# THE PROBLEM OF ELECTROCONVECTION AND HEAT EXCHANGE IN DISPERSED GAS-LIQUID SYSTEMS

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**Abstract:** The problem of momentum, energy, and electrical charge transport is formulated for gas-liquid dispersed systems.

**Key words:** gas-liquid system, electroconvection, electric field, momentum,energy and electrical charge transport

## 1. INTRODUCTION

In a number of branches of industry, such as thermal energy production and chemical and food technology processes in which a gas interacts with a liquid are often used, and they are often carried out under bubbler conditions to intensify them. The main questions involved in study of such processes are the hydrodynamics of the gas-liquid layer, removal of the liquid phase, heat-mass transport and organization of various processes both in the bubble layer and the vapor-gas space.

The action of electric fields can significantly intensify heat exchange in gas-liquid media. To a certain extent such questions have been investigated in bubble boiling [1]. It follows from data available in the literature on the problem of heat exchange in bubble-layer processes that in the dependence of the bubbler plate heat liberation coefficient on gas velocity one can distinguish at least three regions, with the dependence being selfsimilar in two of these [2]. There are studies which indicate that under certain conditions an electric field can intensify interphase heat exchange.

The mechanism of electric-field action in bubbling processes is related to the effect of electroconvection on the hydromechanical state of the phases: the change in the phase thermodynamic properties in an electric field is negligibly small compared to electroconvective phenomena [3].

The absence of a unified viewpoint on this problem and the extreme lack of data, especially experimental, on the effect of an electric field stimulated our study of the question.

## 2. MOMENTUM, ENERGY, AND ELECTRICAL CHARGE TRANSPORT IN GAS-LIQUID DISPERSED SYSTEMS

One method of describing the interrelated processes which occur in such systems is based on the concepts of the mechanics of a continuous medium, although in formulating problems consideration of electrical fields is either a special case [3-5] or absent [6].

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We will consider the system formed upon escape of a gaseous phase into a liquid medium and divided by the free liquid surface into the bubble region **Ω**' and the vapor-gas volume **Ω**", in which the liquid phase is dispersed and carried off. Before the process commences the region **Ω**' is filled with liquid, while **Ω**" is filled by gas, and the regions are in a state of thermodynamic equilibrium which, in particular, assumes absence of macroscopic motion of the phases $(v_1')_0 = (v_1'')_0 = 0$ and uniform temperature distribution over volume $(T_1')_0 = (T_1'')_0 = T_0$.

At the beginning of the process ($t = 0$) a vapor-gas flow with parameters $(G_2')_0$, $(T_2')_0$, $(\rho_{2v}')_0$, and $(n_{2i}')_0$ bubbles through the liquid (region **Ω**') in the form of bubbles with a mean breakaway radius $(a')_0$. On the surface s bubbles are generated in the form of the dispersed system medium **Ω**", while liquid is dispersed into droplets with characteristics $(2a_2'')_S$, $(G_2'')_S$, and $(n'')_S$.

At the initial time ($t = 0$) in region **Ω**" an ionized vapor-gas flow appears with parameters $(G_1'')_0$, $(T_1'')_0$, $(\rho_{1v}'')_0$, and $(n_{1I}'')_0$. From the region **Ω**" a flow of liquid electroaerosol is extracted with phase transitions occurring on the phase boundaries.

We consider the assumptions of [5, 6] and neglect collisions between unipolarly charged droplets and bubbles.

We then require the time and space distribution of $v_i$, $v_{ra}$, $a$, $n$, $\alpha_i$, $j_{12}$, $j_{21}$, $T_i$, $T_\sigma$, $\rho_i$, $P_i$, $P_\sigma$, $E$, $\varphi$, $\rho_{e1}$, and $j_e$ in the regions **Ω**' and **Ω**".

