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ON SOME FORMS OF OPEN FUNCTIONS IN IDEAL TOPOLOGICAL SPACES

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Abstract. We introduce the notion of $mIO(X)$-structures determined by operators Int, Cl and Cl* on an ideal topological space $(X, \tau, \mathcal{I})$. By using $mIO(X)$-structures, we introduce and investigate a function $f : (X, \tau) \to (Y, \sigma, I)$ called mI-open. As special cases of mI-open functions, we obtain properties of semi-I-open [8], pre-I-open [1], α-I-open [1], b-I-open [2], β-I-open [1], weakly semi-I-open [5], and weakly-b-I-open [13] functions.

1. Introduction

The notion of the ideal topological spaces is introduced in [11] and [19]. In [10] the authors introduced the notion of I-open sets in an ideal topological space. As generalizations of I-open sets, the notions of semi-I-open sets [7], pre-I-open sets [4], α-I-open sets [7], β-I-open sets [7] are introduced and used to obtain decompositions of continuity. Recently, these sets are used in various generalizations of open-like functions from a topological space (X, τ) to an ideal topological space (Y, σ, I). In [16] and [17], the present authors introduced and studied the notions of minimal structures and m-spaces as a generalization of topological spaces.

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In this paper, we introduce the notion of $mIO(X)$-structures determined by operators Int, Cl and $\mathrm{Cl}^\star$ on an ideal topological space $(X, \tau, \mathcal{I})$. By using $mIO(X)$-structures, we introduce and investigate a function $f : (X, \tau) \to (Y, \sigma, I)$ called mI-open. As special cases of mI-open functions, we obtain semi-I-open functios [8], pre-I-open functions [1], α-I-open functions [1], b-I-open functions [2], β-I-open functions [1], weakly semi-I-open functions [5], and weakly b-I-open functions [13].

2. Preliminaries

Definition 2.1. Let X be a nonempty set and $\mathcal{P}(X)$ the power set of X. A subfamily m_X of $\mathcal{P}(X)$ is called a *minimal structure* (briefly *m-structure*) on X [16], [17] if $\emptyset \in m_X$ and $X \in m_X$.

By (X, m_X), we denote a nonempty set X with an m-structure m_X on X and call it an m-space. Each member of m_X is said to be *m_X-open* (briefly m-open) and the complement of an m_X-open set is said to be *m_X-closed* (briefly m-closed). Hearafter, m_X is briefly denoted by m.

Definition 2.2. Let X be a nonempty set and m an m-structure on X. For a subset A of X, the *m-closure* of A and the *m-interior* of A are defined in [12] as follows:

$\mathrm{mCl}(A) = \cap\{F : A \subset F, X \setminus F \in m\}$ and $\mathrm{mInt}(A) = \cup\{U : U \subset A, U \in m\}$.

Lemma 2.1. [12] *Let (X, m) be an m-space. For subsets A and B of X, the following properties hold:*

(1) $\mathrm{mCl}(X \setminus A) = X \setminus \mathrm{mInt}(A)$ *and* $\mathrm{mInt}(X \setminus A) = X \setminus \mathrm{mCl}(A)$,

(2) If $(X \setminus A) \in m$, *then* $\mathrm{mCl}(A) = A$ *and if* $A \in m$, *then* $\mathrm{mInt}(A) = A$,

(3) $\mathrm{mCl}(\emptyset) = \emptyset$, $\mathrm{mCl}(X) = X$, $\mathrm{mInt}(\emptyset) = \emptyset$ *and* $\mathrm{mInt}(X) = X$,

(4) If $A \subset B$, *then* $\mathrm{mCl}(A) \subset \mathrm{mCl}(B)$ *and* $\mathrm{mInt}(A) \subset \mathrm{mInt}(B)$,

(5) $\mathrm{mInt}(A) \subset A \subset \mathrm{mCl}(A)$,

(6) $\mathrm{mCl}(\mathrm{mCl}(A)) = \mathrm{mCl}(A)$ *and* $\mathrm{mInt}(\mathrm{mInt}(A)) = \mathrm{mInt}(A)$.

