THEORETICAL ASPECTS REGARDING THE LOADING OF THE ANNULAR PLATES, SIMPLY LEANED ON THE BOTH OUTLINES, UNDER THE ACTION OF SOME BENDING MOMENTS UNIFORMLY DISTRIBUTED

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Abstract. The paper, taking into discussing the loading of the plates due to the presence of some bending moments uniformly distributed on the simply leaned outlines, compare two types of analysis. The appropriate expressions of the vertical displacement and the rotating of the plate, respectively of the radial and annular stresses are shown, for each study case.

Keywords: annular plate, simply leaning, radial and annular stresses.

1. INTRODUCTION

Need to find some appropriate methods for assessing the deformation and stress states of some parts of the structure from the industrial mechanical equipments under pressure, in general, respectively of some specific loads of some practical cases of leaning of the annular plates, in particular, become self-evident. The specialty literature amply reflects the previous statement.

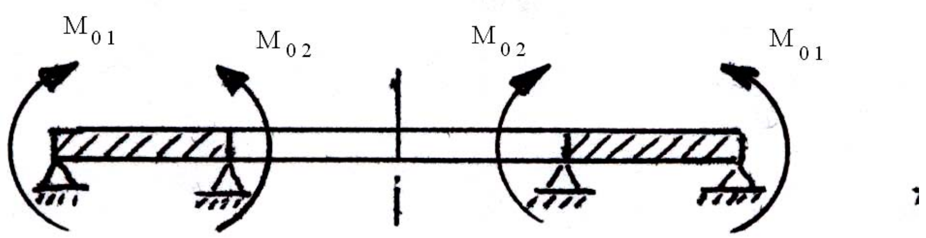

Fig. 1. Plate loaded by radial bending moments uniformly distributed on two concentric outlines simply leaned.

This paper studies the simple leaning on outlines of some annular plates, in two study cases, when the plate is subjected to the action of some radial bending moments, uniformly distributed, Figure 1.

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The loading of the plate is considered in the elastic domain, as a possible influence of the temperature can be overwritten properly. Again, the method to cancel the vertical displacement on a leaning or on other is used, by introducing, each time, of a load uniformly distributed on the considered support [1, 2, 3]. Are envisaged, and this time, too, two variants of calculation.

2. AXISYMMETRIC DEFORMED STATE - Version I

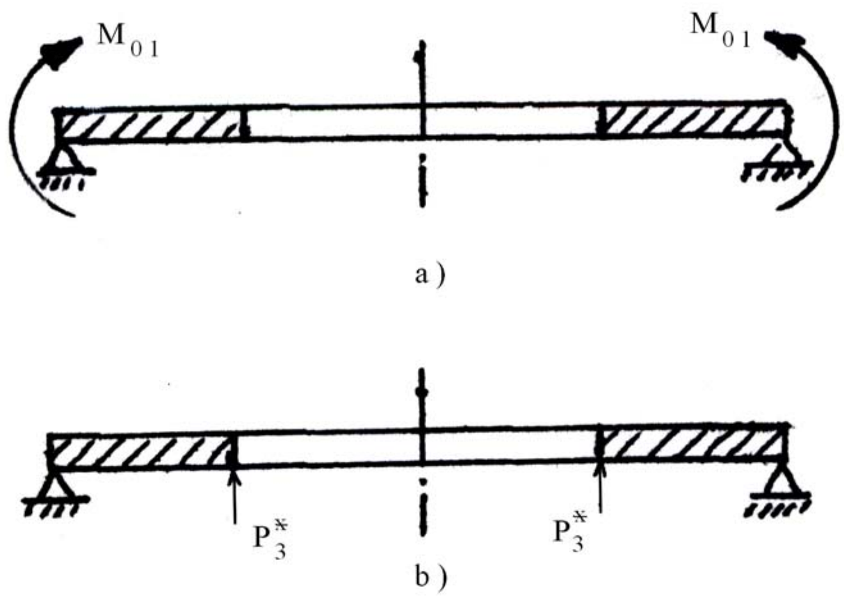

Fig. 2. Analysis scheme - Version I.

It is assumed that the outside leaning exist (Figure 2a), simulating the presence of the second leaning, that inside, by introducing of a uniformly distributed load (Figure 2b), to produce a vertical displacement having a equal value, but opposite to that created of the active bending moment on the outside leaning.

