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Scientific Studies and Research
Series Mathematics and Informatics
Vol. 19 (2009), No. 1, 83 - 92

COMMON FIXED POINT THEOREMS FOR R-WEAKLY COMMUTING MAPPINGS OF TYPE (Ag) IN UNIFORM SPACES

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Abstract. The notion of R-weak commutativity of type (Ag) and that of compatibility of a pair of self mappings is extended to the framework of uniform spaces and fixed point theorems concerning them are proved. The results obtained in the process generalize several known results in the literature including the recent results of V. Pant, R.P. Pant and others and have a bearing on a problem of B.E. Rhoades.

1. INTRODUCTION

There is a close interdependence between commutativity and existence of fixed points of mappings. For example, Pfeffer [10] showed that an involution **r** of a circle S has a fixed point if and only if there exists a free involution ($\neq$ **r**) of S which commutes with **r**. This relationship between commutativity and existence of fixed points was further highlighted by Jungck [3]. Since then several generalizations of commutativity have been formulated and studied and existence of fixed point theorems concerning them have been investigated by several authors. For example, Sessa [13] defined a weak variant of commutativity called *weak commutativity* and obtained fixed point theorems for weakly commuting mappings. Generalizing weak commutativity, Jungck [4] in 1986 introduced the notion of compatibility of pair of mappings and proved fixed point theorems concerning them. Subsequently, in 1994, R.P.Pant [6] defined R-weak commutativity of mappings and obtained common fixed point theorems concerning them.

Keywords and phrases: common fixed point, R-weakly commuting map of type (Ag), compatible map, non-compatible map, Hausdorff uniform space
(2000) Mathematics Subject Classification: 47H10, 54H25

Later on, in 1997, Pathak, Cho and Kang [9] introduced a variant of *R-weak commutativity* called *R-weak commutativity of type (Ag)* and used the same to obtain common fixed point theorems under Lipschitz type analogue of strict contractive condition.

The study of fixed point theorems in metric spaces is an active and classical area of research. In contrast, the study of fixed point theorems in uniform spaces is more general and is of relatively recent vintage and has been investigated by several authors including Tarafdar [14], Acharya [1], Ganguly [2], Tarafdar, Singh and Watson [15], Rhoades [11], Türgkolu [16] and others. The purpose of the present paper is to extend the notions of R-weak commutativity of mappings of type (Ag) and that of compatibility of mappings to the framework of uniform spaces and to use them to obtain common fixed point theorems for mappings defined on uniform spaces. Our results generalize several known results in the literature including those of V. Pant [8], R.P. Pant [7], and others. Moreover, our approach provides a wider framework for the study of existence of fixed points of which metric fixed point theory is a special case. Furthermore, it is interesting to observe that in case of non-compatible mappings the assumption of completeness or continuity of mappings is not necessary for the existence of fixed points as is well illustrated by the results obtained in the paper.

Rhoades [12] raised the problem of existence of sufficient conditions on the maps which ensure the existence of a fixed point but does not force the maps to be continuous at the fixed point. This question was answered in the affirmative by Pant [7] and is further elaborated in the present paper.

2. PRELIMINARIES AND BASIC DEFINITIONS

Let (X, u) be a uniform space, where u denotes a uniformity on X. The uniform topology induced by the uniformity u will be denoted by τ_u. A family $\{p_\alpha : \alpha \in \Lambda\}$ of pseudometrics on X with indexing set Λ, is called an *associated family* for the uniformity u on X, if the family $\{V(\alpha, r): \alpha \in \Lambda,\ r > 0\}$, where $V(\alpha, r) = \{(x, y) : x, y \in X,\ p_\alpha(x, y) < r\}$ is a subbase for the uniformity u . A family $\{p_\alpha : \alpha \in \Lambda\}$ of pseudometrics on X is called an *augmented associated family* for u , if $\{p_\alpha : \alpha \in \Lambda\}$ is an associated family for u and has the additional property that, given $\alpha, \beta \in \Lambda$ there is a $\nu \in \Lambda$ such that $p_\nu(x, y) \geq \max\{p_\alpha(x, y),\ p_\beta(x, y)\}$ for all $(x, y) \in X \times X$.

It is well known that if (X, u) is a uniform space and $\{p_\alpha : \alpha \in \Lambda\}$ is an augmented associated family of pseudometrics for (X, u), then the collection

$\{V(\alpha, r): \alpha\in\Lambda, r>0\}$ is a base for the uniformity u . It is also well known that for each uniformity u on X the family of all pseudometrics on X which are uniformly continuous with respect to the product uniformity on $X\times X$ constitutes an augmented associated family of pseudometrics for the uniformity u (see [5]).