We calculate the distributions of these parameters for the region **Ω**' until the process reaches a steady state, where

$$\frac{\partial T_i}{\partial t} = 0, \quad \frac{\partial P_i}{\partial t} = 0, \quad \frac{\partial \rho_i}{\partial t} = 0 \tag{1}$$

and for the region **Ω**", in addition, until thermodynamic equilibrium is reached between the phases

$$\alpha_2' = \alpha_{2E} \quad \text{or} \quad \rho_V = \rho_{VE}. \tag{2}$$

Within the concepts of [3, 4, 6-8] we represent the problem by the system of equations:

$$\frac{\partial \rho_i}{\partial t} + \nabla(\rho_i \mathbf{v_i}) = \pm n(j_{21} - j_{12}); \quad \frac{\partial n}{\partial t} + \nabla(n\mathbf{v}_2) = \psi; \tag{3}$$

$$\frac{\partial a}{\partial t} = v_{ra} + \frac{j_{12} - j_{21}}{4\pi a^2 \rho_{2a}^0}; \tag{4}$$

$$\frac{\partial v_{ra}}{\partial t} = \frac{1}{a\left(1 - 1.1\alpha_2^{1/3}\right)}\left[\frac{\alpha_1(P_2 - P_1 - 2\sigma/a)}{\rho_1^0} - \frac{4\mu_1\alpha_1}{a\rho_1^0} - \frac{3\left(\alpha_1 - 1.47\alpha_2^{1/3} + 0.33\alpha_2\right)}{2}v_{ra}^2 + \frac{(v_2 - v_1)^2}{4}\right]; \tag{5}$$

$$\rho_i\left[\frac{\partial \mathbf{v_i}}{\partial t} + (\mathbf{v_i} \cdot \nabla)\mathbf{v_i}\right] = \alpha_i\left[-\nabla P^* + \nabla^k \tau_{*1}^k - n(j_{12} - j_{21})(\mathbf{v_2} - \mathbf{v_1})\right] \pm \alpha_1 n \mathbf{F}^* + \rho_i \mathbf{g} + \mathbf{f_{ei}}; \tag{6}$$

$$\mathbf{E} = -\nabla\varphi; \tag{7}$$

$$\varepsilon_0 \nabla(\varepsilon \mathbf{E}) = \sum_{i=1}^{2} \rho_{ei}; \tag{8}$$

$$\mathbf{j_e} = \sigma_e \mathbf{E} + \sum_{i=1}^{2} \rho_{ei} \mathbf{v_i} + \varepsilon_0 \frac{\partial(\varepsilon \mathbf{E})}{\partial t}; \tag{9}$$

$$\sum_{i=1}^{2} \frac{\partial \rho_{ei}}{\partial t} + \nabla \mathbf{j_e} = 0; \tag{10}$$

$$\rho_1 \frac{dk_{v1}}{dt} = -n(\alpha_1 \mathbf{F_m} + \mathbf{F_\mu})(\mathbf{v_2} - \mathbf{v_1}) - 3\rho_1^0 \alpha_2 v_{ra} (\mathbf{v_2} - \mathbf{v_1})^2 - \eta_f n \mathbf{F_\mu} (\mathbf{v_2} - \mathbf{v_1}) - \eta_k \frac{\mu_1}{a^2} k_{v1} -$$
$$- n(j_{12} - j_{21}) \left[ \frac{(\mathbf{v_2} - \mathbf{v_1})^2}{2} - k_{v1} \right]; \tag{11}$$

$$\left[\alpha_i \rho_i^0 c_{pi} + \alpha_i \beta_i P_i (\gamma_i T_i - 1)\right] \left( \frac{\partial T_i}{\partial t} + \mathbf{v_i} \nabla T_i \right) + \frac{\alpha_i \gamma_i P_i T_i}{\rho_i^0} \left( \frac{\partial \rho_i}{\partial t} + \mathbf{v_i} \nabla \rho_i \right) = -4\pi a^2 n \beta_{ti} (T_i - T_\sigma) \mp$$

$$\mp n(j_{12} - j_{21}) \left[ (c_{pi})_a T_\sigma - c_{pi} T_i - \frac{P_{ia}}{\rho_{ia}} + \frac{P_i}{\rho_i^0} \right] + Q_i^*; \tag{12}$$

