

## INTERFACE DAMAGE CHARACTERIZATION THROUGH COHESIVE PARAMETERS

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**Abstract:** Interface damage characterization and interlaminar failure analysis of sandwich specimens with an initial interlaminar delamination in between the face sheet and the core are obtained by using interface cohesive elements. Peel tests reveal interesting particularities on damage localization and strain variation while damage is completed. The critical strain at damage initiation, as well as the critical displacement when damage is finalized, is established by using the digital image correlation method (DIC) in the cohesive zone. After the calibration of the finite element model through experimental results, the use of cohesive elements together with a linear or exponential softening law is analyzing the influence of the critical material parameters on the damage process and of the damping factor of the exponential law on the strain distribution in the core at different locations along the interface.

**Keywords:** sandwich composite, interlaminar damage, cohesive elements, initiation of damage, softening laws

### 1. INTRODUCTION

The laminated and sandwich composite material concept has a huge potential and special attention should be given to possible structural collapse which is often caused by the evolution of different types of damages created in a local zone of the structure. Aeronautical, automotive or naval structural integrity is of great importance and any presence of imperfections can reduce significantly the load bearing capacity. The particular damage modes depend upon loading, lay-up, and stacking sequence. It is essential to understand and control for a laminated or sandwich composite material its lay-ups arrangement for special orthotropy, the laminate design, strain and stress analysis, stiffness, and failure characterization, which is: failure modes and analysis, failure criteria, and edge delamination problems. Without a better understanding of progressive failure, the fracture criteria and predictive capabilities will be limited.

A sandwich structure is a three-layer structure having usually a low density and low modulus core material between two high modulus face sheets. The core mainly acts as a spacer, keeping the face sheets apart and has a thickness 2–10 times the face sheet thickness. This sequence provides a lightweight structure with a high bending stiffness being particularly attractive for industrial applications. For such sandwich materials, debonds (areas between the face sheet and core with no adhesion) constitute an important damage type that can lead to debonding crack growth, that is crack growth in the interface between face sheets and core. Interface cracking is generally mixed mode cracking, as both normal and shear stresses develop just ahead of the crack tip (Williams, [1]; Rice, [2]). Experiments have shown that the fracture energy can depend on the mixity mode as shown by Cao and Evans, [3]; Wang and Suo, [4]; Liechti and Chai, [5]. Suo and Hutchinson [6] analyzed sandwich

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specimens with core layer thickness much smaller than the face sheet layers. They found that for moderate stiffness mismatch, the difference in the mixity mode of the sandwich and the homogeneous specimen (as without the thin core layer) was relatively small. Ostergaard and Sorensen [7] carried out experimental measurements of interface fracture toughness of sandwich structures and found that the mixity mode has a significant influence on the fracture mechanism and the fracture toughness.

A novel experimental technique was initially used for analyzing the strain localization and damage at the interface of a sandwich composite. A detailed literature survey of the history of photogrammetry and digital image correlation (DIC) systems is done by Sutton et al. [8]. In general, DIC is based on the principle of comparing speckle patterns on the surface of the deformed and the undeformed specimen or structural component or between any two deformation states. For this purpose, a virtual grid of subsets of a selected size and shape, consisting of certain pixel grey value distributions, is superimposed on the pre-existing or artificially sprayed on surface pattern and followed during deformation by an optical camera system. In this manner, information on the in-plane local strain distribution is gained without assuming a priori the constitutive behavior of the material. The method finds many applications as in fracture mechanics (Mekky and Nicholson, [9]) or fatigue (Carroll et al., [10]) analyses, and its improvements of the sensitivity are discussed by Sutton et al., [11]. Shen and Paulino [12] provide a full-field DIC algorithm to compute the smooth and continuous displacement field, which is then used as input to a finite element model for inverse analysis through an optimization procedure in order to compute the cohesive properties of a ductile adhesive.

The purpose of this research is to analyze the phenomenon of interface delamination in sandwich specimens with a rigid core (polyurethane foam), to observe the interlaminar damages and failures, and to try to understand most of the local processes. We used the digital image correlation (DIC) method for establishing the three-dimensional displacements of the tested specimens and for monitoring the crack propagation (Constantinescu et al., [13]; Miron, [14]; Miron and Constantinescu [15, 16]. Clearly phenomena were non-linear, and the concept of critical energy release rate used within linear elastic fracture mechanics is doubtful within this context. The use of DIC gave new perspectives in the evaluation of local parameters suitable for damage characterization and interlaminar failure. We proposed as a failure parameter the critical local strain at the crack tip established exactly before the stable crack propagation occurs, which was in our tests 17.5 % (mean value). Of course that such a parameter is specimen and loading dependent, but, however, it can be established for each situation. An alternative parameter can be the critical opening displacement which had an average value of 0.24 mm, when the propagation of the delamination occurs. Further developments were obtained by using numerical simulations and cohesive finite elements in Abaqus [17].