2.1 Definitions: Let (X, u) be a uniform space and let $A^*(u) = \{p_\alpha : \alpha\in\Lambda\}$ be an augmented associated family of pseudometrics for u . Two self mappings f and g defined on X are said to be

(i) compatible if $\lim_{n\to\infty} p_\alpha(fgx_n, gfx_n) = 0$ for each $\alpha\in\Lambda$, whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = t$ for some t in X.

(ii) non-compatible if there exists an $\alpha\in\Lambda$ and a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = t$ for some t in X but either $\lim_{n\to\infty} p_\alpha(fgx_n, gfx_n) \neq 0$ or non-existent .

(iii) R-weakly commuting of type (Ag) at a point $x\in X$ if there exists $R>0$ such that $p_\alpha(ffx, gfx) \le R\, p_\alpha(fx, gx)$ for each $\alpha\in\Lambda$.

(iv) R-weakly commuting of type (Ag) on X if there exists $R>0$ such that $p_\alpha(ffx, gfx) < R\, p_\alpha(fx, gx)$ for each $\alpha\in\Lambda$ and for each $x\in X$.

We now give an example of a pair of compatible mappings in a uniform space.

2.2 Example: Let $X = R\times R$ and define p_α by $p_\alpha(x, y) = |x_1-y_1|$ for all $x, y \in X$ where $x= (x_1, x_2)$, $y = (y_1, y_2)$.

Let f and g be the self mappings defined on X given by $f(x, y) = (x^3, y^3)$ and $g(x, y) = (2 - x, 2 - y)$, respectively for each $(x, y) \in X$.

Let $\{x_n\}$ be the sequence in X, where $x_n = \left(1+\frac{1}{n}, 1\right)$. Then

$$\lim_{n\to\infty} f(x_n) = \lim_{n\to\infty}\left(\left(1+\frac{1}{n}\right)^3, 1\right) = (1, 1)$$

and

$$\lim_{n\to\infty} g(x_n) = \lim_{n\to\infty}\left(\left(1-\frac{1}{n}\right), 1\right) = (1, 1) .$$

Again, $gfx_n = \left(2-\left(1+\frac{1}{n}\right)^3, 1\right)$, $fgx_n = \left(\left(1-\frac{1}{n}\right)^3, 1\right)$, and so

$$\lim_{n\to\infty} p_\alpha(fgx_n, gfx_n) = \lim_{n\to\infty}\left|2-\left(\left(1+\frac{1}{n}\right)^3-\left(1-\frac{1}{n}\right)^3\right)\right| = 0$$

So f and g are compatible mappings on the uniform space X.

3. COMMON FIXED POINT THEOREMS

3.1 Theorem: *Let (X,* u*) be a Hausdorff uniform space and let* $\{p_\alpha : \alpha\in\Lambda\}$ *be an augmented associated family of pseudometrics for* u . *Let f and g be non-compatible self mappings defined on X such that*

(1) $\overline{fX} \subset gX$ *, where* $\overline{fX}$ *denotes the closure of the range of the mapping f.*

(2) For each $\alpha\in\Lambda$,

$$p_\alpha(fx, fy) < \max\{p_\alpha(gx, gy), k[p_\alpha(fx, gx) + p_\alpha(fy, gy)]/2, [p_\alpha(fy, gx) + p_\alpha(fx, gy)]/2\},\ 1\le k<2$$

whenever right hand side is positive. If f and g are R-weakly commuting mappings of type (Ag), then f and g have a unique common fixed point and the fixed point is a point of discontinuity of both the mappings f and g.

Proof: Since f and g are non-compatible mappings, there exists a sequence $\{x_n\}$ in X, such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = t$ for some t in X and there exists an $\alpha\in\Lambda$ such that

Either $\lim_{n\to\infty} p_\alpha(fgx_n, gfx_n)$ is non-zero or the limit does not exist. (A)

Since $\lim_{n\to\infty} fx_n = t$, $t\in \overline{fX}$. Now $\overline{fX} \subset gX$ implies that there exists a point u in X such that t = gu. We claim that fu = gu. If possible, let fu ≠ gu. Since (X, u) is Hausdorff, there exists an $\alpha\in\Lambda$ such that $p_\alpha(fu, gu) > 0$.

