"Vasile Alecsandri" University of Bacău
Faculty of Sciences
Scientific Studies and Research
Series Mathematics and Informatics
Vol. 29 (2019), No. 1, 41-58

GENERALIZED CLOSED SET AND GENERALIZED CONTINUITY IN FUZZY M-SPACE

ANJANA BHATTACHARYYA

Abstract. This paper deals with a generalized version of fuzzy closed sets in fuzzy m-spaces [2]. Using this concept as a basic tool, we introduce a generalized version of closed and open functions in fuzzy m-spaces. Several characterizations of these functions are proved. Moreover, a generalized version of continuity in fuzzy m-space is introduced and studied. Afterwards, several applications of this type of function are given in the study of generalized versions of fuzzy regular, fuzzy normal and fuzzy compact spaces, in the setting of fuzzy m-spaces.

1. Introduction

Alimohammady and Roohi in [1] introduced the notion of fuzzy minimal structure (fuzzy m-structure, for short) as follows : A family $\mathcal{M}$ of fuzzy sets in a non-empty set X is said to be fuzzy minimal structure on X if $\alpha 1_X \in \mathcal{M}$ for every $\alpha \in [0,1]$. Afterwards, a more general version of fuzzy minimal structure (in the sense of Chang) was introduced in [4, 7] as follows : A family $\mathcal{F}$ of fuzzy sets in a non-empty set X is a fuzzy minimal structure on X if $0_X \in \mathcal{F}$ and $1_X \in \mathcal{F}$. In this paper, we use the notion of fuzzy minimal structure in the sense of Chang.

Keywords and phrases: Fuzzy m-closed set, fuzzy generalized m-closed set, fuzzy generalized m-continuous function, fuzzy m-open q-nbd, fuzzy generalized m-regular and m-normal spaces, fuzzy generalized m-compact space.
(2010) Mathematics Subject Classification: 54A40, 54C99

2. Preliminaries

In [9], Zadeh introduced the concept of a fuzzy set A which is a mapping from a non-empty set X into the closed interval $I = [0, 1]$, i.e., $A \in I^X$. The support [9] of a fuzzy set A, denoted by $suppA$ and is defined by $suppA = \{x \in X : A(x) \neq 0\}$. The fuzzy set with the singleton support $\{x\} \subseteq X$ and the value t $(0 < t \leq 1)$ will be denoted by x_t. 0_X and 1_X are the constant fuzzy sets taking values 0 and 1 respectively in X. The complement [9] of a fuzzy set A in X is denoted by $1_X \setminus A$ and is defined by $(1_X \setminus A)(x) = 1 - A(x)$, for each $x \in X$. For any two fuzzy sets A, B in X, $A \leq B$ means $A(x) \leq B(x)$, for all $x \in X$ [9] while AqB means A is quasi-coincident (q-coincident, for short) [8] with B, i.e., there exists $x \in X$ such that $A(x) + B(x) > 1$. The negation of these two statements will be denoted by $A \not\leq B$ and $A \not{q} B$ respectively. For a fuzzy point x_α and a fuzzy set A in X, $x_\alpha \in A$ means $x_\alpha \leq A$, i.e., $A(x) \geq \alpha$.

3. fgm-Closed Set and fgm-Closure Operator

Let X be a non-empty set and $m \subset I^X$ be a fuzzy minimal structure on X. Then (X, m) is called a fuzzy minimal space (fuzzy m-space, for short) [2]. The members of m are called fuzzy m-open sets [2]. The complement of a fuzzy m-open set in a fuzzy m-space is called a fuzzy m-closed set.

Definition 3.1 [2]. Let (X, m) be a fuzzy m-space and $A \in I^X$. Then the fuzzy m-closure and fuzzy m-interior of A, denoted by $mclA$ and $mintA$ respectively, are defined as follows :

$$mclA = \bigwedge \{F : A \leq F, 1_X \setminus F \in m\}$$

$$mintA = \bigvee \{D : D \leq A, D \in m\}$$

It is to be noted that given a fuzzy minimal structure m on X, if $A \in I^X$, then $mintA$ may not be an element of m. But if m satisfies M-condition (i.e., m is closed under arbitrary union) [2], then $mintA$ is an element of m and $mclA$ is fuzzy m-closed.

Proposition 3.2 [2]. Let (X, m) be a fuzzy m-space. Then for any $A \in I^X$, a fuzzy point $x_\alpha \in mclA$ if and only if for any $U \in m$ with $x_\alpha qU$, UqA.

Lemma 3.3 [2]. Let (X, m) be a fuzzy m-space. For $A, B \in I^X$, the following statements are true:
(i) $A \leq B \Rightarrow mintA \leq mintB$, $mclA \leq mclB$,
(ii) $mint0_X = 0_X$, $mint1_X = 1_X$, $mcl0_X = 0_X$, $mcl1_X = 1_X$,
(iii) $mintA \leq A \leq mclA$,
(iv) $mclA = A$ if $1_X \setminus A \in m$, $mintA = A$, if $A \in m$,
(v) $mcl(1_X \setminus A) = 1_X \setminus mintA$, $mint(1_X \setminus A) = 1_X \setminus mclA$,
(vi) $mcl(mclA) = mclA$ and $mint(mintA) = mintA$.

Lemma 3.4 [2]. For $A, B \in I^X$ where (X, m) is a fuzzy m-space,
(i) $mclA \bigvee mclB \leq mcl(A \bigvee B)$
(ii) $mint(A \bigwedge B) \leq mintA \bigwedge mintB$.

Now we introduce generalized version of fuzzy m-closed set.

Definition 3.5. Let (X, m) be a fuzzy m-space and $A \in I^X$. Then A is called fuzzy generalized m-closed (fgm-closed, for short) if $mclA \leq U$ whenever $A \leq U \in m$. The complement of an fgm-closed set in X is called fgm-open set in X.

Remark 3.6. It is clear that every fuzzy m-closed set is fgm-closed, but converse need not be true as it seen from the following example.

Example 3.7. Let $X = \{a, b\}$, $m = \{0_X, 1_X, A\}$ where $A(a) = 0.5, A(b) = 0.6$. Then (X, m) is a fuzzy m-space. Consider the fuzzy set B defined by $B(a) = 0.5, B(b) = 0.3$. Then $B \leq A \in m$. Now $mclB = 1_X \setminus A \leq A \Rightarrow B$ is fgm-closed in X, though B is not fuzzy m-closed in X.

Definition 3.8. Let (X, m) be a fuzzy m-space and $A \in I^X$. Then fuzzy generalized m-closure of A, denoted by $fgmcl(A)$, is defined by $fgmcl(A) = \bigwedge\{F : A \leq F$ and F is fgm-closed in $X\}$.