The issue of strain localization ahead an interface delamination before and during failure in a sandwich component with a mat glass fiber face sheet and a rigid polyurethane core is further analyzed. DIC was used in monitoring the evolution of failure from an initial artificially induced interface defect by using two rows of virtually emulated strain gages of about 1.4 mm gage length at the interface and beneath the interface, in the core material. Strains in different locations ahead the initial delamination are monitored for each test, the number of readings being dependent on the way in which the delamination propagated, in a stable or unstable manner. In this way, substantial information on strain evolution and localization are acquired. The DIC analyses show that critical strain at initiation in the interface cohesive zone resulted as having an average value  $\epsilon_c = 7.46\%$ , which corresponds to an ultimate strength of the bridging tractions  $\sigma_c$ , and the critical displacement for the softening law as  $\delta_c = 0.42$  mm when damage is finalized. With these values a finite element (FE) model with cohesive elements at the interface was calibrated and a linear or exponential softening cohesive law was used. The influence of the critical parameters at the initiation and finalization of damage is numerically obtained by the variation of  $\epsilon_c$  and  $\delta_c$  in order to analyze their influence on the variation of strain fields in the core of the sandwich composite, close to the interface.

## 2. COHESIVE ELEMENTS

During the crack growth process, two new surfaces are created. Before the physical crack is formed, these two surfaces are held together by traction within a cohesive zone. The traction varies in relation to the relative displacement of the surfaces, and a cohesive law describes the phenomena in the cohesive zone in terms of the traction and the separation of the surfaces to be formed under the fracture process. A cohesive law is also denoted a traction-separation law. The concept to describe the cohesive phenomena before fracture has been established for almost half a century ago. The concept of cohesive zones (Dugdale, [18]; Barenblatt, [19]) has

received revived interest and the cohesive zone modeling (CZM) approach has emerged as a powerful analytical tool for nonlinear fracture processes. This model considers the relation between the traction and separation that are normal to the fracture surfaces, and the unphysical stress singularity at the crack tip in the traditional linear elastic fracture mechanics is removed. The cohesive models were later extended to the mode II fracture process, in which the tangential traction and separation are considered instead. As Högberg [20] mentions, experimental observations show distinctive characteristics of the micromechanical failure mechanisms in peel and shear fracture, thus the cohesive behaviour is expected to be mode dependent.

Stigh et al. [21] emphasizes that with cohesive modeling, no extra properties are necessary to simulate crack growth. Only the cohesive law is needed to analyze both initiation and growth of a crack. This is also a drawback in modeling flexibility. Namely, if the fracture toughness changes with crack growth, a conventional cohesive law cannot capture this phenomenon by itself. However, as mentioned by Andersson and Biel [22], cohesive properties vary from specimen to specimen even if the experiments are performed with utmost care. This means that conclusions regarding material properties should be deducible using one specimen and repeated experiments should be performed to learn about the variability in the property between different specimens. And, once again, mixed modes are usually present in an adhesive layer (Högberg et al., [23]). Another issue is the three-dimensional (3D) modeling of the cohesive zone versus the two-dimensional (2D) one. As shown by van den Bosch et al. [24], 3D simulations are computational expensive, and a 2D model reduces the calculation time by a factor of 15-50, depending on the width of the model and thus the element discretization in the width direction. It is concluded that a plane strain model is an attractive alternative to perform qualitative parameter studies and useful in the first steps towards an iterative fitting procedure on quantitative experimental results, but overestimates the quasi-static peel force.

### 3. USE OF COHESIVE ELEMENTS FOR INTERFACE DAMAGE ANALYSIS

The double cantilever beam (DCB) made from a sandwich composite has the experimentally established mechanical properties which are used for the numerical analysis as follows: face sheets made from mat material ( $300 \text{ g/m}^2$  mass per layer) with Young's modulus  $E = 9,000 \text{ MPa}$  and Poisson's ratio  $\nu = 0.33$ ; polyurethane core ( $200 \text{ kg/m}^3$  density) with Young's modulus  $E = 172.1 \text{ MPa}$  and Poisson's ratio  $\nu = 0.37$ ; a bicomponent polyurethane adhesive used to glue the sheet to the core with given properties as  $E = 30 \text{ MPa}$  and  $\nu = 0.37$ . We consider, as established before, the critical strain at initiation  $\varepsilon_c = 7.46 \%$  (or critical bridging traction  $\sigma_c = 2.25 \text{ MPa}$ ), and the critical displacement for the softening law at the finalization of damage as  $\delta_c = 0.42 \text{ mm}$ . The analysis is standard linear elastic and a linear or exponential softening law was assumed for the cohesive elements from Abaqus [16].