Lemma 2.2. [16] *Let (X, m) be an m-space and A a subset of X. Then $x \in \mathrm{mCl}(A)$ if and only if $U \cap A \neq \emptyset$ for each $U \in m$ containing x.*

Definition 2.3. A minimal structure m on a nonempty set X is said to have *property $\mathcal{B}$* [12] if the union of any family of subsets belonging to m belongs to m.

Lemma 2.3. [18] *Let X be a nonempty set and m an m-structure on X satisfying property $\mathcal{B}$. For a subset A of X, the following properties hold:*

(1) $A \in m$ if and only if $\mathrm{mInt}(A) = A$,

(2) A is m-closed if and only if $\mathrm{mCl}(A) = A$,

(3) $\mathrm{mInt}(A) \in m$ and $\mathrm{mCl}(A)$ is m-closed.

Definition 2.4. [15] Let (X, τ) be a topological space and (Y, m) an m-space. A function $f : (X, \tau) \to (Y, m)$ is said to be *m-open* at $x \in X$ if for each open set U of X containing x, there exists $V \in m$ containing $f(x)$ such that $V \subset f(U)$. If f is m-open at each $x \in X$, f is said to be *m-open*.

Theorem 2.1. [15] *For a function $f : (X, \tau) \to (Y, m)$, the following properties are equivalent:*

(1) f is m-open at x;

(2) for each open set U of X containing x, $x \in f^{-1}(\mathrm{mInt}(f(U))$;

(3) If $x \in \mathrm{Int}(A)$ for $A \in \mathcal{P}(X)$, then $x \in f^{-1}(\mathrm{mInt}(f(A)))$;

(4) $x \in \mathrm{Int}(f^{-1}(B))$ for $B \in \mathcal{P}(Y)$, then $x \in f^{-1}(\mathrm{mInt}(B))$;

(5) If $x \in f^{-1}(\mathrm{mCl}(B))$ for $B \in \mathcal{P}(Y)$, then $x \in \mathrm{Cl}(f^{-1}(B))$.

Theorem 2.2. [15] *A function $f : (X, \tau) \to (Y, m)$ is m-open if and only if $f(U) = \mathrm{mInt}(f(U))$ for each open set U of X.*

Remark 2.1. If m has property $\mathcal{B}$, by Lemma 2.3 we obtain that a function $f : (X, \tau) \to (Y, m)$ is m-open if and only if $f(U)$ is m-open for each open set U of X.

Theorem 2.3. [14] *For a function $f : (X, \tau) \to (Y, m_Y)$, the following properties are equivalent:*

(1) f is m-open;

(2) $f(\mathrm{Int}(A)) \subset \mathrm{mInt}(f(A))$ for every subset A of X;

(3) $\mathrm{Int}(f^{-1}(B)) \subset f^{-1}(\mathrm{mInt}(B))$ for every subset B of Y;

(4) $f^{-1}(\mathrm{mCl}(B)) \subset \mathrm{Cl}(f^{-1}(B))$ for every subset B of Y;

(5) for each $x \in X$ and each open set U containing x, there exists an m-open set V containing $f(x)$ such that $V \subset f(U)$.

Theorem 2.4. [14] *For a function $f : (X, \tau) \to (Y, m)$, where m has property ($\mathcal{B}$), the following properties are equivalent:*

(1) f is m-open;

(2) for any subset S of Y and each closed set A of X containing $f^{-1}(S)$, there exists an m-closed set B of Y containing S such that $f^{-1}(B) \subset A$.

