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CHARACTERIZATIONS OF FUNCTIONS WITH STRONGLY α -CLOSED GRAPHS

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Abstract. In this paper, we study some properties of functions with strongly α -closed graphs by utilizing α -open sets and the α -closure operator.

1. INTRODUCTION AND PRELIMINARIES

The notion of α -open sets was introduced by O. Njåstad [20] in 1965. Since then it has been widely investigated in the literature (see, [1], [2], [3], [9], [10], [11], [12], [15], [16], [17], [18], [19], [21], [23], [24], [26], [27], [28]). Functions with strongly closed graphs were introduced by Herrington and Long [7] to characterize H -closed spaces. Properties of such functions were further investigated by Long and Herrington [14] and Noiri [23]. In this paper, we study some properties of functions with strongly α -closed graphs by utilizing α -open sets and the α -closure operator. Throughout this paper, by (X, τ) and (Y, σ) (or X and Y) we always mean topological spaces. Let A be a subset of X . We denote the interior, the closure and the complement of a set A by $Int(A)$, $Cl(A)$ and $X \setminus A$ or A^c respectively. A subset A of a topological space (X, τ) is called α -open [20] (resp. *semi-open* [13]) if $A \subseteq Int(Cl(Int(A)))$ (resp. $A \subseteq Cl(Int(A))$).

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The complement of an α -open (resp. semi-open) set is called α -closed (resp. semi-closed [5]). By $\alpha O(X, \tau)$ (resp. $SO(X, \tau)$, $\alpha C(X, \tau)$), we denote the family of all α -open (resp. semi-open, α -closed) sets of X . We set $\alpha O(X, x) = \{U \mid x \in U \in \alpha O(X, \tau)\}$, $O(X, x) = \{U \mid x \in U \in \tau\}$ and $\alpha C(X, x) = \{U \mid x \in U \in \alpha C(X, \tau)\}$. The intersection of all α -closed (resp. semi-closed) sets containing A is called the α -closure (resp. semi-closure [4]) of A , denoted by $\alpha Cl(A)$ (resp. $sCl(A)$). A set U in a topological space (X, τ) is an α -neighborhood [16] of a point x if U contains an α -open set V such that $x \in V$.

Lemma 1.1. *The intersection of an arbitrary collection of α -closed sets in (X, τ) is α -closed*

Corollary 1.2. [15]. *Let A be a subset of X . Then, $x \in \alpha Cl(A)$ if and only if for any α -open set U in X containing x , $A \cap U \neq \emptyset$.*

Lemma 1.3. *Let A and B be subsets of a space (X, τ) , then the following properties hold:*

- (1) $A \subset \alpha Cl(A)$.
- (2) If $A \subset B$, then $\alpha Cl(A) \subset \alpha Cl(B)$.
- (3) $\alpha Cl(A)$ is α -closed.
- (4) $\alpha Cl(\alpha Cl(A)) = \alpha Cl(A)$.
- (5) A is α -closed if and only if $A = \alpha Cl(A)$.

Corollary 1.4. *Let A_i ($i \in I$) be a subset of a space (X, τ) , then the following properties hold:*

- (1) $\alpha Cl(\cap \{A_i : i \in I\}) \subset \cap \{\alpha Cl(A_i) : i \in I\}$.
- (2) $\alpha Cl(\cup \{A_i : i \in I\}) \supset \cup \{\alpha Cl(A_i) : i \in I\}$.

Definition 1. *A topological space (X, τ) is said to be:*

- (1) α - T_1 [17], *if for any pair of distinct points x and y in X , there exist an α -open set U in X containing x but not y and an α -open set V in X containing y but not x .*
- (2) α - T_2 [15], *if for any pair of distinct points x and y in X , there exist $U \in \alpha O(X, x)$ and $V \in \alpha O(X, y)$ such that $U \cap V = \emptyset$.*

Lemma 1.5. *A topological space (X, τ) is α - T_2 if and only if it is T_2 .*

Proof. This is shown in [27] and a simple proof is given in [[24], Corollary 4.7].

