

## ON MODELING THE LONG TRANSMISSION LINES THROUGH EQUIVALENT $\Pi$ CIRCUITS

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**Abstract :** In this paper the authors present the results obtained in modeling a long transmission line through a sequence of equivalent  $\Pi$  circuits. An equivalent  $\Pi$  circuit is model with transverse elements capacities. Are shown the results obtained for idle regime and for various load regimes. Are compared the results obtained with those theoretical given by the long lines equations. Conclusions are drawn about the usefulness of the model used in the laboratory.

**Keywords:** telegraph equation, long line equation, laboratory stand

### 1. INTRODUCTION

For the long transmission lines, with the scheme given in figure 1, we can write the function equations [1]:

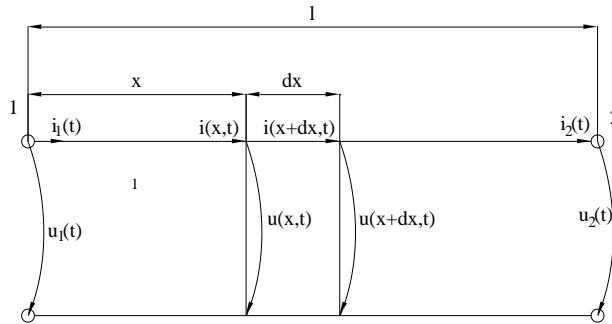


Fig. 1. The explanation of long lines

$$\begin{aligned} -\frac{\partial^2 u}{\partial x^2} &= R_0 \cdot \frac{\partial i}{\partial x} + L_0 \cdot \frac{\partial^2 i}{\partial t \partial x} \\ -\frac{\partial^2 i}{\partial x \partial t} &= G_0 \cdot \frac{\partial u}{\partial t} + C_0 \cdot \frac{\partial^2 u}{\partial t^2} \end{aligned} \quad (1)$$

In sinusoidal regime, the solution of these equations is:

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$$\begin{bmatrix} \underline{U}(x) \\ \underline{I}(x) \end{bmatrix} = \begin{bmatrix} ch(\underline{\gamma} \cdot x) & -\underline{Z}_c \cdot sh(\underline{\gamma} \cdot x) \\ -\frac{sh(\underline{\gamma} \cdot x)}{\underline{Z}_c} & ch(\underline{\gamma} \cdot x) \end{bmatrix} \cdot \begin{bmatrix} \underline{U}_1 \\ \underline{I}_1 \end{bmatrix} \quad (2)$$

or

$$\begin{bmatrix} \underline{U}(x) \\ \underline{I}(x) \end{bmatrix} = \begin{bmatrix} ch(\underline{\gamma} \cdot (l-x)) & \underline{Z}_c \cdot sh(\underline{\gamma} \cdot (l-x)) \\ \frac{sh(\underline{\gamma} \cdot (l-x))}{\underline{Z}_c} & ch(\underline{\gamma} \cdot (l-x)) \end{bmatrix} \cdot \begin{bmatrix} \underline{U}_2 \\ \underline{I}_2 \end{bmatrix} \quad (3)$$

where  $\underline{U}_1$  and  $\underline{I}_1$  are the voltage and the current at the beginning of the line, and  $\underline{U}_2$  and  $\underline{I}_2$  at the end of the line.

## 2. PRESENTATION OF THE STAND USED

The plant scheme is given in figure 2.