For a function $f : (X, \tau) \to (Y, m)$, we denote

$$D_0(f) = \{x \in X\colon f \textit{ is not m-open at } x\}.$$

Theorem 2.5. [15] *For a function $f : (X, \tau) \to (Y, m)$, the following properties hold:*
$D_0(f) = \cup\{U \setminus f^{-1}(\mathrm{mInt}(f(U))) : U \in \tau\}$
$= \cup\{\mathrm{Int}(A) \setminus f^{-1}(\mathrm{mInt}(f(A))) : A \in \mathcal{P}(X)\}$
$= \cup\{\mathrm{Int}(f^{-1}(B)) \setminus f^{-1}(\mathrm{mInt}(B)) : B \in \mathcal{P}(Y)\}$
$= \cup\{f^{-1}(\mathrm{mCl}(B)) \setminus \mathrm{Cl}(f^{-1}(B)) : B \in \mathcal{P}(Y)\}$.

3. Ideal topological spaces

Let (X, τ) be a topological space. The notion of ideals has been introduced in [11] and [19] and further investigated in [10].

Definition 3.1. A nonempty collection I of subsets of a set X is called an *ideal on X* if it satisfies the following two conditions:
(1) $A \in I$ and $B \subset A$ implies $B \in I$,
(2) $A \in I$ and $B \in I$ implies $A \cup B \in I$.

A topological space (X, τ) with an ideal I on X is called an ideal topological space and is denoted by (X, τ, I). Let (X, τ, I) be an ideal topological space. For any subset A of X, $A^\star(I, \tau) = \{x \in X : U \cap A \notin I$ for every $U \in \tau(x)\}$, where $\tau(x) = \{U \in \tau : x \in U\}$, is called the local function of A with respect to τ and I [10]. Hereafter $A^\star(I, \tau)$ is simply denoted by $A^\star$. It is well known that $\mathrm{Cl}^\star(A) = A \cup A^\star$ defines a Kuratowski closure operator on X and the topology generated by $\mathrm{Cl}^\star$ is denoted by $\tau^\star$.

Lemma 3.1. [10] *Let (X, τ, I) be an ideal topological space and A, B be subsets of X. Then the following properties hold:*
(1) $A \subset B$ implies $\mathrm{Cl}^\star(A) \subset \mathrm{Cl}^\star(B)$,
(2) $\mathrm{Cl}^\star(X) = X$ and $\mathrm{Cl}^\star(\emptyset) = \emptyset$,
(3) $\mathrm{Cl}^\star(A) \cup \mathrm{Cl}^\star(B) \subset \mathrm{Cl}^\star(A \cup B)$.

Definition 3.2. Let (X, τ, I) be an ideal topological space. A subset A of X is said to be
(1) *α-I-open* [7] if $A \subset \mathrm{Int}(\mathrm{Cl}^\star(\mathrm{Int}(A)))$,
(2) *semi-I-open* [7] if $A \subset \mathrm{Cl}^\star(\mathrm{Int}(A))$,
(3) *pre-I-open* [4] if $A \subset \mathrm{Int}(\mathrm{Cl}^\star(A))$,
(4) *b-I-open* [3] if $A \subset \mathrm{Int}(\mathrm{Cl}^\star(A)) \cup \mathrm{Cl}^\star(\mathrm{Int}(A))$,
(5) *β-I-open* [7] if $A \subset \mathrm{Cl}(\mathrm{Int}(\mathrm{Cl}^\star(A)))$,
(6) *weakly semi-I-open* [5] if $A \subset \mathrm{Cl}^\star(\mathrm{Int}(\mathrm{Cl}(A)))$,
(7) *weakly b-I-open* [13] if $A \subset \mathrm{Cl}(\mathrm{Int}(\mathrm{Cl}^\star(A))) \cup \mathrm{Cl}^\star(\mathrm{Int}(\mathrm{Cl}(A)))$,
(8) *strongly β-I-open* [6] if $A \subset \mathrm{Cl}^\star(\mathrm{Int}(\mathrm{Cl}^\star(A)))$.