Note 3.9. It is clear that for any $A \in I^X$, $A \leq fgmcl(A)$. If A is fgm-closed, then $A = fgmcl(A)$. But $fgmcl(A)$ may not be fgm-closed follows from the next example.

Example 3.10. Let $X = \{a, b\}$, $m = \{0_X, 1_X, A, B\}$ where $A(a) = 0.5, A(b) = 0.4, B(a) = 0.4, B(b) = 0.5$. Then (X, m) is a fuzzy m-space. Consider two fuzzy sets C and D defined by $C(a) = 0.4, C(b) = 0.6, D(a) = 0.6, D(b) = 0.4$. Then 1_X is the only fuzzy m-open set containing C as well as D and so C and D are fgm-closed in X. Let $E = C \bigwedge D$. Then $E(a) = E(b) = 0.4$. Then $E \leq A \in m$. Now $mclE = (1_X \setminus A) \bigwedge (1_X \setminus B) = F$ where $F(a) = F(b) = 0.5$ and so $F \not\leq A \Rightarrow E$ is not fgm-closed in X. This shows that the intersection of two fgm-closed sets need not be so. Hence the proof.

Proposition 3.11. Le (X, m) be a fuzzy m-space and $A \in I^X$. Then for a fuzzy point x_α in X, $x_\alpha \in fgmclA$ if and only if every fgm-open set U, $x_\alpha qU$ implies UqA.

Proof. Let $x_\alpha \in fgmclA$ and U be fgm-open set in X with $x_\alpha qU$. Then $U(x) + \alpha > 1 \Rightarrow x_\alpha \notin 1_X \setminus U$ which is fgm-closed in X. As $x_\alpha \in fgmclA$, $x_\alpha \in F$, for all fgm-closed set F containing A. So $A \not\leq 1_X \setminus U \Rightarrow$ there is $y \in X$ such that $A(y) > 1 - U(y) \Rightarrow AqU$.

Conversely, let for every fgm-open set U in X, $x_\alpha qU$ imply UqA. We have to prove that $x_\alpha \in F$, for all fgm-closed set F in X containing A. If possible, let $x_\alpha \notin F$ where F is fgm-closed set containing A. Then $x_\alpha q(1_X \setminus F)$ which is fgm-open in X. By hypothesis, $(1_X \setminus F)qA \Rightarrow$ there is $y \in X$ such that $1 - F(y) + A(y) > 1 \Rightarrow A(y) > F(y)$ which contradicts the fact that $F \geq A$.

Theorem 3.12. Let (X, m) be a fuzzy m-space and $A, B \in I^X$. Then the following statements are true :

(i) $fgmcl(0_X) = 0_X$,

(ii) $fgmcl(1_X) = 1_X$,

(iii) if $A \leq B$, then $fgmcl(A) \leq fgmcl(B)$,

(iv) $fgmcl(A \bigvee B) = fgmcl(A) \bigvee fgmcl(B)$,

(v) $fgmcl(A \bigwedge B) \leq fgmcl(A) \bigwedge fgmcl(B)$, equality does not hold, in general, follows from Example 3.10,

(vi) $fgmcl(fgmcl(A)) = fgmcl(A)$.

Proof. (i), (ii) and (iii) are obvious.

(iv) By (iii), $fgmcl(A) \bigvee fgmcl(B) \leq fgmcl(A \bigvee B)$.

To prove the converse, let $x_t \in fgmcl(A \bigvee B)$. Then for any fgm-open set U in X, $x_t qU$ implies $Uq(A \bigvee B)$. Then there exists $y \in X$ such that $U(y) + max\{A(y), B(y)\} > 1 \Rightarrow$ either $U(y) + A(y) > 1$ or $U(y) + B(y) > 1 \Rightarrow$ either UqA or $UqB \Rightarrow$ either $x_t \in fgmcl(A)$ or $x_t \in fgmcl(B) \Rightarrow x_t \in fgmcl(A) \bigvee fgmcl(B)$.

(v) Follows from (iii).
(vi) From (iii) as $A \leq fgmcl(A)$, $fgmcl(A) \leq fgmcl(fgmcl(A))$.
Conversely, let $x_t \in fgmcl(fgmcl(A)) = fgmcl(B)$ where $B = fgmcl(A)$. Let U be any fgm-open set in X with $x_t q U$. Then $UqB \Rightarrow$ there exists $y \in X$ such that $U(y) + B(y) > 1$. Let $B(y) = s$. Then $y_s \in B = fgmcl(A)$. Now $y_s q U$ where U is fgm-open in X and so $UqA \Rightarrow x_t \in fgmcl(A)$ and so $fgmcl(fgmcl(A)) \leq fgmcl(A)$. Hence the proof.

Definition 3.13. Let (X, m) be a fuzzy m-space and $A \in I^X$. Then fuzzy generalized m-interior of A, denoted by $fgmint(A)$, is defined by $fgmint(A) = \bigvee\{G : G \leq A \text{ and } G \text{ is } fgm\text{-open in } X\}$.

Remark 3.14. For every fuzzy set A in a fuzzy m-space (X, m), $fgmint(A) \leq A$. But if A is fgm-open, then $fgmint(A) = A$. In general, $fgmint(A)$ may not by fgm-open follows from Example 3.10. Indeed, here $1_X \setminus C$ and $1_X \setminus D$ are fgm-pen sets in X. Then $(1_X \setminus C) \bigvee (1_X \setminus D) = G$ (say) where $G(a) = G(b) = 0.6$. If we show that $1_X \setminus G$ is not fgm-closed, then G is not fgm-open in X. Now $1_X \setminus G \leq A \in m$. But $mcl(1_X \setminus G) = (1_X \setminus A) \bigwedge (1_X \setminus B) = H$ (say) where $H(a) = H(b) = 0.5$ and so $H \not\leq A$ implies that $1_X \setminus G$ is not fgm-closed in X. Consequently, G is not fgm-open in X.