$$\left[ \frac{\partial \sigma}{\partial T_\sigma} - T_\sigma \frac{\partial \sigma}{\partial T_\sigma} \cdot \frac{\partial^2 \sigma}{\partial \sigma \partial T_\sigma} - T_\sigma \frac{\partial^2 \sigma}{\partial T_\sigma^2} - \frac{\partial T_\sigma}{\partial \sigma} \left( \frac{\partial \sigma}{\partial T_\sigma} \right)^2 \right] \left( \frac{\partial T_\sigma}{\partial t} + \mathbf{v_2} \nabla T_\sigma \right) - \frac{2T_\sigma}{a} \cdot \frac{\partial \sigma}{\partial T_\sigma} \cdot \frac{da}{dt} =$$

$$= \sum_{i=1}^{2} \beta_{ti} (T_i - T_\sigma) + (j_{12} - j_{21}) \cdot \left( \frac{\mu_1 v_{ra}}{\pi a^3 \rho_1^0} \pm \frac{r^*}{4\pi a^2} \right); \tag{13}$$

$$\frac{\partial \rho_{v1}}{\partial t} = D_{v1} \nabla^2 \rho_{v1}; \quad \frac{\partial \rho_{v2}}{\partial t} = D_{v2} \left( \frac{\partial^2 \rho_{v2}}{\partial r^2} + \frac{2}{r} \cdot \frac{\partial \rho_{v2}}{\partial r} \right); \tag{14}$$

$$\rho_2^0 c_{p2} \frac{\partial T_2'}{\partial t} - \frac{\partial P_2'}{\partial t} = \lambda_2 \left( \frac{\partial^2 T_2'}{\partial r^2} + \frac{2}{r} \cdot \frac{\partial T_2'}{\partial r} \right) - (c_{p2} - c_{p1}) \cdot (j_{12} - j_{21}) \cdot \frac{\partial T_2'}{\partial r};$$

$$\rho_2^0 c_{p2} \frac{\partial T_2''}{\partial t} = \lambda_2 \left( \frac{\partial^2 T_2''}{\partial r^2} + \frac{2}{r} \cdot \frac{\partial T_2''}{\partial r} \right). \tag{15}$$

Equation (3) reflects conservation of mass for each phase and conservation of the volume concentration of bubbles and droplets in the dispersed phase, where $\rho_i = \alpha_i \rho_i^0$, $\alpha_1 + \alpha_2 = 1$, and $\alpha_2 = 4\pi a^3 n / 3$. Here and below, the upper sign (+ or -) is taken for the phase $i = 1$, and the lower for the phase $i = 2$, unless otherwise noted. Mass exchange intensity in the region **Ω**' is defined by the diffusion equation within the dispersed particle

$$j_{12}' = -j_{21}' = -\frac{\rho_2^0 D_{v2}}{1 - \rho_{va}} \cdot \left( \frac{\partial \rho_{v2}}{\partial r} \right)_{r=a} + \rho_2^0 \rho_{va} \frac{da}{dt}, \tag{16}$$

which can be solved jointly with the first expressions of Eqs. (14) and (15). In the region **Ω**" the mass exchange intensity is determined by the criterial equation of mass liberation from the phase boundary into the carrier phase [7]:

$$j_{12}'' = -j_{21}'' = -\frac{\rho_1^0 D_{v1} (\rho_{va} - \rho_{v1})}{2a} \cdot \left\{ 2 + 0.625 \cdot \left( \frac{\mu_1}{\rho_1^0 D_{v1}} \right)^{1/3} \left[ (v_2 - v_1) \frac{2a\rho_1^0}{\mu_1} \right]^{1/2} \right\}, \tag{17}$$

which is solved jointly with the second equations of Eqs. (14) and (15).

Equation (4) describes radial motion of the phase boundary of a dispersed particle, pulsations of which are defined by the generalized Rayleigh-Lamb equation (5).

