM XXIII, Issue No. 2 (2016), 57-68.

[9] Bhattacharyya, Anjana; fgs^*-closed sets and fgs^*-continuous functions in fuzzy topological spaces, *Scientific Studies and Research, Series Mathematics and Informatics*, Vol. 26, No. 2 (2016), 95-114.

[10] Bhattacharyya, Anjana; Several concepts of continuity in fuzzy setting, *e-Proceedings, The 10th International Conference MSAST*, Vol. 5 (2016), 282-293.

[11] Bhattacharyya, Anjana; Fuzzy generalized closed sets in a fuzzy topological space, *Jour. Fuzzy Math.*, Vol. 25, No. 2 (2017), 285-301.

[12] Bhattacharyya, Anjana; Some dif and only iferent types of fuzzy normal spaces, *Analele Universitătii Oradea Fasc. Matematica*, Tom XXIV, Issue No. 2 (2017), 151-164.

[13] Bhattacharyya, Anjana; Generalized version of fuzzy δ-preclosed set in fuzzy setting, *Jour. Fuzzy Math.*, Vol. 26, No. 4 (2018), 975-1004.

[14] Bhattacharyya, Anjana; Fuzzy generalized δ-semiclosed set, *Scientific Studies and Research, Series Mathematics and Informatics*, (Accepted for Publication).

[15] Bhattacharyya, Anjana; More on $fgpr$-closed set in a fuzzy topological space, (Communicated).

[16] Bhattacharyya, Anjana; $f\breve{g}p$-closed set and $f\breve{g}p$-continuous function, (Communicated).

[17] Bin Shahna, A.S.; On fuzzy strong semicontinuity and fuzzy precontinuity, *Fuzzy Sets and Systems*, Vol. 44 (1991), 303-308.

[18] Chakraborty, R.P. and Bhattacharyya, Anjana; On irresolute fuzzy multifunctions, *J. Pure Math.*, Vol. 14 (1997), 26-34.

[19] Chakraborty, R.P., Bhattacharyya, Anjana and Mukherjee, M.N.; Semicompactness in fuzzy topological spaces, *Bull. Malays. Math. Sci. Soc.*, Vol. 2 No. 28 (2) (2005), 205-213.

[20] Chang, C.L.; Fuzzy topological spaces, *J. Math. Anal. Appl.*, Vol. 24 (1968), 182-190.

[21] DiConcillio, A. and Gerla, G.; Almost compactness in fuzzy topological spaces, *Fuzzy Sets and Systems*, Vol. 13 (1984), 187-192.

[22] ES, A. Hayder; Almost compactness and near compactness in topological spaces, *Fuzzy Sets and Systems*, Vol. 22 (1987), 289-295.

[23] Fath Alla, M.A.; On fuzzy topological spaces, *Ph.D. Thesis, Assiut Univ., Sohag, Egypt*, (1984).

[24] Ganguly, S. and Saha, S.; A note on δ-continuity and δ-connected sets in fuzzy set theory, *Simon Stevin*, Vol. 62 (1988), 127-141.

[25] Ganguly, S. and Saha, S.; A note on compactness in fuzzy setting, *Fuzzy Sets and Systems*, Vol. 34 (1990), 117-124.

[26] Hutton, B.; Normality in fuzzy topological spaces, *J. Math Anal. Appl.*, Vol. 50 (1975), 74-79.

[27] Hutton, B. and Reilly, I.; Separation axioms in fuzzy topological spaces, *Fuzzy Sets and Systems*, Vol. 31 (1980), 93-104.

[28] Malakar, S.; On fuzzy semi-irresolute and strongly irresolute functions, *Fuzzy Sets and Systems*, Vol. 45 (1992), 239-244.

[29] Mukherjee, M.N. and Ghosh, B.; On nearly compact and θ-rigid fuzzy sets in fuzzy topological spaces, *Fuzzy Sets and Systems*, Vol. 43 (1991), 57-68.

[30] Mukherjee, M.N. and Sinha, S.P.; Almost compact fuzzy sets in fuzzy topological spaces, *Fuzzy Sets and Systems*, Vol. 38 (1990), 389-396.

[31] Nanda, S.; Strongly compact fuzzy topological spaces, *Fuzzy Sets and Systems*, Vol. 42 (1991), 254-262.

[32] Pu, Pao Ming and Liu, Ying Ming; Fuzzy topology I. Neighbourhood structure of a fuzzy point and Moore-Smith Convergence, *J. Math Anal. Appl.*, Vol. 76 (1980), 571-599.

[33] Thakur, S.S. and Saraf, R.K.; Fuzzy pre-semiclosed mappings, Int. Sci. Cong. (CHENNAI) (1997).

[34] Wong, C.K.; Fuzzy points and local properties of fuzzy topology, *J. Math. Anal. Appl.*, Vol. 46 (1974), 316-328.

[35] Zadeh, L.A.; Fuzzy Sets, *Inform. Control*, Vol. 8 (1965), 338-353.

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