The family of all α-I-open (resp. semi-I-open, pre-I-open, b-I-open, β-I-open, weakly semi-I-open, weakly b-I-open, strongly β-I-open) sets in an ideal topological space (X, τ, I) is denoted by $\alpha\text{IO}(X)$ (resp. $\text{SIO}(X)$, $\text{PIO}(X)$, $\text{BIO}(X)$, $\beta\text{IO}(X)$, $\text{WSIO}(X)$, $\text{WBIO}(X)$, $\text{S}\beta\text{IO}(X)$).

Definition 3.3. By $mIO(X)$, we denote each one of the families $\alpha\text{IO}(X)$, $\text{SIO}(X)$, $\text{PIO}(X)$, $\text{BIO}(X)$, $\beta\text{IO}(X)$, $\text{WSIO}(X)$, $\text{WBIO}(X)$, $\text{S}\beta\text{IO}(X)$.

Lemma 3.2. *Let (X, τ, I) be an ideal topological space. Then $mIO(X)$ is a minimal structure and has property $\mathcal{B}$.*

Proof. The proof follows from Lemma 2.1(3)(4) and Lemma 3.1(1)(2).

Remark 3.1. It is shown in Proposition 3.2 of [1] (resp. Theorem 3.4 of [8], Theorem 2.10 of [4], Proposition 3.10 of [2], Theorem 3.2 of [9], Theorem 2.1 of [5], Theorem 2.7 of [13], Proposition 3 of [6]) that $\alpha\text{IO}(X)$ (resp. $\text{SIO}(X)$, $\text{PIO}(X)$, $\text{BIO}(X)$, $\beta\text{IO}(X)$, $\text{WSIO}(X)$, $\text{WBIO}(X)$, $\text{S}\beta\text{IO}(X)$) has property $\mathcal{B}$.

Definition 3.4. Let (X, τ, I) be an ideal topological space. For a subset A of X, $\text{mCl}_\text{I}(A)$ and $\text{mInt}_\text{I}(A)$ are defined as follows:
(1) $\text{mCl}_\text{I}(A) = \cap\{F : A \subset F, X \setminus F \in mIO(\text{X})\}$,
(2) $\text{mInt}_\text{I}(A) = \cup\{U : U \subset A, U \in mIO(\text{X})\}$.

Let (X, τ, I) be an ideal topological space and $mIO(X)$ the m-structure on X. If $mIO(X) = \alpha\text{IO}(X)$ (resp. $\text{SIO}(X)$, $\text{PIO}(X)$, $\text{BIO}(X)$, $\beta\text{IO}(X)$, $\text{WSIO}(X)$, $\text{WBIO}(X)$, $\text{S}\beta\text{IO}(X)$), then we have
(1) $\text{mCl}_\text{I}(A) = \alpha\text{Cl}_\text{I}(A)$ (resp. $\text{sCl}_\text{I}(A)$, $\text{pCl}_\text{I}(A)$, $\text{bCl}_\text{I}(A)$, $\beta\text{Cl}_\text{I}(A)$, $\text{wsCl}_\text{I}(A)$, $\text{wbCl}_\text{I}(A)$, $\text{s}\beta\text{Cl}_\text{I}(A)$,
(2) $\text{mInt}_\text{I}(A) = \alpha\text{Int}_\text{I}(A)$ (resp. $\text{sInt}_\text{I}(A)$, $\text{pInt}_\text{I}(A)$, $\text{bInt}_\text{I}(A)$, $\beta\text{Int}_\text{I}(A)$, $\text{wsInt}_\text{I}(A)$, $\text{wbInt}_\text{I}(A)$, $\text{s}\beta\text{Int}_\text{I}(A)$.