Definition 2. *A function $f : X \rightarrow Y$ is said to be*

- (1) α -continuous [19] *if $f^{-1}(V) \in \alpha O(X)$ for each open set V of Y ;*

(2) weakly α -continuous [23] if for each $x \in X$ and each $V \in O(Y, f(x))$, there exists $U \in \alpha O(X, x)$ such that $f(U) \subset Cl(V)$.

Lemma 1.6. *Let (X, τ) be a topological space. Then $\alpha Cl(V) = Cl(V)$ for each $V \in SO(X)$.*

Proof. For any $V \in SO(X)$, $\alpha Cl(V) = V \cup Cl(Int(Cl(V))) = V \cup Cl(Int(V)) = V \cup Cl(V) = Cl(V)$.

Lemma 1.7. *A function $f : X \rightarrow Y$ is weakly α -continuous if and only if for each $x \in X$ and each $V \in \alpha O(Y, f(x))$, there exists $U \in \alpha O(X, x)$ such that $f(U) \subset \alpha Cl(V)$.*

Proof. Necessity. Let $x \in X$ and $V \in \alpha O(Y, f(x))$. Then $f(x) \in V \subset Int(Cl(Int(V)))$ and there exists $U \in \alpha O(X, x)$ such that $f(U) \subset Cl(Int(Cl(Int(V))))$. By Lemma 1.6, we have $Cl(Int(Cl(Int(V)))) = Cl(Int(V)) = Cl(V) = \alpha Cl(V)$. Therefore, $f(U) \subset \alpha Cl(V)$.

Sufficiency. Let $x \in X$ and $V \in O(Y, f(x))$. There exists $U \in \alpha O(X, x)$ such that $f(U) \subset \alpha Cl(V)$. By Lemma 1.6, we obtain $f(U) \subset Cl(V)$.

2. STRONGLY α -CLOSED GRAPHS

If $f : (X, \tau) \rightarrow (Y, \sigma)$ is any function, then the subset $G(f) = \{(x, f(x)) : x \in X\}$ of the product space $(X \times Y, \tau \times \sigma)$ is called the graph of f [8].

Definition 3. *A function $f : X \rightarrow Y$ has a strongly α -closed (resp. strongly closed [7]) graph if for each $(x, y) \in (X \times Y) \setminus G(f)$, there exist $U \in \alpha O(X, x)$ (resp. $U \in O(X, x)$) and $V \in O(Y, y)$ such that $(U \times Cl(V)) \cap G(f) = \emptyset$.*

Lemma 2.1. *For a function $f : (X, \tau) \rightarrow (Y, \sigma)$, the following properties are equivalent:*

- (1) $G(f)$ is strongly α -closed;
- (2) For each $(x, y) \in (X \times Y) \setminus G(f)$, there exist $U \in \alpha O(X, x)$ and $V \in O(Y, y)$ such that $f(U) \cap Cl(V) = \emptyset$;
- (3) For each $(x, y) \in (X \times Y) \setminus G(f)$, there exist $U \in \alpha O(X, x)$ and $V \in \alpha O(Y, y)$ such that $(U \times \alpha Cl(V)) \cap G(f) = \emptyset$;
- (4) For each $(x, y) \in (X \times Y) \setminus G(f)$, there exist $U \in \alpha O(X, x)$ and $V \in \alpha O(Y, y)$ such that $f(U) \cap \alpha Cl(V) = \emptyset$.

Proof. It is obvious that $(1) \Leftrightarrow (2)$ and $(3) \Leftrightarrow (4)$.

$(1) \Rightarrow (3)$: Since $\tau \subset \alpha O(X) \subset SO(X)$, by Lemma 1.6 the proof is obvious.

$(3) \Rightarrow (1)$: Let $(x, y) \in (X \times Y) \setminus G(f)$. There exist $U \in \alpha O(X, x)$ and $V \in \alpha O(Y, y)$ such that $(U \times \alpha Cl(V)) \cap G(f) = \emptyset$. Put $G = Int(Cl(Int(V)))$. Then $y \in V \subset G \in \sigma$ and $Cl(G) = Cl(V) = \alpha Cl(V)$. Therefore, we obtain $(U \times Cl(G)) \cap G(f) = (U \times \alpha Cl(V)) \cap G(f) = \emptyset$. This shows that $G(f)$ is strongly α -closed.