Lemma 3.15. In a fuzzy m-space (X, m) and $A \in I^X$, the following statements hold:
(i) $fgmcl(1_X \setminus A) = 1_X \setminus fgmint(A)$
(ii) $fgmint(1_X \setminus A) = 1_X \setminus fgmcl(A)$.
Proof (i). Let $x_t \in fgmcl(1_X \setminus A)$. If possible, let $x_t \notin 1_X \setminus fgmint(A)$. Then $1 - (fgmint(A))(x) < t \Rightarrow [fgmint(A)](x) + t > 1 \Rightarrow fgmint(A)qx_t \Rightarrow$ there exists at least one fgm-open set $F \leq A$ with $x_t qF \Rightarrow x_t qA$. As $x_t \in fgmcl(1_X \setminus A), Fq(1_X \setminus A) \Rightarrow Aq(1_X \setminus A)$, a contradiction. Hence

$$fgmcl(1_X \setminus A) \leq 1_X \setminus fgmint(A)...(1)$$

Conversely, let $x_t \in 1_X \setminus fgmint(A)$. Then $1 - [(fgmint(A)](x) \geq t \Rightarrow x_t \not{q}(fgmint(A)) \Rightarrow x_t \not{q}F$ where F is fgm-open set contained in A ... (2).
Let U be any fgm-closed set in X such that $1_X \setminus A \leq U$. Then $1_X \setminus U \leq A$. Now $1_X \setminus U$ is fgm-open set in X contained in A. By

(2), $x_t \not q (1_X \setminus U) \Rightarrow x_t \in U \Rightarrow x_t \in fgmcl(1_X \setminus A)$ and so

$$1_X \setminus fgmint(A) \leq fgmcl(1_X \setminus A) ...(3).$$

Combining (1) and (3), (i) follows.
(ii) Putting $1_X \setminus A$ for A in (i), we get $fgmcl(A) = 1_X \setminus fgmint(1_X \setminus A) \Rightarrow fgmint(1_X \setminus A) = 1_X \setminus fgmcl(A)$.

4. $fg(m, m_1)$-Closed ($fg(m, m_1)$-Open) Function : Some Properties

We first recall the following definitions from [3].

Definition 4.1 [3]. A function $f : (X, m) \to (Y, m_1)$ is called fuzzy (m, m_1)-closed (resp., fuzzy (m, m_1)-open) if $f(U)$ is fuzzy m_1-closed (resp., fuzzy m_1-open) set in Y for every fuzzy m-closed (resp., fuzzy m-open) set U in X.

Definition 4.2 [3]. A function $f : (X, m) \to (Y, m_1)$ is called fuzzy (m, m_1)-continuous if $f^{-1}(U)$ is fuzzy m-open in X for every fuzzy m_1-open set U in Y.

Let us now introduce the following concept.

Definition 4.3. A function $f : (X, m) \to (Y, m_1)$ is called fuzzy generalized (m, m_1)-closed ($fg(m, m_1)$-closed, for short) if $f(F)$ is fgm-closed set in Y for every fuzzy m-closed set F in X.

Remark 4.4. Since every fuzzy m-closed set is fgm-closed, we can easily conclude that fuzzy (m, m_1)-closed function is $fg(m, m_1)$-closed But the converse need not be true as it seen from the following example.

Example 4.5. Let $X = \{a, b\}$, $m = \{0_X, 1_X, B\}$, $m_1 = \{0_X, 1_X, A\}$ where $A(a) = 0.5, A(b) = 0.6, B(a) = 0.5, B(b) = 0.7$. Then (X, m) and (X, m_1) are fuzzy m-spaces. Consider the identity function $i : (X, m) \to (X, m_1)$ and the fuzzy set $1_X \setminus B$. Then $1_X \setminus B \in m^c$. Now $i(1_X \setminus B) = 1_X \setminus B \notin m_1^c \Rightarrow i$ is not fuzzy (m, m_1)-closed function. Also $1_X \setminus B \leq A$ and $m_1cl(1_X \setminus B) = 1_X \setminus A \leq A \Rightarrow 1_X \setminus B$ is fgm_1-closed set in $(X, m_1) \Rightarrow i$ is $fg(m, m)$-closed function.

Remark 4.6. Composition of two $fg(m, m_1)$-closed functions may not be so as it seen from the following example.

Example 4.7. Let $X = \{a, b\}$, $m = \{0_X 1_X, A\}$, $m_1 = \{0_X, 1_X\}$, $m_2 = \{0_X, 1_X, B\}$ where $A(a) = 0.5, A(b) = 0.6, B(a) = 0.5, B(b) = 0.4$. Then (X, m) and (X, m_1) are fuzzy m-spaces. Consider two identity functions $i_1 : (X, m) \to (X, m_1)$ and $i_2 : (X, m_1) \to (X, m_2)$. Clearly i_1 and i_2 are $fg(m, m_1)$-closed and $fg(m_1, m_2)$-closed functions respectively. Now $1_X \setminus A \in m^c$. Then $(i_2 \circ i_1)(1_X \setminus A) = 1_X \setminus A \leq B$. But $m_2cl(1_X \setminus A) = 1_X \setminus B \not\leq B \Rightarrow 1_X \setminus A$ is not fgm_2-closed set in $(X, m_2) \Rightarrow i_2 \circ i_1$ is not $fg(m, m_2)$-closed function.

Theorem 4.8. If $f : (X, m) \to (Y, m_1)$ is a fuzzy (m, m_1)-closed function and $g : (Y, m_1) \to (Z, m_2)$ is a $fg(m_1, m_2)$-closed function, then $g \circ f : (X, m) \to (Z, m_2)$ is $fg(m, m_2)$-closed function.

Proof. Let A be fuzzy m-closed in X. Then $f(A)$ is fuzzy m_1-closed in Y. By hypothesis, $g(f(A)) = (g \circ f)(A)$ is fgm_2-closed in $Z \Rightarrow g \circ f$ is $fg(m, m_2)$-closed function.

Theorem 4.9. An injective function $f : (X, m) \to (Y, m_1)$ is $fg(m, m_1)$-closed if and only if for each $S \in I^Y$ and each fuzzy m-open set U in X with $f^{-1}(S) \leq U$, there exists fgm_1-open set V in Y such that $S \leq V$ and $f^{-1}(V) \leq U$.