4. mI-OPEN FUNCTIONS

Definition 4.1. Let $f : (X, \tau) \to (Y, \sigma, I)$ be a function, where (Y, σ, I) be an ideal topological space. A function $f : (X, \tau) \to (Y, \sigma, I)$ is said to be *α-I-open* [1] (resp. *semi-I-open* [8], *pre-I-open* [1], *b-I-open* [2], *β-I-open* [1], *weakly semi-I-open* [5], *weakly b-I-open* [13]) if $f(U)$ is α-I-open (resp. semi-I-open, pre-I-open, b-I-open, β-I-open, weakly semi-I-open, weakly b-I-open) in (Y, σ, I) for every open set U of (X, τ).

Definition 4.2. A function $f : (X, \tau) \to (Y, \sigma, I)$ is said to be *mI-open* if $f(U)$ is mI-open in (Y, σ, I) for every open set U of (X, τ).

By Remark 2.1, $f : (X, \tau) \to (Y, \sigma, I)$ is mI-open if and only if $f : (X, \tau) \to (Y, mIO(Y))$ is m-open. In the following theorems, if we put $mIO(Y) = \alpha\mathrm{IO}(Y)$ (resp. $\mathrm{SIO}(Y)$, $\mathrm{PIO}(Y)$, $\mathrm{BIO}(Y)$, $\beta\mathrm{IO}(Y)$, $\mathrm{WSIO}(Y)$, $\mathrm{WBIO}(Y)$, $\mathrm{S}\beta\mathrm{IO}(Y)$), then we can obtain the characterizations of α-I-open (resp. semi-I-open, pre-I-open, b-I-open, β-I-open, weakly semi-I-open, weakly b-I-open, strongly β-I-open) functions.

By Theorem 2.1, we have the following characterizations of mI-open functions.

Theorem 4.1. *For a function $f : (X, \tau) \to (Y, \sigma, I)$, the following properties are equivalent:*
(1) f is mI-open at x;
(2) for each open set U of X containing x, $x \in f^{-1}(\mathrm{mInt_I}(f(U))$;
(3) If $x \in \mathrm{Int}(A)$ for $A \in \mathcal{P}(X)$, then $x \in f^{-1}(\mathrm{mInt_I}(f(A)))$;
(4) $x \in \mathrm{Int}(f^{-1}(B))$ for $B \in \mathcal{P}(Y)$, then $x \in f^{-1}(\mathrm{mInt_I}(B))$;
(5) If $x \in f^{-1}(\mathrm{mCl_I}(B))$ for $B \in \mathcal{P}(Y)$, then $x \in \mathrm{Cl}(f^{-1}(B))$.

By Lemma 3.2, $mIO(X)$ has property $\mathcal{B}$ and by Theorem 2.2 and Lemma 2.3 we obtain the following theorem.

Theorem 4.2. *A function $f : (X, \tau) \to (Y, \sigma, I)$ is mI-open if and only if $f(U)$ is mI-open in Y for each open set U of X.*

Let $mIO(Y) = \mathrm{SIO}(Y)$, then we obtain the characterizations of semi-I-open functions.

Corollary 4.1. *A function $f : (X, \tau) \to (Y, \sigma, I)$ is semi-I-open if and only if $f(U)$ is semi-I-open in Y for each open set U of X.*

By Theorem 2.3, we obtain the following theorem.

Theorem 4.3. *For a function $f : (X, \tau) \to (Y, \sigma, I)$, the following properties are equivalent:*
(1) f is mI-open;
(2) $f(\mathrm{Int}(A)) \subset \mathrm{mInt_I}(f(A))$ for every subset A of X;
(3) $\mathrm{Int}(f^{-1}(B)) \subset f^{-1}(\mathrm{mInt_I}(B))$ for every subset B of Y;
(4) $f^{-1}(\mathrm{mCl_I}(B)) \subset \mathrm{Cl}(f^{-1}(B))$ for every subset B of Y;
(5) for each $x \in X$ and each open set U containing x, there exists an mI-open set V containing f(x) such that $V \subset f(U)$.

Let $mIO(Y) = \mathrm{PIO}(Y)$, then we obtain the characterizations of pre-I-open functions.