The hydrodynamic state of each phase is defined by Navier-Stokes equation (6), where

$$\tau_{*1}^{kl} = 2\mu_{ef1}\left(e^{kl} - \frac{1}{3}\delta^{kl}\nabla\mathbf{v_1}\right) - \frac{1}{2}\rho_1^0\alpha_2\left[(\mathbf{v_2}-\mathbf{v_1})^k \cdot (\mathbf{v_2}-\mathbf{v_1})^l - \frac{1}{3}\delta^{kl}(\mathbf{v_2}-\mathbf{v_1})^2\right]; \quad (18)$$

$$e^{kl} = \frac{1}{2}\left(\frac{\partial \mathbf{v}_1^k}{\partial x_*^l} + \frac{\partial \mathbf{v}_1^l}{\partial x_1^k}\right); \quad (19)$$

$$P^* = \alpha_1 P_1 + \alpha_2(P_2 - 2\sigma/a) + \rho_1^0\alpha_2\left[v_{ra}^2 + \frac{(\mathbf{v_2}-\mathbf{v_1})^2}{6}\right]; \quad (20)$$

$$\mathbf{F^*} = \mathbf{F_\mu} + \mathbf{F_m} + \mathbf{F_\omega} + \mathbf{F_{\Delta P}} + \mathbf{F_B}; \quad (21)$$

$$\mathbf{f_{e1}} = \rho_{e1}\mathbf{E} - \frac{1}{2}\varepsilon_1 E^2\nabla\varepsilon_1 + \frac{1}{2}\varepsilon_0\nabla\left[\rho_1^0 E^2\left(\frac{\partial\varepsilon_1}{\partial\rho_1^0}\right)_{T_1}\right]; \;\; \mathbf{f_{e2}} = \rho_{e2}\mathbf{E}. \quad (22)$$

Equation (18) is a reduced viscous stress tensor, where the external deformation rate tensor is described by Eq. (19). Equations (20)-(22) describe generalized interphase pressure and force, as well as electrical force density within the phases. Equations for the forces written in Eq. (21) were given in [6]; their relative values can be estimated from the criterion

$$\mathrm{L} = 2a\sqrt{\frac{\rho_1^0}{\mu_1 t_*}} = \sqrt{\mathrm{St}\cdot\mathrm{Re}}\ . \quad (23)$$

For $\mathrm{L} \ll 1$ (Re << 1), $F_m \ll F_B \ll F_\mu$ and we may limit ourselves to the viscous friction force, while for L>> 1 (Re << 1), $F_m \gg F_B \gg F_\mu$ and we consider only "combined mass" force.

The volume charge density Pel appearing in Eq. (22) has a Boltzmann distribution, distorted by the presence of excess charge of one sign $n_i^\pm$ from ionized gas flows

$$\rho_{e1} = -2(\rho_{e1})_0 \,\mathrm{sh}\left(\frac{q_I\varphi}{kT_1}\right) \pm \left(n_{1I}^\pm\right)_0 q_I \cdot \exp\left(\mp\frac{q_I\varphi}{kT_1}\right), \quad (24)$$

where in the region Ω', $\left(n_{1I}^\pm\right)_0 = (n_{1I}')_0$, and in the region Ω", $\left(n_{1I}^\pm\right)_0 = (n_{1I}'')_0$. Here $(n_{1I})_0$ defines charge diffusion from the dispersed phase volume into the carrier phase volume. If $n_I^\pm = 0$, the kinetics of dispersed phase particle charging are defined by the expression

$$\rho_{e2} = [q_\infty + (q_0 - q_\infty)\cdot\exp(-t/\tau)]\cdot n\ , \quad (25)$$