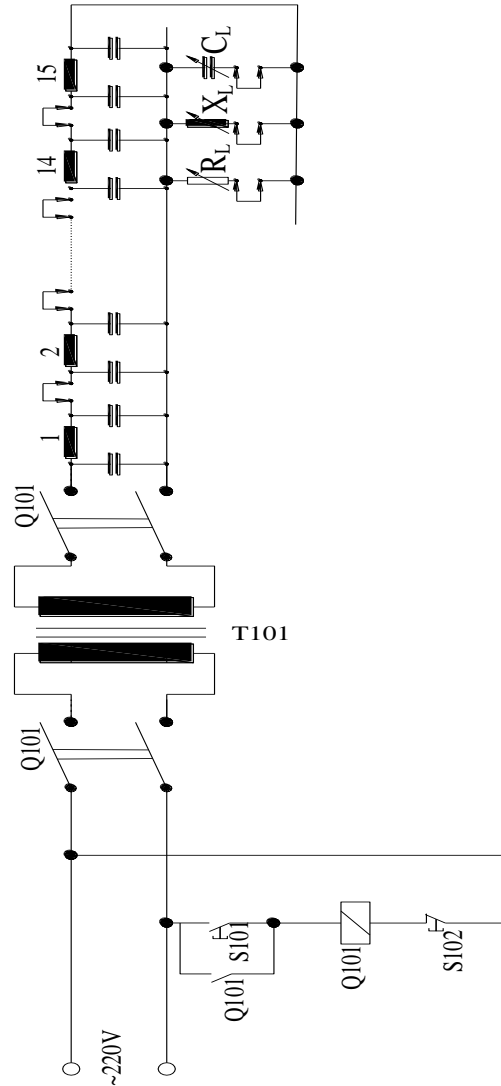


Fig. 2. The stand scheme used for tests

The elements from figure 2 have the following significance:

- T101 – Power transformer with voltage 230/10 V ac, 65 VA;
- Q101 – Contactor 230 V ac;
- S101 – Connecting button;
- S102 – Disconnecting button;
- 1,2,...,15 – Equivalent  $\Pi$  circuits with the following characteristics
  - $\underline{Z}=3.9+j.39 \Omega$
  - $C=2 \times 0.47 \mu F$  ( $B=295.3 \mu S$ )
  - a  $\Pi$  circuit models 100 km line
- $X_L$  – Inductive receptor (coil),  $9.7 \Omega$ , 1.152 H;
- $R_L$  – Resistive receptor (potentiometer), with domain  $135 \div 730 \Omega$ ;
- $C_L$  - Capacitive receptor (0.8-30)  $\mu F$

An equivalent  $\Pi$  circuit models 100 km line. Characteristic sizes  $\underline{Z}_0$  (specific impedance),  $\underline{Y}_0$  (specific admittance),  $\underline{Z}_c$  (characteristic impedance) and  $\underline{\gamma}$  (phase constant) are:

$$\underline{Z}_0 = \frac{\underline{Z}}{100} = 0.039 + j0.39 \Omega / km$$

$$\underline{Y}_0 = j \cdot B / 100 = j2.953 \cdot 10^{-6} S / km$$

$$\underline{Z}_c = \sqrt{\frac{\underline{Z}_0}{\underline{Y}_0}} = 363.86 - j18.148 \Omega$$

$$\underline{\gamma} = \sqrt{\underline{Z}_0 \cdot \underline{Y}_0} = 5.359 \cdot 10^{-5} + j1.075 \cdot 10^{-3}$$

### 3. OBTAIN RESULTS

#### 3.1. Idle regime

The regime has drawn for 1000 km, to avoid the 1500 km resonance, which is dangerous for the laboratory plant. For drawing the theoretical curve it is utilize the (2) relation, with  $l=1000$  km. From the second relation in (2), taking account that  $\underline{I}(l)=0$ , results:

$$\underline{I}_1(x=l) = \frac{\underline{U}_1}{\underline{Z}_c} \cdot \frac{sh(\underline{\gamma} \cdot l)}{ch(\underline{\gamma} \cdot l)} \quad (4)$$

$$\underline{I}(x) = -\frac{sh(\underline{\gamma} \cdot x)}{\underline{Z}_c} \cdot \underline{U}_1 + ch(\underline{\gamma} \cdot x) \cdot \underline{I}_1(l) \quad (5)$$

In figure 3 are shown the calculated and measured values for voltages, and in figure 4 the calculated and measured values for currents.