Corollary 4.2. *For a function $f : (X, \tau) \to (Y, \sigma, I)$, the following properties are equivalent:*
(1) f is pre-I-open;
(2) $f(\mathrm{Int}(A)) \subset \mathrm{pInt_I}(f(A))$ for every subset A of X;
(3) $\mathrm{Int}(f^{-1}(B)) \subset f^{-1}(\mathrm{pInt_I}(B))$ for every subset B of Y;
(4) $f^{-1}(\mathrm{pCl_I}(B)) \subset \mathrm{Cl}(f^{-1}(B))$ for every subset B of Y;
(5) for each $x \in X$ and each open set U containing x, there exists a pre-I-open set V containing f(x) such that $V \subset f(U)$.

Remark 4.1. By Theorem 4.3(5), we obtain a characterization of semi-I-open (resp. b-I-open, weakly semi-I-open, weakly b-I-open) functions in Theorem 5.1 of [8] (resp. Theorem 4.31 of [2], Theorem 3.1 of [5], Theorem 4.5 of [13]).

Theorem 4.4. *For a function $f : (X, \tau) \to (Y, \sigma, I)$, the following properties are equivalent:*
(1) f is mI-open;
(2) for any subset S of Y and each closed set F of X containing $f^{-1}(S)$, there exists an mI-closed set H of Y containing S such that $f^{-1}(H) \subset F$.

Let $mIO(Y) = \beta\mathrm{IO}(Y)$, then we obtain the characterizations of β-I-open functions.

Corollary 4.3. *For a function $f : (X, \tau) \to (Y, \sigma, I)$, the following properties are equivalent:*
(1) f is β-I-open;
(2) for any subset S of Y and each closed set F of X containing $f^{-1}(S)$, there exists an β-I-closed set H of Y containing S such that $f^{-1}(H) \subset F$.

Remark 4.2. By Theorem 4.4, we obtain a characterization of α-I-open (resp. semi-I-open, pre-I-open, b-I-open, β-I-open, weakly semi-I-open, weakly b-I-open) functions in Theorem 5.2 of [1] (resp. Theorem 5.3 of [1], Theorem 5.3 of [1], Theorem 4.32 of [2], Theorem 5.3 of [1], Theorem 3.2 of [5], Theorem 4.6 of [13]).

For a function $f : (X, \tau) \to (Y, \sigma, I)$, we denote

$$D_{mIO}(f) = \{x \in X\colon f \textit{ is not mI-open at } x\}.$$

By Theorem 2.5, we obtain the following theorem.

Theorem 4.5. *For a function $f : (X, \tau) \to (Y, \sigma, I)$, the following properties hold:*

$$D_{mIO}(f) = \cup\{U \setminus f^{-1}(\mathrm{mInt_I}(f(U))) : U \in \tau\}$$
$$= \cup\{\mathrm{Int}(A) \setminus f^{-1}(\mathrm{mInt_I}(f(A))) : A \in \mathcal{P}(X)\}$$
$$= \cup\{\mathrm{Int}(f^{-1}(B)) \setminus f^{-1}(\mathrm{mInt_I}(B)) : B \in \mathcal{P}(Y)\}$$
$$= \cup\{f^{-1}(\mathrm{mCl_I}(B)) \setminus \mathrm{Cl}(f^{-1}(B)) : B \in \mathcal{P}(Y)\}.$$

Let $mIO(Y) = \mathrm{BIO}(Y)$, then we obtain the properties of b-I-open functions.