where $q_0 = 3\cdot(\varepsilon_1 - \varepsilon_2)\cdot\varepsilon_0\pi a^2 E/(\varepsilon_2 + 2\varepsilon_1)$ and $q_\infty = 3\cdot(\sigma_{e1} - \sigma_{e2})\cdot\varepsilon_0\pi a^2 E/(\sigma_{e2} + 2\sigma_{e1})$ are, respectively, the initial and limiting particle charges; $\tau = \varepsilon_0\cdot(\varepsilon_2 + 2\varepsilon_1)/(\sigma_{e2} + 2\sigma_{e1})$ is the characteristic time of the charging process. For low conductivities $\sigma_{e1}$ and $\sigma_{e2}$ and $n_I^{\pm}=0$, the dispersed phase charging kinetics are defined by [6]

$$\rho_{e2}=4\pi\varepsilon_0 Ena^2\left(1+2\frac{\varepsilon_2-\varepsilon_1}{\varepsilon_2+2\varepsilon_1}\right)\cdot\frac{q_I\left(n_{1I}^{\pm}\right)_0 b_I t}{4\varepsilon_0+q_I\left(n_{1I}^{\pm}\right)_0 b_I t}\ . \tag{26}$$

For $\operatorname{rot}\mathbf{E}=0$, the relationship between electric field intensity and potential is determined by Eq. (7), while they themselves are found with Poisson equation (8).

The transport current density in Eq. (9) is composed of the mixture conductivity current $\sigma_e\mathbf{E}$, the convection current $\sum_{i=1}^{2}\rho_{ei}\mathbf{v_i}$, and the displacement current $\varepsilon_0\cdot\partial(\varepsilon\mathbf{E})/\partial t$. In the nonsteady case the equation of continuity of the total current has the form of Eq. (10).

The energy equations for the system include Eq. (11) for the kinetic energy $k_{v1}$ of fine-scale motion produced by noncoincidence in phase velocities, while Eqs. (12) and (13) are written for each phase, including the σ-phase, where the additional term is

$$Q_1{*}=\nabla(\lambda_1\nabla T_1)+2\mu_{ef1}\left(e^{kl}-\frac{1}{3}\delta^{kl}\nabla\mathbf{v_1}\right)\cdot e^{kl}+\eta_f n\mathbf{F_\mu}(\mathbf{v_1}-\mathbf{v_2})+\eta_k\frac{\mu_1}{a^2}k_{v1}+\frac{12\alpha_1\alpha_2\mu_1 v_{ra}^2}{a^2};\ \ Q_2{*}=0. \tag{27}$$

The coefficients Sti can be calculated from the criterial equation of dispersed particle heat liberation into the surrounding medium [7] and energy [Eq. (15)]:

$$\beta_{t1}=\frac{\lambda_1}{2a}\left\{2+0.46\cdot\left[\frac{2a\rho_1^0}{\mu_1}(v_1-v_2)^{0.55}\cdot\left(\frac{\mu_1 c_{p1}}{\lambda_1}\right)^{0.33}\right]\right\};\ \ \beta_{t2}=\frac{\lambda_2}{T_2-T_\sigma}\left(\frac{\partial T_2}{\partial r}\right)_{r=a}\ . \tag{28}$$

The sign before the term $r{*}/4\pi a^2$ in Eq. (13) is positive for the region **Ω**" and negative for **Ω**'. The intensity of mass and heat exchange is determined by Eqs. (14)-(17).

System (3)-(15) is completed by dependences of the physical properties of the phases on their state parameters. It should be noted that for greater precision it is necessary to consider the distribution of particles over size, although this complicates description of this class of phenomena even more.

The conditions required for uniqueness may vary depending on the concrete problem considered. For example, as initial conditions we specify thermodynamic and hydrostatic equilibrium of the media in the absence of a dispersed phase and electric field. The boundary conditions include those at the electrodes - the attachment condition, wall temperature, and electrical potential (in addition, on the lower electrode we specify specific flow rate, temperature, pressure, concentrations of the vapor-gas flow ions and bubbling liquid, and the diameter of the perforations through which the flow is supplied); on the free liquid surface we have conditions for the change in aggregate state of the phases, including the function $\psi$ for the dispersed phase in the last expression of Eq. (3), discontinuities in phase velocity and volume content, as well as refraction conditions for the electric field intensity and thermodynamic parameters; on the phase boundaries we have boundary conditions of the fourth sort, attachment conditions, and values of excess charge surface density; on the remaining boundaries we have attachment conditions and boundary conditions of the first sort.