Theorem 2.2. *If $f : X \rightarrow Y$ is a function with the strongly α -closed graph, then for each $x \in X$, $f(x) = \cap \{\alpha Cl(f(U)) : U \in \alpha O(X, x)\}$.*

Proof. Suppose the theorem is false. Then there exists a $y \neq f(x)$ such that $y \in \cap \{\alpha Cl(f(U)) : U \in \alpha O(X, x)\}$. This implies that $y \in \alpha Cl(f(U))$ for every $U \in \alpha O(X, x)$. So $V \cap f(U) \neq \emptyset$ for every $V \in \alpha O(Y, y)$. This, in its turn, indicates that $\alpha Cl(V) \cap f(U) \supset V \cap f(U) \neq \emptyset$ which contradicts the hypothesis that f is a function with strongly α -closed graph. Hence the theorem holds.

Theorem 2.3. *If $f : X \rightarrow Y$ is α -continuous and Y is T_2 , then $G(f)$ is strongly α -closed.*

Proof. Let $(x, y) \in (X \times Y) \setminus G(f)$. The T_2 -ness of Y gives the existence of a set $V \in O(Y, y)$ such that $f(x) \notin Cl(V)$. Now $Y \setminus Cl(V) \in O(Y, f(x))$. Therefore, by the α -continuity of f there exists $U \in \alpha O(X, x)$ such that $f(U) \subset Y \setminus Cl(V)$. Consequently, $f(U) \cap Cl(V) = \emptyset$ and therefore $G(f)$ is strongly α -closed.

It is shown in ([14], Theorem 3) and ([22], Theorem 2) that if $f : X \rightarrow Y$ is surjective and $G(f)$ is strongly closed, then Y is Hausdorff. The following theorem is a slight improvement of this result.

Theorem 2.4. *If $f : X \rightarrow Y$ is surjective and has a strongly α -closed graph $G(f)$, then Y is both T_2 and α - T_1 .*

Proof. Let y_1, y_2 ($y_1 \neq y_2$) $\in Y$. The surjectivity of f gives a $x_1 \in X$ such that $f(x_1) = y_1$. Now $(x_1, y_2) \in (X \times Y) \setminus G(f)$. The strongly α -closedness of $G(f)$ provides $U \in \alpha O(X, x_1)$, $V \in O(Y, y_2)$ such that $f(U) \cap Cl(V) = \emptyset$, whence one infers that $y_1 \notin Cl(V)$. This means that there exists $W \in O(Y, y_1)$ such that $W \cap V = \emptyset$. So, Y is T_2 and T_2 -ness always guarantees α - T_1 -ness. Hence Y is α - T_1 .

Theorem 2.5. *A space X is T_2 if and only if the identity function $id : X \rightarrow X$ has a strongly α -closed graph $G(id)$.*

Proof. Necessity. Let X be T_2 . Since the identity function $id : X \rightarrow X$ is continuous, it follows from Theorem 2.4 that $G(id)$ is strongly α -closed.

Sufficiency. Let $G(id)$ be a strongly α -closed graph. Then the surjectivity of id and strong α -closedness of $G(id)$ together imply, by Theorem 2.4, that X is T_2 .

Theorem 2.6. *If $f : X \rightarrow Y$ is an injection and $G(f)$ is strongly α -closed, then X is α - T_1 .*

Proof. Since f is injective, for any pair of distinct points $x_1, x_2 \in X$, $f(x_1) \neq f(x_2)$. Then $(x_1, f(x_2)) \in (X \times Y) \setminus G(f)$. Since $G(f)$ is strongly α -closed, there exist $U \in \alpha O(X, x_1)$, $V \in O(Y, f(x_2))$ such that $f(U) \cap Cl(V) = \emptyset$. Therefore $x_2 \notin U$. Pursuing the same reasoning as before we obtain a set $W \in \alpha O(X, x_2)$ such that $x_1 \notin W$. Hence Y is α - T_1 .