A 2D FE model with the hypothesis of plane stress is preferred as to reduce the computational time, [24]; as the 2D cohesive elements have four nodes the finite elements used for the sheet and the core are also four noded. The sandwich model has a length of 200 mm, width 20 mm, and thickness 16.8 mm. Each adhesive layer is considered to have 0.295 mm (as measured through a SEM analysis). The core was modeled with elements of length 0.295 mm. This mesh considers a variable dimension of the core elements. The face sheets are meshed in square elements of 0.32 mm and the tabs in square elements of 1 mm (Figure 1). Length of initial delamination is 40 mm and 20 mm has the tab through which the imposed displacement is applied. Another mesh, as in Figure 2, considers uniform size length of elements both in interface and in the core of 0.5 mm. The height of the elements is 0.3 mm in the interface, 0.6 mm in the sheets, and 0.5 mm in the core. The elements are 0.5 x 0.5 mm in the tabs. This time the length of the initial delamination is 55 mm with the same length of the tab as 20 mm.

As boundary conditions, the center of the lower tab was fixed and the center of the upper tab was loaded through an imposed vertical displacement. Displacement is imposed progressively till the value of 25 mm (observed to be sufficient in the experimental analysis, Figure 3) by considering in most of the simulations 200 loading intervals with a step of 0.005. Of course that the number of loading intervals can be increased, which will refine the numerical analysis. These boundary and loading conditions are very close to the ones which resulted during the experimental testing.

The force-displacement response of the DCB specimens for three stable propagations in Test 1, Test 3, and Test 5, is compared in Figure 3 to the FE simulated force-displacement curve; best approximation is obtained for Test 3. Quite different behaviours are to be seen in other tests where unstable crack propagation occurs and big differences in between the experimental and numerical maximum force at the initiation of delamination is to be

obtained. It is clear that for such tests there are various factors which influence the cohesive damage: on one hand possible “barriers” which retard the propagation, and on the other hand interface voids and defects which favor unstable propagation.

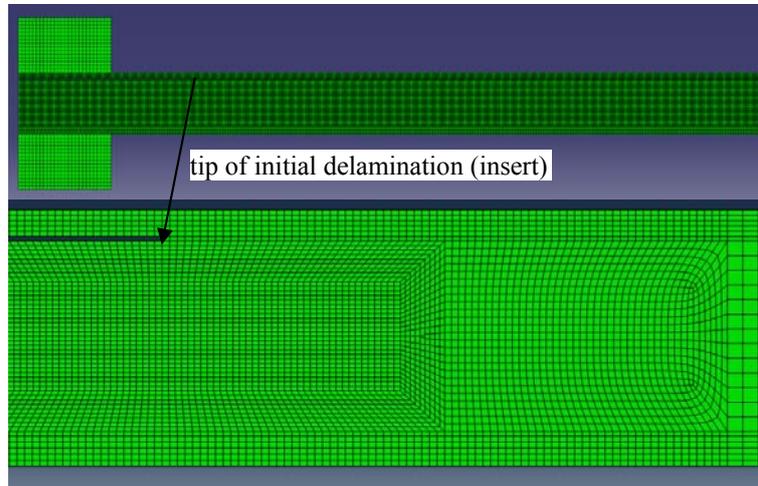


Fig. 1. Model with variable FE mesh of the DCB.

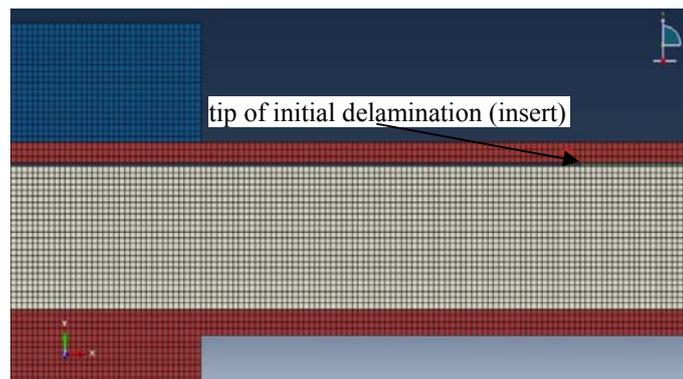


Fig. 2. Model with uniform FE mesh of the DCB.