## 3. CONCLUSION

The high efficiency of electric field action on gas-liquid systems has stimulated everincreasing interest of researchers and practicians in the problem of heat mass transport under electrical convection conditions; no less important are the advantages of the latter over natural and, often, other forms of forced convection, its applicability under conditions of weightlessness, the low energy and metal mass requirements of the technologies, and its ecological nature.

## NOTATION

$v$ (or $\mathbf{v}$), $v_{ra}$, and $G$ - linear, radial, and reduced velocity, [m/s]; $T$ - temperature, [K]; $\rho$ - density, [kg/m$^3$] or relative mass concentration; $n_I$ and $n$ - volume density of ions and numerical particle concentration, [m$^{-3}$]; $\rho_e$ - space charge density, [C/m$^3$]; $a$ - particle radius, [m]; $\alpha_i$ - volume content of phase *i*; $j_{12}$ and $j_{21}$ - phase transition intensity, [kg/(m$^2\cdot$s)]; $P$ - pressure, [*Pa*]; $E$ (or $\mathbf{E}$) and $\varphi$, electric field intensity, [V/m] and potential, [V]; $j_e$ (or $\mathbf{j_e}$) - transport current density, [A/m$^2$]; $\psi$ - function which considers processes of breakup, agglomeration, efflux and influx of particles, [(m$^3\cdot$s)$^{-1}$]; $t$ - time, [s]; $D$ - diffusion coefficient, [m$^2$/s]; $\sigma$ - liquid surface tension, [N/m]; $\mu$ - dynamic viscosity coefficient, [*Pa*.s]; $\delta^{kl}$ - Kronecker delta; $x_*$ - generalized coordinate, [m] or [rad]; $\mathbf{F_\mu}$, $\mathbf{F_m}$, $\mathbf{F_\omega}$, $\mathbf{F_{\Delta P}}$, $\mathbf{F_B}$ - interphase viscous friction force, "combined mass" force, force produced by velocity gradient in carrier phase, excess pressure head force for accelerated motion of carrier force, and Basset force, [N]; $\gamma$ and $\beta$, thermal coefficients of pressure and volume expansion, [K$^{-1}$]; $\lambda$ and $\beta_t$ - thermal conductivity, [W/(m$\cdot$K)], and heat liberation, [W/(m$^2\cdot$K)], coefficients; $c_p$ - isobaric specific heat, [J/(kg$\cdot$K)]; $\eta_f$ and $\eta_k$ - energy dissipation coefficients for translational fine-scale motion in boundary layer and Stokes quasistationary flow over particle; $r^*$ - heat of phase transition, [J/kg]; $r$ - spherical coordinate, [m]; $g$ - acceleration of gravity, [m/s$^2$]; St and Re - Struchal and Reynolds numbers; $\varepsilon$ - relative dielectric permittivity; $\varepsilon_0$ - electrical constant, [F/m]; $q_I$ excess charge, [C]; $b_I$ - ion mobility, [m$^2$/(V$\cdot$s)]; $k$ - Boltzmann's constant, [J/K]; $t_*$, $\tau$ and $t_0$ - characteristic time for change in velocity of sphere motion, electrical relaxation time, and characteristic time for change in field induction, [s]; Nu, Ar, K, $\mathrm{Pr}_l$ similarity criteria used in [2] for processing of data on heat liberation under bubbling conditions; $\mathrm{Ar_e}$ - analog of Archimedes number in electrical field. Indices: ' and ", parameters in regions $\Omega$' and $\Omega$"; *k* and $l$ -tensor components; $i = 1$, 2, $\sigma$ - parameters of carrier, dispersed, and "sigma" [4] phases; $s$, 0, $v$ and *a*, parameters at free liquid surface, beginning of the process, vapor component, and particle surface; $E$ and $ef$ - equilibrium and effective values.

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