Theorem 2.7. *If $f : X \rightarrow Y$ is a bijection with the strongly α -closed graph, then both X and Y are α - T_1 .*

Proof. The proof is an immediate consequence of Theorems 2.4 and 2.6.

Theorem 2.8. *If a function $f : X \rightarrow Y$ is a weakly α -continuous injection with the strongly α -closed graph $G(f)$, then X is T_2 .*

Proof. Since f is injective, for any pair of distinct points $x_1, x_2 \in X$, $f(x_1) \neq f(x_2)$. Therefore $(x_1, f(x_2)) \in (X \times Y) \setminus G(f)$. Since $G(f)$ is strongly α -closed, there exist $U \in \alpha O(X, x_1)$, $V \in O(Y, f(x_2))$ such that $f(U) \cap Cl(V) = \emptyset$; hence $U \cap f^{-1}(Cl(V)) = \emptyset$. Consequently, $f^{-1}(Cl(V)) \subset X \setminus U$. Since f is weakly α continuous, there exists $W \in \alpha O(X, x_2)$ such that $f(W) \subset Cl(V)$. From this and the foregoing it follows that $W \subset f^{-1}(Cl(V)) \subset X \setminus U$; hence $W \cap U = \emptyset$. Thus for the pair of distinct points $x_1, x_2 \in X$, there exist $U \in \alpha O(X, x_1)$, $W \in \alpha O(X, x_2)$ such that $W \cap U = \emptyset$. By Lemma 1.5, this guarantees the T_2 -ness of X .

Corollary 2.9. *If a function $f : X \rightarrow Y$ is an α -continuous injection with the strongly α -closed graph, then X is T_2 .*

Proof. The proof follows from Theorem 2.9 and the fact that every α -continuous is weakly α -continuous.

Remark 2.10. *If f is not T_2 in Corollary 2.9, then even α -continuity need not imply a strongly α -closed graph. For example, let X be a topological space containing more than one point with the indiscrete topology and let $id : X \rightarrow X$ the identity function. Then id is certainly α -continuous, but the graph of id is not strongly α -closed because $X \times X$ has the indiscrete topology and hence the graph of id being the diagonal set, which is different from the whole space, is not strongly α -closed.*

Theorem 2.11. *If $f : X \rightarrow Y$ is a weakly α -continuous bijection with the strongly α -closed graph, then both X and Y are T_2 .*

Proof. The proof follows from Theorems 2.8 and 2.4.

Lemma 2.12. *Every clopen subset of a quasi H -closed space X is quasi H -closed relative to X .*

Proof. Let B be any clopen subset of a quasi H -closed space X . Let $\{O_\lambda : \lambda \in \Omega\}$ be any cover of B by open sets in X . Then the family $F = \{O_\lambda : \lambda \in \Omega\} \cup \{X \setminus B\}$ is a cover of X by open sets in X . Because of quasi H -closedness of X there exists a finite subfamily $F^* = \{O_{\lambda_i} : 1 \leq i \leq n\} \cup \{X \setminus B\}$ of F whose closure covers X . So, because of clopenness of B we now infer that the family $\{Cl(O_{\lambda_i}) : 1 \leq i \leq n\}$ covers B . Therefore, B is quasi H -closed relative to X .

Theorem 2.13. *If Y is a quasi H -closed extremally disconnected space, then a function $f : X \rightarrow Y$ with the strongly α -closed graph $G(f)$ is weakly α -continuous.*

Proof. Let $x \in X$ and $V \in O(Y, f(x))$. Take any $y \in Y \setminus Cl(V)$. Then $(x, y) \in (X \times Y) \setminus G(f)$. Now the strong α -closedness of $G(f)$ induces the existence of $U_y(x) \in \alpha O(X, x)$, $V_y \in O(Y, y)$ such that $f(U_y(x)) \cap Cl(V_y) = \emptyset \dots (*)$.