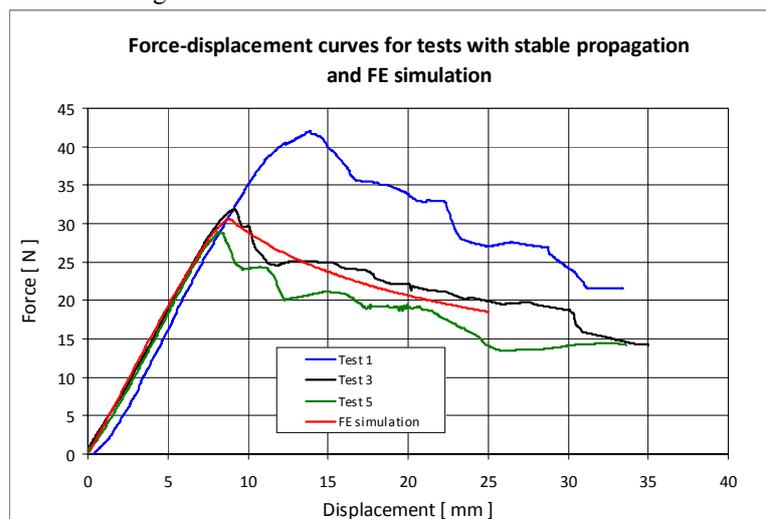


Fig. 3. Experimental and numerically simulated force-displacement curves.

For the numerical model we have chosen initially the critical strain at damage initiation in the interface as being  $\epsilon_c = 7.46\%$ , which is an average value obtained from experimental tests. Therefore, the experimental results

should be slightly different from the numerical ones due to following: the virtual strain gage emulated i has about 1.4 mm and opening strains are averaged; during the experimental testing not always the damage starts to initiate at the interface when strain is 7.46 %, as happens in the numerical model.

In Figure 4 is shown the distribution of the opening strain in the core of the sandwich after the initial delamination has propagated. At the tip of the crack the part in tension is in red colour, and ahead it a zone in compression in blue colour is evident.

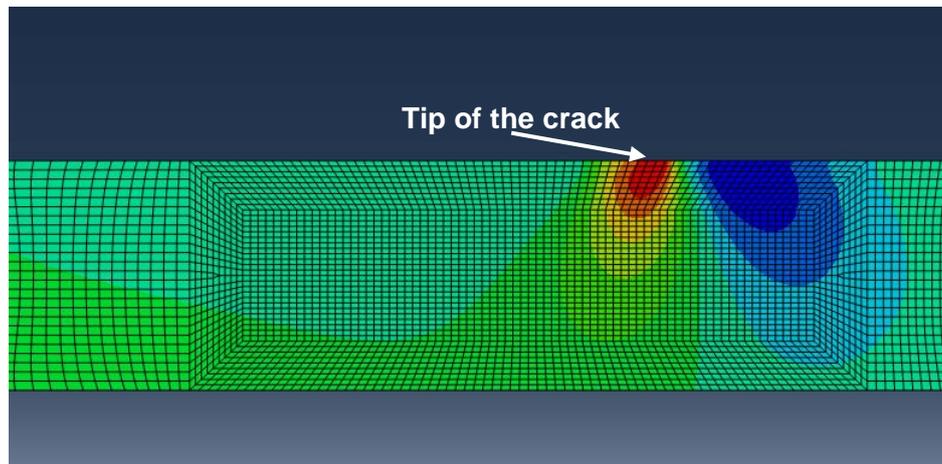


Fig. 4. Distribution of opening strain ahead the tip of a stable propagating crack.

### 3. INFLUENCE OF CRITICAL PARAMETERS ON DAMAGE CHARACTERIZATION

Cohesive strength and fracture energy are believed to have greater importance with respect to the specific shape chosen for the cohesive model (Kafkalidis et al., [25]). Therefore, many traction-separation relations have been employed in the literature, e.g. the potential based exponential model (Xu and Needleman, [26]), the trapezoidal model (e.g. Tvergaard and Hutchinson, [27]) and the bilinear model (e.g. Zhang and Paulino, [28]; Liljedhal et al., [29]) are perhaps the most widely adopted. In particular, the sensitivity of cohesive zone parameters in predicting the mechanical response of the specimen is examined and the results obtained using widely adopted traction-separation relations (i.e. trapezoidal, bilinear and exponential model) are compared. Most damage models, such as the Progressive Damage Model for Composites provided in Abaqus [16] and typical cohesive elements (Camanho et al., [30]; Turon et al., [31]; Dávila et al., [32]), represent the evolution of damage with linear softening laws that are described by a maximum traction and a critical energy release rate.