Proof. Let f be $fg(m, m_1)$-closed function. Let $S \in I^Y$ and U be a fuzzy m-open set in X such that $f^{-1}(S) \leq U$. Then $1_X \setminus f^{-1}(S) \geq 1_X \setminus U \Rightarrow f(1_X \setminus U) \leq f(1_X \setminus f^{-1}(S)) = 1_Y \setminus f(f^{-1}(S)) = 1_Y \setminus S$ (as f is injective). Now $1_X \setminus U$ is fuzzy m-closed in X. Then $f(1_X \setminus U)$ is fgm_1-closed in Y (as f is $fg(m, m_1)$-closed function). Let $V = 1_Y \setminus f(1_X \setminus U)$. Then V is fgm_1-open in Y. Now $S \leq 1_Y \setminus f(1_X \setminus U) = V$ and $f^{-1}(V) = f^{-1}(1_Y \setminus f(1_X \setminus U)) = 1_X \setminus f^{-1}(f(1_X \setminus U)) \leq U$.
Conversely, let F be a fuzzy m-closed set in X and O be a fuzzy m-open set in Y such that

$$f(F) \leq O....(i)$$

Then $f^{-1}(1_Y \setminus f(F)) = 1_X \setminus f^{-1}(f(F)) \leq 1_X \setminus F$ which is fuzzy m-open in X. By hypothesis, there exists an fgm_1-open set V in Y such that

$$1_Y \setminus f(F) \leq V...(ii)$$

and

$$f^{-1}(V) \leq 1_X \setminus F...(iii)$$

Therefore, $F \leq 1_X \setminus f^{-1}(V)$ implies that

$$f(F) \leq f(1_X \setminus f^{-1}(V)) = 1_Y \setminus V...(iv)$$

(as f is injective). From (i), $1_Y \setminus O \leq 1_Y \setminus f(F)$, $f^{-1}(1_Y \setminus O) \leq f^{-1}(1_Y \setminus f(F)) \leq f^{-1}(V)$ (by (ii)) $\leq 1_X \setminus F$ (by (iii)). Then $F \leq 1_X \setminus f^{-1}(V) \leq 1_X \setminus f^{-1}(1_Y \setminus f(F)) \leq 1_X \setminus f^{-1}(1_Y \setminus O) \Rightarrow f(F) \leq f(1_X \setminus f^{-1}(1_Y \setminus O)) = 1_Y \setminus f(f^{-1}(1_Y \setminus O)) = O$ (as f is injective). As $1_Y \setminus V$ is fgm_1-closed in Y, $m_1cl(f(F)) \leq m_1cl(1_Y \setminus V)$ (by (iv)) $= 1_Y \setminus V \leq O$ (as by (i) and (ii), $1_Y \setminus V \leq f(F) \leq O \Rightarrow 1_Y \setminus V \leq O$) $\Rightarrow m_1cl(f(F)) \leq O$ whenever $f(F) \leq O \Rightarrow f(F)$ is fgm_1-closed in Y. Consequently, f is $fg(m, m_1)$-closed function.

Theorem 4.10. If a function $f : (X, m) \to (Y, m_1)$ is fuzzy (m, m_1)-continuous and $fg(m, m_1)$-closed function and A is an fgm-closed set in X, then $f(A)$ is fgm_1-closed set in Y.

Proof. Let O be a fuzzy m_1-open set in Y with $f(A) \leq O$. Then $A \leq f^{-1}(O)$ which is fuzzy m-open set in X as f is fuzzy (m, m_1)-continuous. Since A is fgm-closed, $mclA \leq f^{-1}(O) \Rightarrow f(mclA) \leq O$. As f is $fg(m, m_1)$-closed function, $f(mclA)$ is fgm_1-closed in Y and so $m_1cl(f(mclA)) \leq O$. Since $f(A) \leq f(mclA)$, $m_1cl(f(A)) \leq m_1cl(f(mclA)) \leq O \Rightarrow f(A)$ is fgm_1-closed in Y.

Theorem 4.11. If $h : (X, m) \to (Y, m_1)$ is $fg(m, m_1)$-closed function where m satisfies M-condition, then $fgm_1cl(h(A)) \leq h(mclA)$, for all $A \in I^X$.

Proof. Let $A \in I^X$. Then $mclA$ is fuzzy m-closed in X as m satisfies M-condition. By hypothesis, $h(mclA)$ is fgm_1-closed in Y and so $fgm_1cl(h(A)) \leq fgm_1cl(h(mclA)) = h(mclA)$.

Definition 4.12. A function $h : (X, m) \to (Y, m_1)$ is called fuzzy generalized (m, m_1)-open ($fg(m, m_1)$-open, for short) function if $h(U)$ is fgm_1-open in Y for each fgm-open set U in X.

Theorem 4.13. For a bijective function $h : (X, m) \to (Y, m_1)$ where m-satisfies M-condition, the following statements are equivalent:

(i) h is $fg(m, m_1)$-open,

(ii) $h(mintA) \leq fgm_1int(h(A))$, for all $A \in I^X$,

(iii) for each fuzzy point x_t in X and each fuzzy m-open set U in X containing x_t, there exists an fgm_1-open set V containing $h(x_t)$ such that $V \leq h(U)$.

Proof (i) $\Rightarrow$ (ii). Let $A \in I^X$. Then $mintA$ is fuzzy m-open in X (as m satisfies M-condition). By (i), $h(mintA)$ is fgm_1-open in Y. Since $mintA \leq A$, $h(mintA) \leq h(A)$ and $fgm_1int(h(A))$ is the union of all fgm_1-open sets contained in $h(A)$, we have $h(mintA) \leq fgm_1int(h(A))$.
(ii) $\Rightarrow$ (i). Let U be a fuzzy m-open set in X. Then $h(U)$ is fuzzy m_1-open in Y and so $h(U) = h(mintU) \leq fgm_1int(h(U))$ (by (ii)) $\Rightarrow h(U)$ is fgm_1-open set in Y.
(ii) $\Rightarrow$ (iii). Let x_t be a fuzzy point in X and U, a fuzzy m-open set in X such that $x_t \in U$. Then $h(x_t) \in h(U) = h(mintU) \leq fgm_1int(h(U))$. Then $h(U)$ is fgm_1-open in Y. Let $V = f(U)$. Then $h(x_t) \in V$ and $V \leq f(U)$.
(iii) $\Rightarrow$ (i). Let U be any fuzzy m-open set in X and y_t be any fuzzy point in $h(U)$, i.e., $y_t \in h(U)$. Then there exists $x \in X$ such that $h(x) = y$ (as h is bijective). Then $[h(U)](y) \geq t \Rightarrow U(h^{-1}(y)) \geq t \Rightarrow U(x) \geq t \Rightarrow x_t \in U$. By (iii), there exists an fgm_1-open set V in Y such that $h(x_t) \in V$ and $V \leq h(U)$. Then $h(x_t) \in V = fgm_1int(V) \leq fgm_1int(h(U))$. Since x_t is taken arbitrarily and $h(U)$ is the union of all fuzzy points in $h(U)$, $h(U) \leq fgm_1int(h(U)) \Rightarrow h(U)$ is fgm_1-open in $Y \Rightarrow h$ is $fg(m, m_1)$-open function.

Theorem 4.14. If $h : (X, m) \to (Y, m_1)$ is $fg(m, m_1)$-open function where m satisfies M-condition, then the following statements are true :
(i) for each fuzzy point x_t in X and each fuzzy m-open set U in X with x_tqU, there exists fgm_1-open set V with $h(x_t)qV$ such that $V \leq h(U)$,
(ii) $h^{-1}(fgm_1cl(B)) \leq mcl(h^{-1}(B))$, for all $B \in I^Y$.