Now extremal disconnectedness of Y induces the clopenness of $Cl(V)$ and hence $Y \setminus Cl(V)$ is also clopen. Now $\{V_y : y \in Y \setminus Cl(V)\}$ is a cover of $Y \setminus Cl(V)$ by open sets in Y . By Lemma 2.12, there exists a finite subfamily $\{V_{y_i} : 1 \leq i \leq n\}$ such that $Y \setminus Cl(V) \subset \bigcup_{i=1}^n Cl(V_{y_i})$.

Let $W = \bigcap_{i=1}^n U_{y_i}(x)$, where $U_{y_i}(x)$ are α -open sets in X satisfying

$(*)$. Also, $W \in \alpha O(X, x)$. Now $f(W) \cap (Y \setminus Cl(V)) \subset f[\bigcap_{i=1}^n U_{y_i}(x)] \cap$

$(\bigcup_{i=1}^n Cl(V_{y_i})) \subset \bigcup_{i=1}^n (f[U_{y_i}(x)] \cap Cl(V_{y_i})) = \emptyset$, by (*). Therefore, $f(W) \subset Cl(V)$ and this indicates that f is weakly α -continuous.

Noiri [22] showed that if $G(f)$ is strongly closed then f has the following property:

(P) For every set B which is quasi H -closed relative to Y , $f^{-1}(B)$ is a closed set of X .

Analogously, we have the following theorem.

Theorem 2.14. *If a function $f : X \rightarrow Y$ has a strongly α -closed graph $G(f)$, then f enjoys the following property:*

(P*) *For every set F which is quasi H -closed relative to Y , $f^{-1}(F)$ is α -closed in X .*

Proof. Let $f^{-1}(F)$ be not α -closed in X . Then there exists $x \in \alpha Cl(f^{-1}(F)) \setminus f^{-1}(F)$. Let $y \in F$. Then $(x, y) \in (X \times Y) \setminus G(f)$. Strong α -closedness of $G(f)$ gives the existence of $U_y(x) \in \alpha O(X, x)$ and $V_y \in O(Y, y)$ such that $f(U_y(x)) \cap Cl(V_y) = \emptyset \dots (*)$.

Clearly $\{V_y : y \in F\}$ is a cover of F by open sets in Y . Since F is quasi H -closed relative to Y , there exist a finite number of open sets

$V_{y_1}, V_{y_2}, \dots, V_{y_n}$ in Y such that $F \subset \bigcup_{i=1}^n Cl(V_{y_i})$.

Let $U = \bigcap_{i=1}^n U_{y_i}(x)$, where $U_{y_i}(x)$ are the α -open sets in X satisfying

(*). Also $U \in \alpha O(X, x)$. Now $f(U) \cap F \subset f[\bigcap_{i=1}^n U_{y_i}(x)] \cap (\bigcup_{i=1}^n Cl(V_{y_i})) \subset$

$\bigcup_{i=1}^n (f[U_{y_i}(x)] \cap Cl(V_{y_i})) = \emptyset$. But since $x \in \alpha Cl(f^{-1}(F))$, $U \cap f^{-1}(F) \neq \emptyset$; hence $f(U) \cap F \neq \emptyset$. This is a contradiction. Hence the result holds.

3. ADDITIONAL PROPERTIES

Lemma 3.1. *For a topological space X , the following properties are equivalent:*

- (1) X is Urysohn;
- (2) For every pair of distinct points $x, y \in X$, there exist $U \in \alpha O(X, x)$, $V \in \alpha O(X, y)$ such that $Cl(U) \cap Cl(V) = \emptyset$;
- (3) For every pair of distinct points $x, y \in X$, there exist $U \in \alpha O(X, x)$, $V \in \alpha O(X, y)$ such that $\alpha Cl(U) \cap \alpha Cl(V) = \emptyset$.

Proof. (1) \Rightarrow (2): This is obvious.

(2) \Rightarrow (3): Since $\alpha Cl(U) = Cl(U)$ for each $U \in \alpha(X)$ by Lemma 1.6, this is obvious.