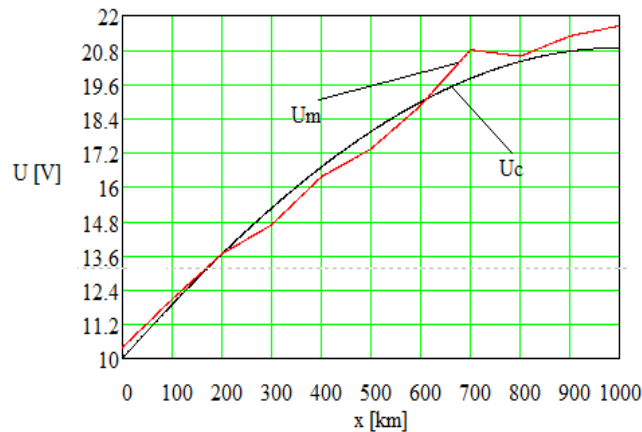


Fig. 3. Measured ( $U_m$ ) and calculated ( $U_c$ ) voltage variation for  $l=1000$  km, idle

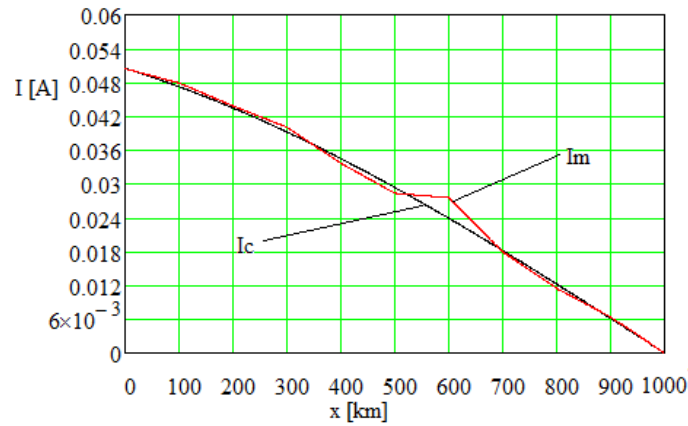


Fig. 4. Measured ( $I_m$ ) and calculated ( $I_c$ ) current variation for  $l=1000$  km, idle

### 3.2. The regime with load equal with natural power

For drawing the theoretical characteristic, we use the (3) relation, in which  $\underline{U}_2/\underline{I}_2=\underline{Z}_c$ .

In figure 5 are shown the calculated and measured values for voltages.

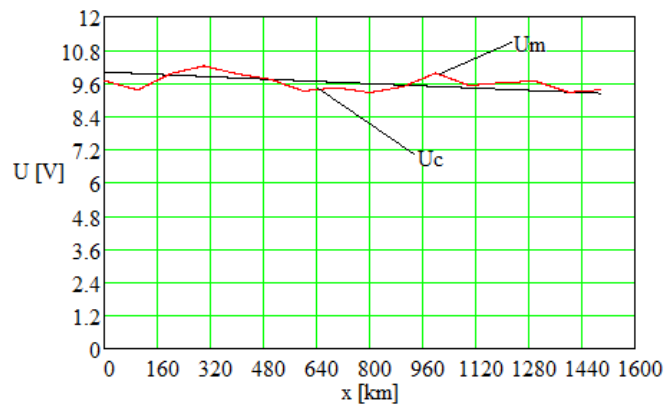


Fig. 5. Measured ( $U_m$ ) and calculated ( $U_c$ ) voltage variation for  $l=1500$  km, natural power

## 4. CONCLUSIONS

From the obtained values results the following:

- The correspondence between experimental and theoretical results is very good in all cases, the differences are less than 6%.
- The idle regime highlights the resonance phenomenon at  $l=1500$  km, the overvoltage value at 1000 km being greater than 2.
- For natural load regime, are obtained approximately constant voltage and current, a small decrease is due pure resistive load ( $\underline{Z}_c=364 \Omega$ ).
- The stand allows for a wide range of samples.

## REFERENCES

- [1] Renato Orta, *Transmission Line Theory*, Department Of Electronics and Telecommunications, Politecnico di Torino, November 2012
- [2] Göran Andersson -*Modelling and Analysis of Electric Power Systems*, EEH - Power Systems Laboratory, ETH Zürich, September 2008