Proof (i). Let x_t be any fuzzy point in X and U be any fuzzy m-open set in X with $x_tqU = mintU \Rightarrow h(x_t)qh(mintU) \leq fgm_1int(h(U))$ (by Theorem 4.13) $\Rightarrow h(x_t)qfgm_1int(h(U)) \Rightarrow$ there exists an fgm_1-open set V in Y such that $h(x_t)qV$ and $V \leq h(U)$.
(ii) Let x_t be any fuzzy point in X such that $x_t \notin mcl(h^{-1}(B))$ for any $B \in I^Y$. Then there exists a fuzzy m-open set U in X with x_tqU, $U \not{q} h^{-1}(B)$. Now

$$h(x_t)qh(U)...(i)$$

where $h(U)$ is fgm_1-open set in Y (as h is $fg(m, m_1)$-open function). Now $h^{-1}(B) \leq 1_X \setminus U \Rightarrow B \leq h(1_X \setminus U) \leq 1_Y \setminus h(U) \Rightarrow B \not{q} h(U)$. Let $V = 1_Y \setminus h(U)$. Then V is fgm_1-closed set in Y with $B \leq V$. We claim that $h(x_t) \notin V$. If possible, let $h(x_t) \in V = 1_Y \setminus h(U)$.

Then $1 - [h(U)](h(x)) \geq t \Rightarrow h(U) \ /q h(x_t)$, contradicts (i). So $h(x_t) \notin V \Rightarrow h(x_t) \notin fgm_1cl(B) \Rightarrow x_t \notin h^{-1}(fgm_1cl(B)) \Rightarrow h^{-1}(fgm_1cl(B)) \leq mcl(h^{-1}(B))$.

Theorem 4.15. If $h : (X, m) \to (Y, m_1)$ is injective, $fg(m, m_1)$-open function, $B \in I^Y$ and F is a fuzzy m-closed set in X with $h^{-1}(B) \leq F$, there exists an fgm_1-closed set V in Y such that $B \leq V$ and $h^{-1}(V) \leq F$.

Proof. Let $B \in I^Y$ and F be a fuzzy m-closed set in X with $h^{-1}(B) \leq F$. Then $1_X \setminus h^{-1}(B) \geq 1_X \setminus F$ where $1_X \setminus F$ is fuzzy m-open in $X \Rightarrow h(1_X \setminus F) \leq h(1_X \setminus h^{-1}(B)) \leq 1_Y \setminus B$ (as h is injective) where $h(1_X \setminus F)$ is fgm_1-open in Y. Let $V = 1_Y \setminus h(1_X \setminus F)$. Then V is fgm_1-closed in Y such that $B \leq 1_Y \setminus h(1_X \setminus F) = V$. Now $h^{-1}(V) = h^{-1}(1_Y \setminus h(1_X \setminus F)) = 1_X \setminus h^{-1}(h(1_X \setminus F)) \leq F$.

5. $fg(m, m_1)$-Continuous Function

Definition 5.1. A function $f : (X, m) \to (Y, m_1)$ is called fuzzy generalized (m, m_1)-continuous ($fg(m, m_1)$-continuous, for short) function if $f^{-1}(F)$ is fuzzy m-closed in X for every fuzzy m_1-closed set F in Y.

Theorem 5.2. Let $h : (X, m) \to (Y, m_1)$ be a function where m_1 satisfies M-condition. Then the following statements are equivalent:
(i) h is $fg(m, m_1)$-continuous,
(ii) for each fuzzy point x_t in X and each fuzzy m_1-open set V in Y containing $h(x_t)$, there exists an fgm-open set U in X containing x_t such that $h(U) \leq V$,
(iii) $h(fgmcl(A)) \leq m_1cl(h(A))$, for all $A \in I^X$,
(iv) $fgmcl(h^{-1}(B)) \leq h^{-1}(mclB)$, for all $B \in I^Y$.

Proof (i) $\Rightarrow$ (ii). Let x_t be a fuzzy point in X and V be any fuzzy m_1-open set in Y with $h(x_t) \in V$. Then $x_t \in h^{-1}(V)$. Let $U = h^{-1}(V)$. Then U is fgm-open in X (by (i)) with $x_t \in U$ and $h(U) \leq V$.
(ii) $\Rightarrow$ (i). Let A be any fuzzy m_1-open set in Y and x_t be a fuzzy point in X such that $x_t \in h^{-1}(A)$. Then $h(x_t) \in A$. By (ii), there exists an fgm-open set U in X with $x_t \in U$ such that $h(U) \leq A$. Then $x_t \in U \leq h^{-1}(A)$. Then $x_t \in U = fgmint(U) \leq fgmint(h^{-1}(A))$. Since x_t is taken arbitrarily and $h^{-1}(U)$ is the union of all fuzzy points in $h^{-1}(A)$, $h^{-1}(A) \leq fgmint(h^{-1}(A)) \Rightarrow h^{-1}(A)$ is fgm-open in $X \Rightarrow h$ is $fg(m, m_1)$-continuous function.

(i) $\Rightarrow$ (iii). Let $A \in I^X$. Since m_1 satisfies M-condition, then $m_1cl(h(A))$ is fuzzy m_1-closed set in Y. Now $A \leq h^{-1}(h(A)) \leq h^{-1}(m_1cl(h(A)))$ which is fgm-closed in X (by (i)) and so $fgmcl(A) \leq h^{-1}(m_1cl(h(A))) \Rightarrow h(fgmcl(A)) \leq m_1cl(h(A))$.
(iii) $\Rightarrow$ (i). Let V be a fuzzy m_1-closed set in Y. Put $U = h^{-1}(V)$. By (iii), $h(fgmcl(U)) \leq m_1cl(h(U)) = m_1cl(h(h^{-1}(V))) \leq m_1clV = V \Rightarrow fgmcl(U) \leq h^{-1}(V) = U \Rightarrow U$ is fgm-closed in $X \Rightarrow h$ is $fg(m, m_1)$-continuous function.
(iii) $\Rightarrow$ (iv). Let $B \in I^Y$ and $A = h^{-1}(B)$. Then $A \in I^X$. By (iii), $h(fgmcl(A)) \leq m_1cl(h(A)) \Rightarrow h(fgmcl(h^{-1}(B))) \leq m_1cl(h(h^{-1}(B))) \leq m_1clB \Rightarrow fgmcl(h^{-1}(B)) \leq h^{-1}(m_1clB)$.
(iv) $\Rightarrow$ (iii). Let $A \in I^X$. Then $h(A) \in I^Y$. By (iv), $fgmcl(h^{-1}(h(A))) \leq h^{-1}(m_1cl(h(A))) \Rightarrow fgmcl(A) \leq fgmcl(h^{-1}(h(A))) \leq f^{-1}(m_1cl(h(A))) \Rightarrow h(fgmcl(A)) \leq m_1cl(h(A))$.