(3) \Rightarrow (1): Suppose that (3) holds. For every pair of distinct points x, y , there exist $U \in \alpha O(X, x)$, $V \in \alpha O(X, y)$ such that $\alpha Cl(U) \cap \alpha Cl(V) = \emptyset$. Now, put $G = Int(Cl(Int(U)))$ and $H = Int(Cl(Int(V)))$, then G and H are open sets containing x and y , respectively. Furthermore, $Cl(G) \cap Cl(H) = Cl(U) \cap Cl(V) = \alpha Cl(U) \cap \alpha Cl(V) = \emptyset$. Therefore, X is Urysohn.

Recall, that a function $f : X \rightarrow Y$ is said to be α -open [19] if $f(A) \in \alpha O(Y)$ for all open set A of X .

Lemma 3.2. *Let a bijection $f : X \rightarrow Y$ be α -open. Then for any closed set B of X , $f(B) \in \alpha C(Y)$.*

Urysohn spaces remain invariant under certain bijective function as is shown in the next theorem.

Theorem 3.3. *If a bijection $f : X \rightarrow Y$ is α -open and X is Urysohn, then Y is Urysohn.*

Proof. Let $y_1, y_2 \in Y$ and $y_1 \neq y_2$. Since f is bijective, $f^{-1}(y_1), f^{-1}(y_2) \in X$ and $f^{-1}(y_1) \neq f^{-1}(y_2)$. The Urysohn property of X gives the existence of sets $U \in O(X, f^{-1}(y_1))$, $V \in O(X, f^{-1}(y_2))$ such that $Cl(U) \cap Cl(V) = \emptyset$. As $Cl(U)$ is a closed set in X , then by the bijectivity and α -openness of f together then indicate, by Lemma 3.2 that $f(Cl(U)) \in \alpha C(Y)$. Therefore by the injectivity of f , $\alpha Cl(f(U)) \cap \alpha Cl(f(V)) \subset f(Cl(U)) \cap f(Cl(V)) = f(Cl(U) \cap Cl(V)) = \emptyset$. Thus α -openness of f gives the existence of two sets $f(U) \in \alpha O(Y, y_1)$, $f(V) \in \alpha O(Y, y_2)$, with $\alpha Cl(f(U)) \cap \alpha Cl(f(V)) = \emptyset$. By Lemma 3.1, Y is Urysohn.

Theorem 3.4. *If $f : X \rightarrow Y$ is weakly α -continuous and Y is Urysohn, then $G(f)$ is strongly α -closed.*

Proof. Let $(x, y) \in (X \times Y) \setminus G(f)$. Then $y \neq f(x)$. Since Y is Urysohn, there exist $V \in O(Y, y)$, $W \in O(Y, f(x))$ such that $Cl(V) \cap Cl(W) = \emptyset$. Since f is weakly α -continuous, there exists $U \in \alpha O(X, x)$ such that $f(U) \subset Cl(W)$. This, therefore, implies that $f(U) \cap Cl(V) = \emptyset$. So by Lemma 2.2, $G(f)$ is strongly α -closed.

Theorem 3.5. *Let X be a Urysohn space. Then any α -open bijection $f : X \rightarrow Y$ has a strongly α -closed graph.*

Proof. Let $(x, y) \in (X \times Y) \setminus G(f)$. Then $y \neq f(x)$ and $y \neq f^{-1}(y)$, where $f^{-1}(y)$ is a singleton. Since X is Urysohn, there exist open sets U_x and U_y such that $x \in U_x$, $f^{-1}(y) \in U_y$ and $Cl(U_x) \cap Cl(U_y) = \emptyset$. Since f is α -open, $f(U_x) \in \alpha O(Y, f(x))$, $f(U_y) \in \alpha O(Y, y)$ and $f(U_x) \cap \alpha Cl(f(U_y)) \subset \alpha Cl(f(U_x)) \cap \alpha Cl(f(U_y)) \subset f(Cl(U_x)) \cap f(Cl(U_y)) = \emptyset$. Therefore, by Lemma 2.2, $G(f)$ is strongly α -closed.

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