Now consider the inequality

$$p_\alpha(fx_n, fu) < \max\{p_\alpha(gx_n, gu), k[p_\alpha(fx_n, gx_n) + p_\alpha(fu, gu)]/2, [p_\alpha(fu, gx_n) + p_\alpha(fx_n, gu)]/2\},$$

Letting $n\to\infty$, we obtain

$$p_\alpha(gu, fu) \le \max\{p_\alpha(gu, gu), k[p_\alpha(gu, gu) + p_\alpha(fu, gu)]/2, [p_\alpha(fu, gu) + p_\alpha(gu, gu)]/2\}$$
$$= k/2\ p_\alpha(fu, gu) < p_\alpha(fu, gu) \qquad \text{(since } k< 2\text{)},$$

which is a contradiction. Therefore fu = gu.

Now, since f and g are R-weakly commuting mappings of type (Ag),

$p_\alpha(ffu, gfu) \le R\ p_\alpha(fu, gu) = 0$ for every $\alpha\in\Lambda$

Therefore, $p_\alpha(ffu, gfu) = 0$ for every $\alpha\in\Lambda$, which in view of Hausdorffness of X implies that ffu =gfu. We claim that ffu = fu. For if ffu ≠ fu, then there exists $\beta\in\Lambda$ such that $p_\beta(fu, ffu) > 0$. Using condition (2) we obtain that

$$p_\beta(fu, ffu) < \max\{p_\beta(gu, gfu), k[p_\beta(fu, gu) + p_\beta(ffu, gfu)]/2, [p_\beta(ffu, gu) + p_\beta(fu, gfu)]/2\}$$
$$= p_\beta(fu, ffu)$$

that is $p_\beta(fu, ffu) < p_\beta(fu, ffu)$, a contradiction. Thus fu = ffu = gfu. So fu is a common fixed point of f and g.

Proof of uniqueness is easy and hence omitted. We now show that the mappings f and g are discontinuous at the common fixed point fu = gu = t. If possible, suppose that f is continuous. Now considering the sequence $\{x_n\}$ of (A), we get that $\lim_{n\to\infty} ffx_n = ft = t$ and $\lim_{n\to\infty} fgx_n = ft = t$. Now, since f and g are R-weakly commuting mappings of type (Ag), we have

$$p_\alpha(ffx_n, gfx_n) \le R\ p_\alpha(fx_n, gx_n), \quad \text{for each } \alpha\in\Lambda$$

Taking limit as $n\to\infty$, we obtain that $\lim gfx_n = ft$. This in turn yields that $\lim_{n\to\infty} p_\alpha(fgx_n, gfx_n) = p_\alpha(ft, ft) = 0$, for each $\alpha\in\Lambda$. This contradiction to the fact that the mappings f and g are non-compatible, proves that f is discontinuous at the fixed point.

To show that g is discontinuous at the fixed point, assume contrary. Then for the sequence $\{x_n\}$ of (A), we obtain $\lim_{n\to\infty} gfx_n = gt$ and $\lim_{n\to\infty} ggx_n = gt$.

Now, consider the inequality

$$p_\alpha(ft, fgx_n) < \max\{p_\alpha(gt, ggx_n), k[p_\alpha(ft, gt) + p_\alpha(fgx_n, ggx_n)]/2, [p_\alpha(fgx_n, gt) + p_\alpha(ft, ggx_n)]/2\}$$

Taking limit as $n\to\infty$ we obtain that

$$\lim_{n\to\infty} p_\alpha(ft, fgx_n) \le \max\{p_\alpha(gt, gt), k[p_\alpha(ft, gt) + \lim_{n\to\infty} p_\alpha(fgx_n, ggx_n)]/2, [\lim_{n\to\infty} p_\alpha(fgx_n, gt) + p_\alpha(ft, gt)]/2\}.$$

Thus $\lim_{n\to\infty} p_\alpha(ft, fgx_n) \le k/2 \lim_{n\to\infty} p_\alpha(fgx_n, ft)$, which yields a contradiction unless $\lim_{n\to\infty} fgx_n = ft = gt$. But then it contradicts the non-compatibility of the mappings f and g. Thus both f and g are discontinuous at their common fixed point.

3.2 Example

Let $X = [1, 7) \times [1, 7)$ and define pseudometrics p_1 and p_2 on X as $p_1(x, y) = | x_1 - y_1 |$ and $p_2(x, y) = | x_2 - y_2 |$ for each x, y $\in$ X, where x = (x_1, x_2), y = (y_1, y_2). Let u be the uniformity generated by p_1 and p_2. Then (X, u) is a Hausdorff uniform space.