Theorem 5.3. If $h : (X, m) \to (Y, m_1)$ is fuzzy (m, m_1)-closed, $fg(m, m_1)$-continuous, injective function where m_1 satisfies M-condition, then $h^{-1}(B)$ is fgm-closed in X for every fgm_1-closed set B in Y.

Proof. Let B be fgm_1-closed set in Y and let $h^{-1}(B) \leq U$ where U is fuzzy m-open set in X. As h is fuzzy (m, m_1)-closed, injective function, by Theorem 4.9, there exists a fuzzy m_1-open set V in Y with $B \leq V$ and $h^{-1}(V) \leq U$. As B is fgm_1-closed set in Y, $m_1clB \leq V \Rightarrow h^{-1}(m_1clB) \leq h^{-1}(V) \leq U$. Since m_1 satisfies M-condition, m_1clB is fuzzy m_1-closed set in Y. Then as h is $fg(m, m_1)$-continuous, $h^{-1}(m_1clB)$ is fgm-closed set in $X \Rightarrow mcl(h^{-1}(m_1clB)) \leq U \Rightarrow mcl(h^{-1}(B)) \leq mcl(h^{-1}(m_1clB)) \leq U \Rightarrow h^{-1}(B)$ is fgm-closed in X.

Remark 5.4. Composition of two $fg(m, m_1)$-continuous functions need not be so, as it seen from the following example.

Example 5.5. Let $X = \{a, b\}$, $m_1 = \{0_X, 1_X, A\}$, $m_2 = \{0_X, 1_X\}$, $m_3 = \{0_X, 1_X, B\}$ where $A(a) = 0.5, A(b) = 0.4$, $B(a) = 0.5, B(b) = 0.6$. Then (X, m_1), (X, m_2) and (X, m_3) are fuzzy m-spaces. Consider two identity functions $i_1 : (X, m_1) \to (X, m_2)$ and $i_2 : (X, m_2) \to (X, m_3)$. Clearly i_1 and i_2 are $fg(m_1, m_2)$-continuous and $fg(m_2, m_3)$-continuous functions respectively. Now $1_X \setminus B \in m_3^c$, $(i_2 \circ i_1)^{-1}(1_X \setminus B) = 1_X \setminus B \leq A \in m_1$. But

$m_1cl(1_X \setminus B) = 1_X \setminus A \not\leq A \Rightarrow 1_X \setminus B$ is not fgm_1-closed in $(X, m_1) \Rightarrow i_2 \circ i_1$ is not $fg(m_1, m_3)$-continuous function.

Theorem 5.6. If $h : (X, m) \to (Y, m_1)$ is $fg(m, m_1)$-continuous and $g : (Y, m_1) \to (Z, m_2)$ is fuzzy (m_1, m_2)-continuous function, then $g \circ f : (X, m_1) \to (Z, m_2)$ is $fg(m, m_2)$-continuous.
Proof. Obvious.

Theorem 5.7. If $h : (X, m) \to (Y, m_1)$ is $fg(m, m_1)$-continuous and fuzzy (m, m_1)-closed injective function and $g : (Y, m_1) \to (Z, m_2)$ is $fg(m_1, m_2)$-continuous function where m_1 satisfies M-condition, then $g \circ h : (X, m) \to (Z, m_2)$ is $fg(m, m_2)$-continuous.
Proof. Let F be a fuzzy m_2-closed set in Z. Then $g^{-1}(F)$ is fgm_1-closed in Y. By Theorem 5.3, $h^{-1}(g^{-1}(F)) = (g \circ h)^{-1}(F)$ is fgm-closed in $X \Rightarrow g \circ h$ is $fg(m, m_2)$-continuous.

6. fgm-Regular, fgm-Normal and fgm-Compact Spaces and Application of $fg(m, m_1)$-Continuous Function

Let us now recall the following definitions from [3] for ready references.

Definition 6.1 [3]. A fuzzy m-space (X, m) is said to be fuzzy m-regular space if for any fuzzy point x_α in X and each fuzzy m-closed set F with $x_\alpha \notin F$, there exist two fuzzy m-open sets U, V in X such that $x_\alpha \in U, F \leq V$ and $U \not q V$.

Definition 6.2 [3]. A fuzzy m-space (X, m) is said to be fuzzy m-normal if for each pair of fuzzy m-closed sets A, B in X with $A \not q B$, there exist two fuzzy m-open sets U, V in X such that $A \leq U, B \leq V$ and $U \not q V$.

Let us now introduce the following concept.

Definition 6.3. A fuzzy set A in a fuzzy m-space (X, m) is called a fuzzy generalized m-open q-nbd (fgm-open q-nbd, for short) of a fuzzy point x_α if there is a fuzzy m-open set U in X such that $x_\alpha q U$.

Definition 6.4. A fuzzy m-space (X, m) is said to be fuzzy generalized m-regular (fgm-regular, for short) space if for any fuzzy point x_t in X and each fgm-closed set F with $x_t \notin F$, there exist two fuzzy m-open sets U, V in X such that $x_t \in U, F \leq V$ and $U \not{q} V$.

Theorem 6.5. In a fuzzy m-space (X, m) where m satisfies M-condition, then the following statements are equivalent:
(i) X is fgm-regular,
(ii) for each fuzzy point x_t in X and any fgm-open q-nbd U of x_t, there exists a fuzzy m-open set V in X such that $x_t \in V$ and $mclV \leq U$,
(iii) for each fuzzy point x_t in X and each fgm-closed set A of X with $x_t \notin A$, there exists a fuzzy m-open set U in X with $x_t \in U$ such that $mclU \not{q} A$.