Taking into consideration the above said, after equalizing the vertical displacements produced of the bending moment M_{01} and the load $P_3^{\bullet}$ uniformly distributed on the inside outline of the plate:

$$w_{M,\,x=1}^{(f)} = w_{P^{\bullet},\,x=1}^{(b)}\,, \tag{1}$$

we reach at the expression:

$$P_3^{\bullet} = 16\cdot\pi\cdot\frac{C_{d,\,M,\,x=1}^{(f)}}{C_{d,\,P^{\bullet},\,x=\alpha}^{(a)}}\cdot M_{0,1}\,, \tag{2}$$

where we noted:

$$C_{d,\,M,\,x=1}^{(f)} = \frac{1}{\alpha^2 - 1}\cdot\left[\frac{1-\alpha^2}{2\cdot\left(1+\nu_p\right)} - \frac{\ln\alpha}{1-\nu_p}\right]. \tag{3}$$

2.1. Deformations expressions
The rotating of the median surface of the plate on the inside leaning has a formula:

$$\varphi^{(A)}_{M,x=1} = \frac{M_{0,1} \cdot r^2_{c,r}}{\Re_p \cdot r_0} \cdot \left[C^{(f)}_{r,M,x=1} - 4 \cdot \frac{C^{(e)}_{d,M,x=1}}{C^{(a)}_{d,P^\bullet,x=\alpha}} \cdot C^{(a)}_{r,P^\bullet,x=\alpha} \right], \tag{4}$$

with the notation:

$$C^{(f)}_{r,M,x=1} = \frac{2}{\left(\alpha^2 - 1\right) \cdot \left(1 - \nu^2_p\right)}, \tag{5}$$

while for the rotation on the outside leaning we find the expression:

$$\varphi^{(A)}_{M,x=\alpha} = \frac{M_{0,1} \cdot r^2_{c,r}}{\Re_p \cdot r_0} \cdot \left[C^{(f)}_{r,M,x=\alpha} - 4 \cdot \frac{C^{(f)}_{d,M,x=1}}{C^{(a)}_{d,P^\bullet,x=\alpha}} \cdot C^{(a)}_{r,P^\bullet,x=\alpha} \right], \tag{6}$$

where:

$$C^{(f)}_{r,M,x=\alpha} = \frac{1}{\alpha^2 - 1} \cdot \left[\frac{\alpha}{1 + \nu_p} + \frac{1}{\left(1 - \nu_p\right) \cdot \alpha} \right]. \tag{7}$$

The current vertical displacement of the points of the median surface of the plate has the form:

$$w^{(A)}_{M,x} = \frac{M_{01} \cdot r^2_{cr}}{\Re_p} \cdot \left[\begin{array}{c} C^{(f)}_{d,M,x=1} - C^{(f)}_{d,M,x} - \\ - \dfrac{C^{(f)}_{d,M,x=1}}{C^{(a)}_{d,P^\bullet,x=\alpha}} \cdot \left(C^{(a)}_{d,P^\bullet,x=\alpha} - C^{(a)}_{d,P^\bullet,x} \right) \end{array} \right], \tag{8}$$

where the notation is used:

$$C^{(f)}_{d,M,x} = \frac{1}{\alpha^2 - 1} \cdot \left[\frac{1 - x^2}{2 \cdot \left(1 + \nu_p\right)} - \frac{\ln x}{1 - \nu_p} \right]. \tag{9}$$

2.2. Stresses expressions
The final radial stresses are derived as difference between the values given of the bending moment M_{01} uniformly distributed on the outside outline and the equivalent load from the inside outline, having the expression:

$$\sigma^{(A)}_{r,M,x} = \frac{6 \cdot M_{01}}{\delta^2_p} \cdot \left[C^{(f)}_{r,\sigma,M} - 4 \cdot \frac{C^{(f)}_{d,M,x=1}}{C_{d,P^\bullet,x=\alpha}} \cdot C^{(a)}_{r,\sigma,P^\bullet} \right], \tag{10}$$

with the notation:

$$C^{(f)}_{r,\sigma,M} = \frac{x^2 - 1}{\alpha^2 - 1} \cdot \left(\frac{\alpha}{x} \right)^2. \tag{11}$$