As discussed by Dávila et al. [32] the shape of the softening law, e.g., linear or exponential, is generally assumed to be inconsequential for the prediction of fracture for small-scale bridging conditions, but plays a fundamental role in the prediction of fracture under large-scale bridging conditions, where the process zone length may be large relative to other length scales in the problem (Bao and Suo, [33]; Foote et al., [34]; Sorensen and Jacobsen, [35]). When crack propagation includes different energy dissipation mechanisms that act over different length scales, the nature of these mechanisms must be accounted for in the cohesive law. Linear and linear-exponential softening laws were compared by Koerber and Camanho [36], for the simulation of the tensile fracture of a composite bolted joint. It was shown that using the linear softening law for fiber fracture results in an over-prediction of the peak load of the component and an erroneous path for the fracture plane, whereas the linear-exponential softening law results in a good prediction for both the peak load and the fracture plane. Clearly, the determination of the softening law required trial-and-error fitting of the results of analyses for measured strengths of panels of different sizes and crack lengths.

Established through experimental analysis, the critical strain at damage initiation corresponds to a critical bridging traction  $\sigma_c = 2.25$  MPa. When choosing in between a linear or an exponential softening law, we have to mention that the damage law is defined by the scalar variable  $D$  – the interface material loses its rigidity and  $D$  will be 1 when the cohesive element is damaged. In Figure 5 is shown the influence of a non-dimensional material parameter which defines the rate of damage evolution (notated  $\alpha$  in Abaqus), which is zero for the linear

softening; as softening becomes exponential and  $\alpha$  increases its value, damage is produced more rapidly. Therefore a linear softening is less conservative, damage being produced in a longer time.

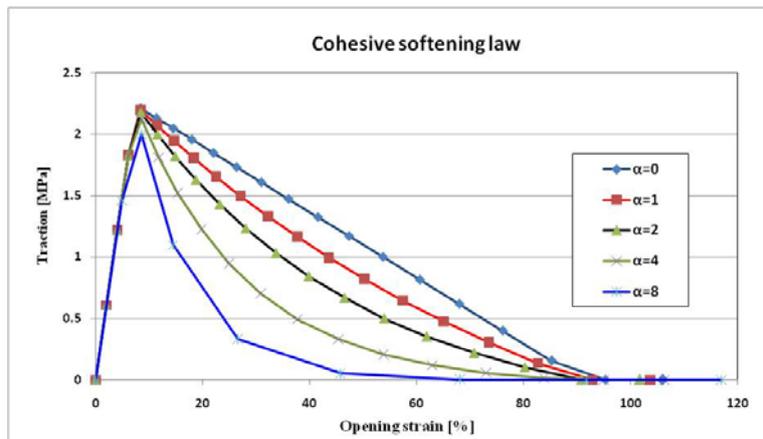
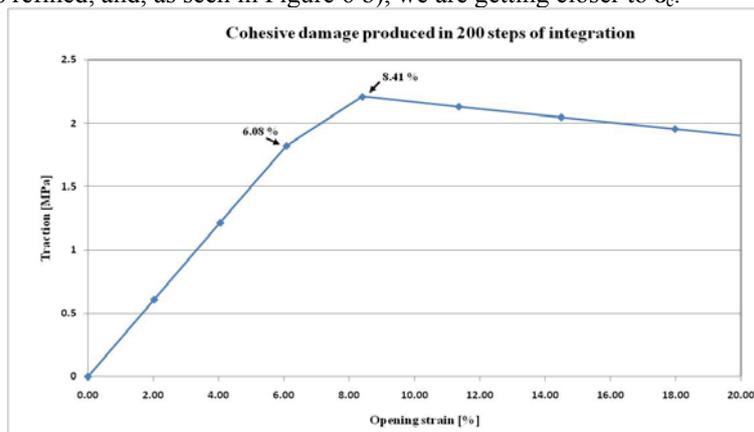
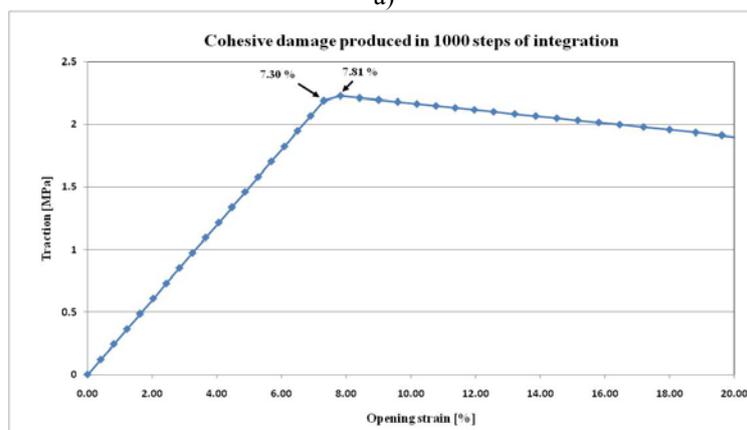


Fig. 5. Linear and exponential softening law in cohesive elements.