Define f, g: $X \to X$ by

$f(1, 1) = (1, 1)$ $\quad$ $g(1, 1) = (1, 1)$

$f(1, y) = (2, 2)$, if $1 < y < 7$ $\quad$ $g(1, y) = (4, 4)$, if $1 < y < 7$

$f(x, y) = (3, 1)$, if $1 < x \le 2$; $1 \le y < 7$ $\quad$ $g(x, y) = (5, 5)$, if $1 < x \le 2$; $1 \le y < 7$

$f(x, y) = (1, 1)$, if $x > 2$; $1 \le y < 7$ $\quad$ $g(x, y) = (x/2, (y+1)/2)$, if $x > 2$; $1 \le y < 7$

Now it is easily verified that the mappings f and g satisfy the hypothesis of the Theorem 3.1. The point (1, 1) is the unique common fixed point of f and g and is also a point of discontinuity of f and g. To see the non-compatibility of f and g, consider the sequence $\{x_n = (2 + 1/n, 1): n \ge 1\}$ in X. Then $fx_n = (1, 1)$, $gx_n = (1 + 1/2n, 1)$, $fgx_n = (3, 1)$ and $gfx_n = (1, 1)$.

We observe that, $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = (1,1)$ but

$$\lim_{n\to\infty} p_1(fgx_n, gfx_n) = | 3 - 1 | = 2 \ne 0,$$

which shows that f and g are non-compatible mappings. Further, it is easily verified that the mappings f and g are R-weakly commuting mappings of type (Ag).

3.3 Corollary (V. Pant [8,Theorem 1]): *Let f and g be non-compatible self mappings defined on a metric space (X, d) such that*

(1) $\overline{fX} \subset gX$ *, where* $\overline{fX}$ *denotes the closure of the range of the mapping f.*

(2) $d(fx, fy) < \max \{d(gx, gy), k[d(fx, gx) + d(fy, gy)] /2, [d(fy, gx) + d(fx, gy)] / 2\}, \quad 1 \le k < 2$

whenever right hand side is positive. If f and g are R-weakly commuting mappings of type (Ag), then f and g have a unique common fixed point and the fixed point is a point of discontinuity of both the mappings f and g.

Proof: Let Λ be a singleton and use Theorem 3.1.

3.4 Corollary (R.P.Pant [7,Theorem 2]): *Let f and g be non-compatible self mappings defined on a metric space (X, d) such that*

(1) $\overline{fX} \subset gX$ *, where* $\overline{fX}$ *denotes the closure of the range of the mapping f.*

(2) $d(fx, fy) < \max\{d(gx, gy), [d(fx, gx) + d(fy, gy)]/2, [d(fy, gx) + d(fx, gy)]/2\}$,

whenever right hand side is positive. If f and g are R-weakly commuting mappings of type (Ag), then f and g have a unique common fixed point and the fixed point is a point of discontinuity of both the mappings f and g.

Proof: Let Λ be a singleton, k =1 and use Theorem 3.1.

3.5 Theorem: *Let (X,* u*) be a Hausdorff uniform space and let $\{ p_\alpha : \alpha\in\Lambda\}$ be an augmented associated family of pseudometrics for* u *. Let f and g be non-compatible self mappings defined on X satisfying the conditions that*

(1) $\lim_{n\to\infty} ffx_n = ft$ and $\lim_{n\to\infty} gfx_n = gt$, *whenever $\{x_n\}$ is a sequence in X such that* $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = t$ *for some t in X and*

(2) For each $\alpha\in\Lambda$,

$$p_\alpha(fx, fy) < \max\{ p_\alpha(gx, gy), k[p_\alpha(fx, gx) + p_\alpha(fy, gy)]/2, [p_\alpha(fy, gx) + p_\alpha(fx, gy)]/2\}, \quad 1 \le k < 2$$

whenever right hand side is positive. If f and g are R-weakly commuting mappings of type (Ag), then f and g have a unique common fixed point and the fixed point is a point of discontinuity of both the mappings f and g.

Proof: Since f and g are non-compatible mappings there exists a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = u$ for some u in X, but there exists an $\alpha\in\Lambda$ such that $\lim_{n\to\infty} p_\alpha(fgx_n, gfx_n)$ is either non-zero or non-existent. (B)

Now, condition (1) implies that $\lim_{n\to\infty} ffx_n = fu$ and $\lim_{n\to\infty} gfx_n = gu$. Since f and g are R-weakly commuting of type (Ag), we have $p_\alpha(ffx_n, gfx_n) \le R\, p_\alpha(fx_n, gx_n)$ for each $\alpha\in\Lambda$. So on letting $n\to\infty$, this yields that fu = gu.