Proof (i) $\Rightarrow$ (ii). Let x_t be a fuzzy point in X and U, any fgm-open q-nbd of x_t. Then $x_t q U \Rightarrow U(x) + t > 1 \Rightarrow x_t \notin 1_X \setminus U$ which is fgm-closed in X. By (i), there exist two fuzzy m-open sets V, W in X such that $x_t \in V, 1_X \setminus U \leq W$ and $V \not{q} W$. Then $V \leq 1_X \setminus W \Rightarrow mclV \leq mcl(1_X \setminus W) = 1_X \setminus W \leq U$.
(ii) $\Rightarrow$ (iii). Let x_t be a fuzzy point in X and A, an fgm-closed set in X with $x_t \notin A$. Then $A(x) < t \Rightarrow x_t q(1_X \setminus A)$ which is fgm-open set in X and so $1_X \setminus A$ is fgm-open q-nbd of x_t. By (ii), there exists a fuzzy m-open set V in X such that $x_t \in V$ and $mclV \leq 1_X \setminus A \Rightarrow mclV \not{q} A$.
(iii) $\Rightarrow$ (i). Let x_t be a fuzzy point in X and F be any fgm-closed set in X with $x_t \notin F$. Then by (iii), there exists a fuzzy m-open set U in X such that $x_t \in U$ and $mclU \not{q} F \Rightarrow F \leq 1_X \setminus mclU$ (=W, say). Then W is fuzzy m-open in X (as m satisfies M-condition) and $U \not{q} W$ (as $U \not{q}(1_X \setminus mclU)$) and so X is fgm-regular space.

Remark 6.6. It is clear from definitions that fgm-regular space is fuzzy m-regular.

Theorem 6.7. Let $h : (X, m) \to (Y, m_1)$ be $fg(m, m_1)$-continuous, fuzzy (m, m_1)-open, bijective function from an fgm-regular space X onto Y. Then Y is fuzzy m-regular.

Proof. Let y_α be any fuzzy point in Y and F be any fuzzy m_1-closed set in Y with $y_\alpha \notin F$. Then there exists unique $x \in X$ such that $h(x) = y$ (as h is bijective). Now $x_\alpha \notin h^{-1}(F)$ where $h^{-1}(F)$ is fgm-closed set in X as h is $fg(m, m)$-continuous function. As X is fgm-regular, there exist two fuzzy m-open sets U, V in X such that $x_\alpha \in U$, $h^{-1}(F) \leq V$ and $U \not{q} V$. As h is fuzzy (m, m_1)-open function, $h(U)$ and $h(V)$ are fuzzy m_1-open sets in Y. Then $h(x_\alpha) \in h(U)$,

$h(h^{-1}(F)) = F$ (as h is bijective) $\leq h(V)$ and $h(U) \not{q} h(V)$ which shows that Y is fuzzy m-regular.

Definition 6.8. A fuzzy m-space (X, m) is called fuzzy generalized m-normal (fgm-normal, for short) if for each pair of fgm-closed sets A, B with $A \not{q} B$, there exist two fuzzy m-open sets U, V in X such that $A \leq U, B \leq V$ and $U \not{q} V$.

Remark 6.9. It is clear that fgm-normal space is fuzzy m-normal.

Theorem 6.10. A fuzzy m-space (X, m) is fgm-normal where m satisfies M-condition if and only if for every fgm-closed set F and fgm-open set G with $F \leq G$, there exists a fuzzy m-open set H in X such that $F \leq H \leq mclH \leq G$.

Proof. Let X be fgm-normal. Let F be fgm-closed set and G be fgm-open set with $F \leq G$. Then $F \not{q} (1_X \setminus G)$ where $1_X \setminus G$ is fgm-closed in X. By hypothesis, there exist two fuzzy m-open sets H, T in X such that $F \leq H, 1_X \setminus G \leq T$ and $H \not{q} T$. Then $H \leq 1_X \setminus T \Rightarrow mclH \leq mcl(1_X \setminus T) = 1_X \setminus T \leq G \Rightarrow F \leq H \leq mclH \leq G$.

Conversely, let A, B be two fgm-closed sets in X with $A \not{q} B$. Then $A \leq 1_X \setminus B$. By hypothesis, there exists a fuzzy m-open set H in X such that $A \leq H \leq mclH \leq 1_X \setminus B$. So $B \leq 1_X \setminus mclH = mint(1_X \setminus H)(= U$, say). Then as m satisfies M-condition, U is fuzzy m-open in X. So $A \leq H, B \leq U$ and $H \not{q} U$ (as $H \not{q} (1_X \setminus mclH)$). Hence X is fgm-normal.

Theorem 6.11. Let $h : (X, m) \to (Y, m_1)$ be an $fg(m, m_1)$-continuous, fuzzy (m, m_1)-open, bijective function from an fgm-normal space X onto Y. Then Y is fuzzy m-normal.

Proof. Let A, B be two fuzzy m-closed sets in Y with $A \not{q} B$. As h is $fg(m, m_1)$-continuous function, $h^{-1}(A), h^{-1}(B)$ are fgm-closed sets in X with $h^{-1}(A) \not{q} h^{-1}(B)$. Since X is fgm-normal, there exist two fuzzy m-open sets U, V in X such that $h^{-1}(A) \leq U, h^{-1}(B) \leq V$ and $U \not{q} V$. As h is fuzzy (m, m_1)-open function, $h(U), h(V)$ are fuzzy m_1-open sets in Y. Then $A \leq h(U), B \leq h(V)$ (as h is bijective) and $h(U) \not{q} h(V)$ which proves that Y is fuzzy m-normal space.

Let us now introduce fuzzy generalized m-irresolute function under which fgm-regularity and fgm-normality remain invariant.

Definition 6.12. A function $f : (X, m) \to (Y, m_1)$ is said to be fuzzy generalized (m, m_1)-irresolute ($fg(m, m_1)$-irresolute, for short) if $f^{-1}(V)$ is fgm-closed in X for all fgm_1-closed set V in Y.

We now state the following two theorems the proofs of which are similar to that of Theorem 6.7 and Theorem 6.11 respectively.

Theorem 6.13. Let $h : (X, m) \to (Y, m_1)$ be an $fg(m, m)$-irresolute, fuzzy (m, m_1)-open, bijective function from an fgm-regular space X onto Y. Then Y is fgm-regular.

Theorem 614. Let $h : (X, m) \to (Y, m_1)$ be an $fg(m, m)$-irresolute, fuzzy (m, m_1)-open, bijective function from an fgm-normal space X onto Y. Then Y is fgm-normal.

Remark 6.15. It is clear that $fg(m, m_1)$-irresolute function is $fg(m, m_1)$-continuous function, but not conversely follows from the following example.

Example 6.16. $fg(m, m_1)$-continuity $\not\Rightarrow fg(m, m_1)$-irresoluteness

Let $X = \{a, b\}$, $m = \{0_X, 1_X, A\}$, $m_1 = \{0_X, 1_X\}$ where $A(a) = 0.5, A(b) = 0.4$. Then (X, m) and (X, m_1) are fuzzy m-spaces. Consider the identity function $i : (X, m) \to (X, m_1)$. Then as 0_X and 1_X are the only fuzzy m-closed sets in (X, m_1), i is clearly $fg(m, m_1)$-continuous function. Now A is fgm_1-closed in (X, m_1). $i^{-1}(A) = A \leq A \in m$. But $mclA = 1_X \setminus A \not\leq A \Rightarrow A$ is not fgm-closed in (X, m). Consequently, i is not $fg(m, m_1)$-irresolute.