Accepting the same procedure for the annular stresses, too, we get the formula:

$$\sigma_{\theta,x}^{(M_{0,1})} = \frac{6 \cdot M_{01}}{\delta_p^2} \cdot \left[C_{\theta,\sigma,M}^{(f)} - 4 \cdot \frac{C_{d,M,x=1}^{(f)}}{C_{d,P^\bullet,x=\alpha}} \cdot C_{\theta,\sigma,P^\bullet}^{(a)} \right], \tag{12}$$

with the notation:

$$C_{\theta,\sigma,M}^{(f)} = \frac{x^2+1}{\alpha^2-1} \cdot \left(\frac{\alpha}{x}\right)^2 . \tag{13}$$

3. AXISYMMETRIC DEFORMED STATE - Version II

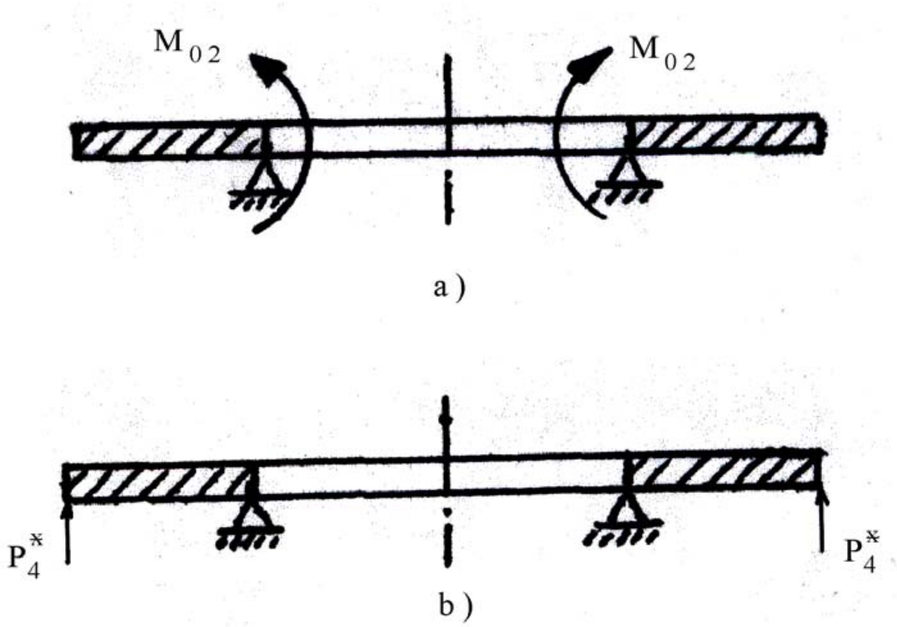

Fig. 3. Analysis scheme - Version II.

This time it is envisaged the inside leaning as basis (Figure 3a), the uniformly distributed load being present on the outside outline of the plate (Figure 3b). The simulation of the outside leaning of the annular plate is obtained by the presence of a load uniformly distributed on the corresponding circumference, so equalizing the vertical displacement with that produced by a radial bending moment:

$$w_{M,x=\alpha}^{(h)} = w_{P^\bullet,x=\alpha}^{(a)} , \tag{14}$$

it is deduced:

$$P_4^\bullet = 16 \cdot \pi \cdot M_{02} \cdot \frac{C_{d,M,x=\alpha}^{(h)}}{C_{d,P^\bullet,x=\alpha}^{(a)}} , \tag{15}$$

where the notation is used:

$$C_{d,M,x=\alpha}^{(h)} = \frac{1}{\alpha^2 - 1} \cdot \left[\frac{\alpha^2 - 1}{2 \cdot \left(1 + \nu_p\right) \cdot \alpha^2} + \frac{1}{1 - \nu_p} \cdot \ln \alpha \right]. \quad (16)$$