Corollary 4.4

Corollary 4.4. *For a function $f : (X, \tau) \to (Y, \sigma, I)$, the following properties hold:*

$$D_{BIO}(f) = \cup\{U \setminus f^{-1}(\mathrm{bInt_I}(f(U))) : U \in \tau\}$$
$$= \cup\{\mathrm{Int}(A) \setminus f^{-1}(\mathrm{bInt_I}(f(A))) : A \in \mathcal{P}(X)\}$$
$$= \cup\{\mathrm{Int}(f^{-1}(B)) \setminus f^{-1}(\mathrm{bInt_I}(B)) : B \in \mathcal{P}(Y)\}$$
$$= \cup\{f^{-1}(\mathrm{bCl_I}(B)) \setminus \mathrm{Cl}(f^{-1}(B)) : B \in \mathcal{P}(Y)\}.$$

References

[1] A. Açikgöz, T. Noiri and Ş. Yüksel, *On α-I-continuous functions and α-I-open functions*, Acta Math. Hungar. **105**(1-2) (2004), 27–37.

[2] M. Akdag, *On b-I-open sets and b-I-continuous functions*, Int. J. Math. Math. Sci. Volume 2007, Article ID 75721.

[3] G. Aslim and A. Caksu Guler, *b-I-open sets and decompositions of continuity via idealization*, Proc. Inst. Math. Mech. Nat. Acad. Sci. Azerbaijan **22** (2003), 27–32.

[4] J. Dontchev, *On pre-$\mathcal{I}$-open sets and a decomposition of $\mathcal{I}$-continuity*, Banyan Math. J. **2** (1996).

[5] E. Hatir and S. Jafari, *On weakly semi-I-open sets and another decomposition of continuity via ideals*, Sarajevo J. Math. **2**(1) (2006), 107–114.

[6] E. Hatir, A. Keskin and T. Noiri, *On a new decomposition of continuity via idealization*, JP J. Geometry Toplogy **3**(1) (2003), 53–64.

[7] E. Hatir and T. Noiri, *On decompositions of continuity via idealization*, Acta Math. Hungar. **96**(4) (2002), 341–349.

[8] E. Hatir and T. Noiri, *On semi-I-open sets and semi-I-continuous functions*, Acta Math. Hungar. **107**(4) (2005), 345–353.

[9] E. Hatir and T. Noiri, *On β-I-open sets and a decomposition of almost-I-continuity*, Bull. Malays. Math. Sci. Soc. (2) **29**(1) (2006), 119–124.

[10] D. Janković and T. R. Hamlett, *New topologies from old via ideals*, Amer. Math. Monthly **97** (1990), 295–310.

[11] K. Kuratowski, Topology, Vol. I, Acadmic Press, New York, 1966.

[12] H. Maki, C. K. Rao and A. Nagoor Gani, *On generalizing semi-open and preopen sets*, Pure Appl. Math. Sci. **49** (1999), 17–29.

[13] J. M. Mustafa, S. Al Ghour and K.Al Zoubi, *Weakly b-I-open sets and weakly b-I-continuous functiions*, Ital. J. Pure Appl. Math. **N.30** (2013), 23–32.

[14] T. Noiri and V. Popa, *Minimal structures, m-open functions and bitopological spaces*, Creative Math. Inf. **18**(1) (2009), 50–56.

[15] T. Noiri and V. Popa, *Minimal structures, punctually m-open functions and bitopological spaces*, Sci. Stud. Res. Ser. Math. Inf. **19**(1) (2009), 145–154.

[16] V. Popa and T. Noiri, *On M-continuous functions,* Anal. Univ. "Dunărea de Jos" Galaţi, Ser. Mat. Fiz. Mec. Teor., Fasc. II **18** (**23**) (2000), 31–41.

[17] V. Popa and T. Noiri, *On the definitions of some generalized forms of continuity under minimal conditions*, Mem. Fac. Sci. Kochi Univ. Ser. Math. **22** (2001), 9–18.

[18] V. Popa and T. Noiri, *A unified theory of weak continuity for functions,* Rend Circ. Mat. Palermo (2) **51** (2002), 439–464.

[19] R. Vaidyanathaswani, *The localization theory in set-topology*, Proc. Indian Acad. Sci. **20** (1945), 51–62.

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