The number of steps of integration will influence the shape of the damage curve till the critical bridging traction is reached. For a softening law, if we represent the opening strain in the interface only up to 20 % we can notice better in Figure 6 that the exact critical strain at damage initiation of value  $\epsilon_c = 7.46\%$  cannot be reached, as being in between two consecutive integration steps. When increasing the number of steps of integration the numerical calculus is refined, and, as seen in Figure 6 b), we are getting closer to  $\epsilon_c$ .



a)



b)

Fig. 6. Influence of the variation of integration steps on reaching critical initiation strain: a) 200; b) 1000.

Of course that increasing the number of integration steps will produce a longer time consuming analysis; that's why for obtaining most of our results we prefer to use only 200 steps on integration. It is also preferable to use an uniform size of elements in the interface and in the core as in Figure 2; in this way the convergence during the numerical analysis is better achieved as compared to the variable size of elements mesh chosen initially as in Figure 1.

The variation of the critical strain at the initiation of damage gives an opening strain variation as shown in Figure 7. The value of 7.46 % is the one established experimentally with DIC. In this case the maximum opening strain in the core is 0.8 % and damage is finalized in about 100 steps of integration. In this figure and the next one a linear softening law is assumed. However, in the numerical response, the damage completion is not perfectly linear due to the reduced number of integration steps.

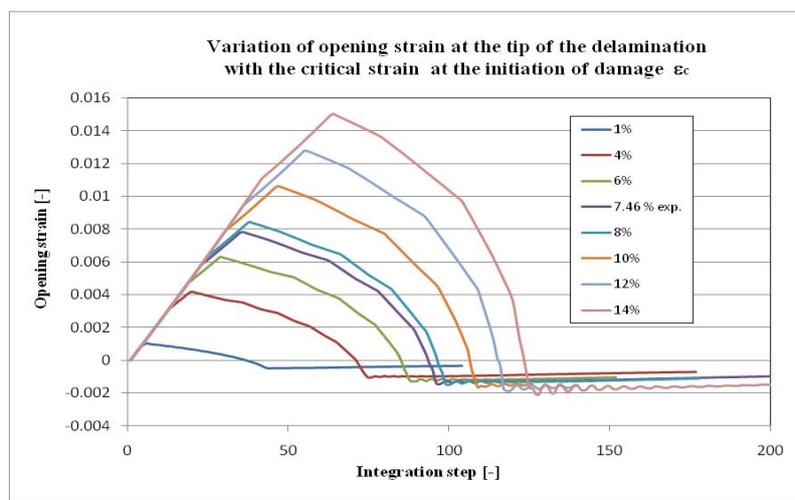


Fig. 7. Variation of opening strains at different critical strains of damage initiation.

In a similar way, when the critical traction at damage initiation is chosen as a damage parameter the variation of the opening strains is shown in Figure 8, and 7.46 % corresponds to 2.25 MPa, as established before. In fact the superposition of the two curves is evident. This also shows that the response of the specimen is linear as the imposed FE analysis is a linear one.

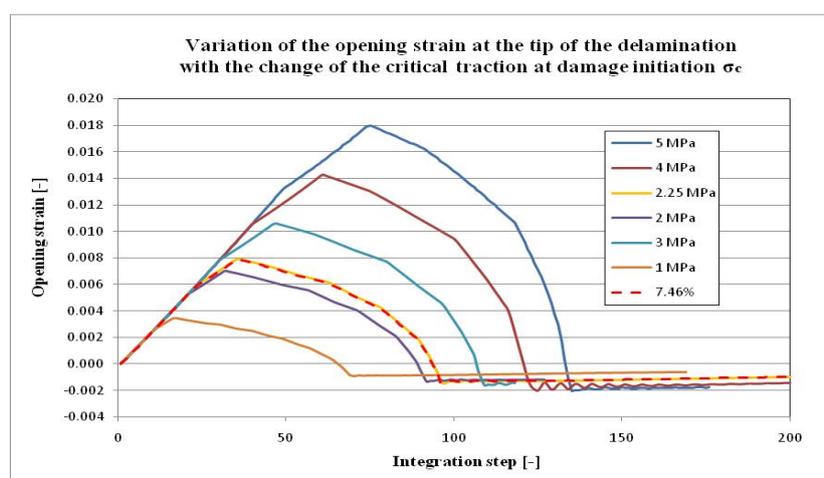


Fig. 8. Variation of opening strains at different critical tractions of damage initiation.