3.1. Deformations expressions

The rotating of the median surface of the plate on the inside leaning is expressed as:

$$\varphi_{M,x=1}^{(B)} = \frac{M_{02} \cdot r_{cr}^2}{\Re_p \cdot r_0} \cdot \left[C_{r,M,x=1}^{(h)} - 4 \cdot \frac{C_{d,M,x=\alpha}^{(h)}}{C_{d,P^\bullet,x=\alpha}^{(a)}} \cdot C_{r,P^\bullet,x=1}^{(a)} \right], \quad (17)$$

using the notation:

$$C_{r,M,x=1}^{(h)} = \frac{1}{\alpha \cdot \left(\alpha^2 - 1\right)} \cdot \left[\frac{1}{\left(1 + \nu_p\right) \cdot \alpha} + \frac{1}{1 - \nu_p} \right], \quad (18)$$

while the rotating on the outside leaning is:

$$\varphi_{M,x=\alpha}^{(B)} = \frac{M_{02} \cdot r_{cr}^2}{\Re_p \cdot r_0} \cdot \left[C_{r,M,x=\alpha}^{(h)} - 4 \cdot \frac{C_{d,M,x=\alpha}^{(h)}}{C_{d,P^\bullet,x=\alpha}^{(a)}} \cdot C_{r,P^\bullet,x=\alpha}^{(a)} \right], \quad (19)$$

with the notation:

$$C_{r,M,x=\alpha}^{(h)} = \frac{2}{\alpha \cdot \left(\alpha^2 - 1\right) \cdot \left(1 - \nu_p^2\right)}. \quad (20)$$

The values of the vertical displacements of the points $P_3^\bullet$ of the neutral surface of the annular plate can be calculated using the formula:

$$w_{M,x}^{(B)} = \frac{M_{02} \cdot r_{cr}^2}{\Re_p} \cdot \left[C_{d,M,x=\alpha}^{(h)} - C_{d,M,x}^{(h)} - \frac{C_{d,M,x=\alpha}^{(h)}}{C_{d,P^\bullet,x=\alpha}^{(a)}} \cdot C_{d,P^\bullet,x}^{(a)} \right], \quad (21)$$

in the structure of which the notation was used:

$$C_{d,M,x}^{(h)} = \frac{1}{\alpha^2 - 1} \cdot \left[\frac{\alpha^2 - x^2}{2 \cdot \left(1 + \nu_p\right) \cdot \alpha^2} + \frac{1}{1 - \nu_p} \cdot \ln \frac{\alpha}{x} \right]. \quad (22)$$

3.2. Stresses expressions

The values of the radial stresses σ developed in the annular plate under the action of the radial bending moment M_{02} can be established based on the expression (taking into account the type of leaning of the annular plate and of the proper load):

$$\sigma^{(B)}_{r,M,x} = \frac{6 \cdot M_{02}}{\delta_p^2} \cdot \left[C^{(h)}_{r,\sigma,M} - 4 \cdot \frac{C^{(h)}_{d,M,x=\alpha}}{C^{(a)}_{d,P^{\bullet},x=\alpha}} \cdot C^{(a)}_{r,\sigma,P^{\bullet}} \right], \tag{23}$$

with the notation:

$$C^{(h)}_{r,\sigma,M} = \frac{1}{\alpha^2 - 1} \cdot \left[\left(\frac{\alpha}{x} \right)^2 - 1 \right]. \tag{24}$$

The same logic for determining the expression of the annular stress leads to:

$$\sigma^{(B)}_{\theta,M,x} = \frac{6 \cdot M_{02}}{\delta_p^2} \cdot \left[C^{(h)}_{\theta,\sigma,M} - 4 \cdot \frac{C^{(h)}_{d,M,x=\alpha}}{C^{(a)}_{d,P^{\bullet},x=\alpha}} \cdot C^{(a)}_{\theta,\sigma,P^{\bullet}} \right], \tag{25}$$

where the notation is found:

$$C^{(h)}_{\theta,\sigma,M} = \frac{1}{\alpha^2 - 1} \cdot \left[\left(\frac{\alpha}{x} \right)^2 + 1 \right]. \tag{26}$$

When the analyzed annular plate is under the simultaneous action of the radial bending moments (Figure 1), both the values of rotations, the vertical displacements and the radial and annular stresses are evaluated by superposing the effects, given the senses of action and the loading in the elastic domain characteristic to the construction materials.

4. CONCLUSIONS

The paper analyzes the state of deformation and stress developed in an annular plate, simply leaned, loaded with bending moments uniformly distributed along the two outlines. The loading is considered in the elastic domain. The influence of the temperature in this study is not considered. Such influence can be assessed by superposing the appropriate effects. The study is carried out in two versions, introducing a time, instead of the leanings, inside and outside, uniformly distributed loads to develop vertical displacements equal in value with those given by the bending moments, also uniformly distributed across an outline or another.

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