Definition 6.17 [5]. Let A be a fuzzy set in a non-empty set X. A collection $\mathcal{U}$ of fuzzy sets in X is called a fuzzy cover of A if $sup\{U(x) : U \in \mathcal{U}\} = 1$, for each $x \in suppA$. If, in addition, $A = 1_X$, we get the definition of fuzzy cover of X.

Definition 6.18 [5, 6]. A fuzzy cover $\mathcal{U}$ of a fuzzy set A in a non-empty set X is said to have a finite subcover $\mathcal{U}_0$, if $\mathcal{U}_0$ is a finite subcollection of $\mathcal{U}$ such that $\bigcup \mathcal{U}_0 \geq A$. If, in particular, $A = 1_X$, then the requirement on $\mathcal{U}_0$ is $\bigcup \mathcal{U}_0 = 1_X$.

Definition 6.19 [3]. A fuzzy set A in a fuzzy m-space (X, m) is said to be fuzzy m-compact if every fuzzy covering $\mathcal{U}$ of A by fuzzy m-open sets of X has a finite subcovering $\mathcal{U}_0$. In particular, if

$A = 1_X$, we get the definition of fuzzy m-compact space.

Let us now introduce the following concept.

Definition 6.20. A fuzzy set A in a fuzzy m-space (X, m) is said to be fuzzy generalized m-compact (fgm-compact, for short) if every fuzzy covering $\mathcal{U}$ of A by fgm-open sets of X has a finite subcovering $\mathcal{U}_0$. In particular, if $A = 1_X$, we get the definition of fgm-compact space.

Remark 6.21. It is clear from definitions that fgm-compact space is fuzzy m-compact. But as fgm-open set may not be fuzzy m-open, the converse may not hold, in general.

Theorem 6.22. Every fgm-closed set in an fgm-compact space is fgm-compact.

Proof. Let A be an fgm-closed set in an fgm-compact space (X, m). Let $\mathcal{U}$ be a fuzzy cover of A by fgm-open sets of X. Then $\mathcal{U} \bigcup (1_X \setminus A)$ ($=\mathcal{V}$, say) is an fgm-open cover of X. As X is fgm-compact space, there exists a finite subcollection $\mathcal{V}_0$ of $\mathcal{V}$ which also covers X. If $\mathcal{V}_0$ contains $1_X \setminus A$, we omit it and get a finite subcovering of A. Consequently, A is fgm-compact.

Theorem 6.23. Let $h : (X, m) \to (Y, m_1)$ be an $fg(m, m_1)$-continuous function. If a fuzzy set A is fgm-compact relative to X, then $h(A)$ is fuzzy m_1-compact relative to Y.

Proof. Let $\mathcal{U} = \{U_\alpha : \alpha \in \Lambda\}$ be a fuzzy cover of $h(A)$ by fuzzy m_1-open sets of Y. Then $h(A) \leq \bigcup\limits_{\alpha \in \Lambda} U_\alpha \Rightarrow A \leq h^{-1}(\bigcup\limits_{\alpha \in \Lambda} U_\alpha) = \bigcup\limits_{\alpha \in \Lambda} h^{-1}(U_\alpha)$. Let $\mathcal{V} = \{h^{-1}(U_\alpha) : \alpha \in \Lambda\}$. Then $\mathcal{V}$ is a fuzzy cover of A by fgm-open sets of X (as h is $fg(m, m_1)$-continuous function). Since A is fgm-compact relative to X, there exists a finite subset Λ_0 of Λ such that $\mathcal{V}_0 = \{h^{-1}(U_\alpha) : \alpha \in \Lambda_0\}$ is again a fuzzy cover of A. Then $A \leq \bigcup\limits_{\alpha \in \Lambda_0} h^{-1}(U_\alpha) \Rightarrow h(A) \leq h(\bigcup\limits_{\alpha \in \Lambda_0} h^{-1}(U_\alpha)) = \bigcup\limits_{\alpha \in \Lambda_0} h(h^{-1}(U_\alpha)) \leq \bigcup\limits_{\alpha \in \Lambda_0} U_\alpha)$ which shows that $h(A)$ is fuzzy m-compact set relative to Y.

In a similar manner we can prove the following theorem easily.

Theorem 6.24. Let $h : (X, m) \to (Y, m_1)$ be an $fg(m, m_1)$-irresolute function. If a fuzzy set A is fgm-compact relative to X, then $h(A)$ is fgm_1-compact relative to Y.

References

[1] Alimohammady, M. and Roohi, M.; **Fuzzy minimal structure and fuzzy minimal vector spaces**, *Chaos, Solitons and Fractals*, **27** (2006), 599-605.

[2] Bhattacharyya, Anjana; **Fuzzy upper and lower M-continuous multifunctions**, *Scientific Studies and Research, Series Mathematics and Informatics*, **25** (**1**) (2015), 125-144.

[3] Bhattacharyya, Anjana; **Several concepts of continuity in fuzzy m-space**, *Annals of Fuzzy Mathematics and Informatics*, **13** (**2**) (2017), 5-21.

[4] Brescan, M.; **On quasi-irresolute function in fuzzy minimal structures**, Bul. Univ. "Petrol-Gaze" Ploieşti, Vol. LXII, (**No. 1**) (2010), 19-25.

[5] Chang, C.L.; **Fuzzy topological spaces**, *J. Math. Anal. Appl.*, **24** (1968), 182-190.

[6] Ganguly, S. and Saha, S.; **A note on compactness in fuzzy setting**, *Fuzzy Sets and Systems*, **34** (1990), 117-124.

[7] Nematollahi, M.J. and Roohi, M.; **Fuzzy minimal structures and fuzzy minimal subspaces**, Italian Journal of Pure and Applied Mathematics **27** (2010), 147-156.

[8] Pu, Pao Ming and Liu, Ying Ming; **Fuzzy topology I. Neighbourhood structure of a fuzzy point and Moore-Smith convergence**, *Jour. Math. Anal. Appl.*, **76** (1980), 571-599.

[9] Zadeh, L.A.; **Fuzzy Sets**, *Inform. Control*, **8** (1965), 338-353.

Victoria Institution (College),
Department of Mathematics,
78B, A.P.C. Road,
Kolkata-700009, India
e-mail: anjanabhattacharyya@hotmail.com