When analyzing the variation of the opening strain in the first row of finite elements close to the interface, in the core of the sandwich, in different locations as distances from the initial delamination, a plot shown as in Figure 9

is to be obtained. Here a linear law of softening is assumed, and critical strain at damage initiation is chosen as being 7.46 %, as from DIC.

The total loading, as imposed displacement of 25 mm, is applied through 200 steps. Meanwhile the initial delamination starts to propagate. In Figure 9 four specific locations were chosen. At the tip of the initial delamination the core is loaded in tension – meanwhile in the other three points where strain is measured, the core is in compression. As the delamination starts to propagate in the other three points at 5 mm, 8.5 mm, and 17 mm, tensile strains start to develop. Damage is completed when strain is constant, with a small negative value as to be seen in Figure 9. In about 180 steps of integration the crack propagates through all the four locations.

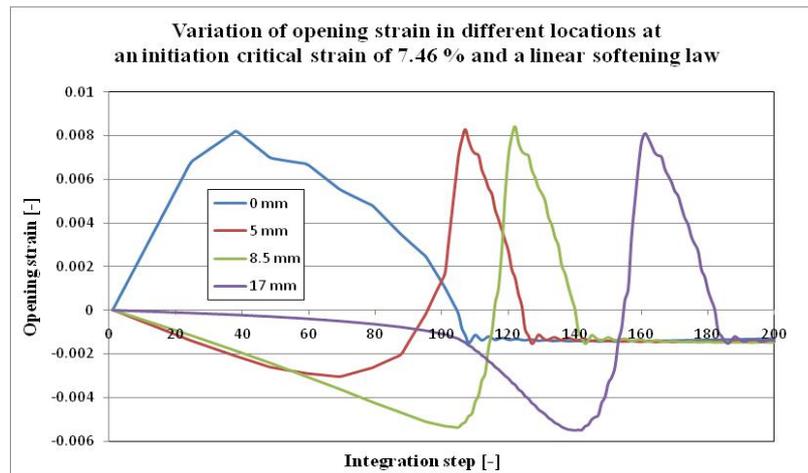


Fig. 9. Variation of opening strains in different locations.

If we keep the same critical strain at damage initiation but we change the damage law from linear to exponential, some differences are noticed. In Figure 10 we consider the exponent of the damage law (non-dimensional material parameter notated  $\alpha$  in Abaqus) as  $\alpha = 2$  and, as defined in Abaqus, the damping factor used from the previous general step. We keep the same location in which we measure the opening strain in the core. Now damage is completed faster, and in less than 160 steps of integration the whole analysis is finished.

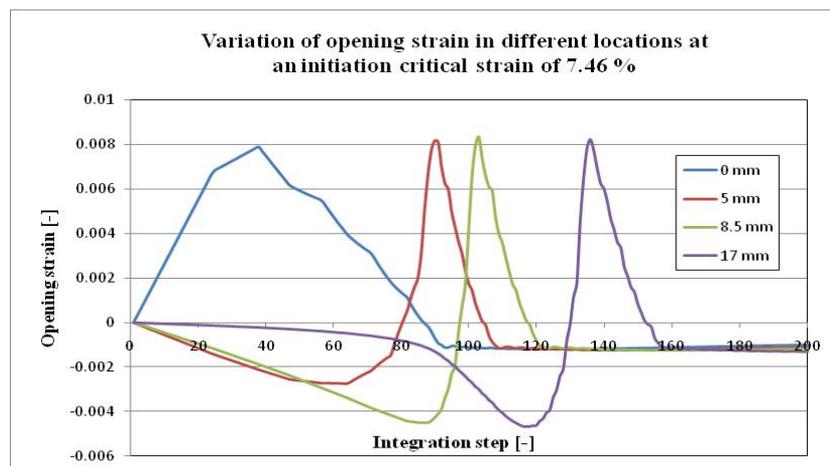


Fig. 10. Variation of opening strains in different locations with a damping factor used from the previous general step and  $\alpha = 2$ .

If  $\alpha = 4$ , that is a higher exponent as before is chosen, an accelerated damage is produced and the variation of the strain fields looks like in Figure 11. Damage is completed in less than 140 steps of integration, as increasing the material parameter  $\alpha$  accelerate damage completion.

If for the exponential softening law of softening we come back to the previous value of  $\alpha = 2$ , but with a specified damping factor equal to 0.0002 (standard value in Abaqus) the whole process of crack propagation through the four locations is produced much faster, in less than 80 steps of integration, as seen in Figure 12. Thus, the choice of the damage law and its parameters is crucial for the damage finalization process.

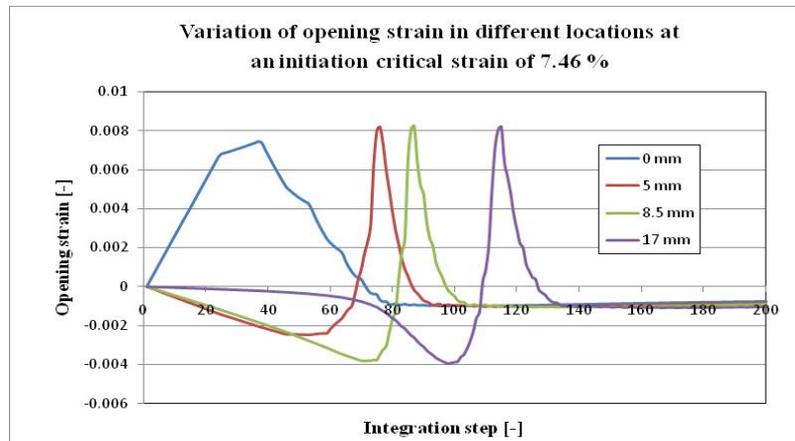


Fig. 11. Variation of opening strains in different locations with a damping factor used from the previous general step and  $\alpha = 4$ .

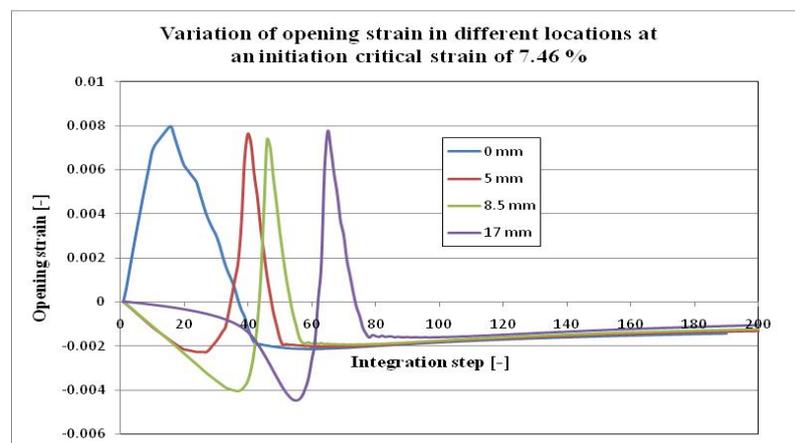


Fig. 12. Variation of opening strains in different locations with a damping factor equal to a standard value in Abaqus and  $\alpha = 2$ .

Damage is produced now in half of the integration steps which were needed when the damping factor was used from the previous general step (Figure 9). Therefore the use of a damping factor equal to a standard value (here value is only 0.0002) is seriously influencing the whole damage process.

#### 4. CONCLUSIONS

The problem of an interface failure in a sandwich component with a mat glass fiber face sheet and a rigid polyurethane core is analyzed by using cohesive elements defined in Abaqus. The critical parameters at damage initiation and damage completion were established previously by using DIC and virtual strain gages emulated at the interface and beneath it, in the material of the core. The moment of damage initiation and its evolution are monitored by measuring the local opening strains with the GOM ARAMIS system.

Having the experimentally established critical parameters as a reference, numerical simulations are done in order to study the influence of the mesh topology, the number of steps and integration, and the variation of the critical parameters on the damage evolution. The linear and exponential softening laws are considered; for the last one

the influence of the material parameter  $\alpha$  and the choice of the damping factor prove to have great importance on the damage process and on the distribution of the strain fields in different locations ahead the initial delamination.

The strain localization phenomena observations enlarge the perspectives of the analyses on successive failures at the interface during the same test. Cohesive zone critical parameters are essentially specimen and material dependent, but the proposed hybrid procedure gives a reliable perspective on the evaluation of interlaminar damage initiation and finalization. Cohesive damage is influenced by local interface imperfections, and, therefore, extensive experimental testing has to be carried on till average critical parameters are established for being used in numerical simulations.

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