Again, using R-weak commutativity of type (Ag) of the mappings f and g, we obtain that $p_\alpha(ffu, gfu) \le R\, p_\alpha(fu, gu) = 0$ for each $\alpha\in\Lambda$. This implies that ffu = gfu.

We claim that fu = ffu. For if, fu $\neq$ ffu, then there exists an $\alpha\in\Lambda$ such that $p_\alpha(fu, ffu) > 0$. Using condition (2), we get that

$$\begin{aligned} p_\alpha(fu, ffu) &< \max\{ p_\alpha(gu, gfu), k[p_\alpha(fu, gu) + p_\alpha(ffu, gfu)]/2, \\ &\qquad [p_\alpha(fu, gfu) + p_\alpha(ffu, gu)]/2]\}, \\ &= p_\alpha(fu, ffu). \end{aligned}$$

That is p_α(fu, ffu) < p_α(fu, ffu), which is a contradiction. Thus fu is a common fixed point of f and g.

Proof of uniqueness is easy and hence omitted. We now show that the mappings f and g are discontinuous at the common fixed point t = fu = gu. If possible, suppose f is continuous. Then considering the sequence $\{x_n\}$ of (B), we get $\lim_{n\to\infty} ffx_n = fu = t$ and $\lim_{n\to\infty} fgx_n = fu = t$. Again R-weak commutativity of type (Ag) of mappings f and g implies that $p_\alpha(ffx_n, gfx_n) \le R\, p_\alpha(fx_n, gx_n)$ for each $\alpha\in\Lambda$. On letting $n\to\infty$, this yields $\lim_{n\to\infty} gfx_n = fu = t$. This, in turn yields that $\lim_{n\to\infty} p_\alpha(fgx_n, gfx_n) = 0$ for each $\alpha\in\Lambda$. This contradiction to the non-compatibility of the mappings f and g proves that f is discontinuous at the fixed point. Similarly, to prove that g is discontinuous at the fixed point, assume contrary. Then for the sequence $\{x_n\}$ of (B), we get that $\lim_{n\to\infty} gfx_n = gu = t$ and $\lim_{n\to\infty} ggx_n = gu = t$.

From inequality (2), we obtain that

$$p_\alpha(ft, fgx_n) < \max\{p_\alpha(gt, ggx_n), k[p_\alpha(ft, gt) + p_\alpha(fgx_n, ggx_n)]/2, [p_\alpha(fgx_n, gt) + p_\alpha(ft, ggx_n)]/2\}.$$

Taking limit as $n\to\infty$, this yields a contradiction unless $\lim_{n\to\infty} fgx_n = ft = t$. But then it contradicts the fact that the mappings f and g are non-compatible. Thus both f and g are discontinuous at their common fixed point.

3.6 Remark: Example 3.2 satisfies the hypothesis of Theorem 3.5 and the point (1, 1) is unique common fixed point of the mappings f and g which are discontinuous at the point (1, 1).

3.7 Corollary (V. Pant [8, Theorem 2]): *Let f and g be two non-compatible self mappings defined on a metric space (X, d) satisfying the conditions that*

(1) $\lim_{n\to\infty} ffx_n = ft$ and $\lim_{n\to\infty} gfx_n = gt$, *whenever $\{x_n\}$ is a sequence in X such that* $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = t$ *for some t in X and*

(2) $d(fx, fy) < \max\{d(gx, gy), k[d(fx, gx) + d(fy, gy)]/2, [d(fy, gx) + d(fx, gy)]/2\}, \quad 1 \le k < 2$

whenever right hand side is positive. If f and g are R-weakly commuting mappings of type (Ag), then f and g have a unique common fixed point and the fixed point is a point of discontinuity of both the mappings f and g.

Proof: Let Λ be a singleton and use Theorem 3.5.

3.8 Theorem: *Let (X,* u*) be a Hausdorff uniform space and let* $\{p_\alpha : \alpha \in \Lambda\}$ *be an augmented associated family of pseudometrics for* u *. Let f and g be non-compatible self mappings defined on X satisfying the conditions that*

(1) $\lim_{n\to\infty} ffx_n = ft$ *and* $\lim_{n\to\infty} gfx_n = gt$ *, whenever* $\{x_n\}$ *is a sequence in X such that* $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = t$ *for some t in X*

and

(2) For each $\alpha \in \Lambda$,

$p_\alpha(fx, fy) < \max\{p_\alpha(gx, gy), p_\alpha(fx, gx), p_\alpha(fy, gy), [p_\alpha(fy,gx)+ p_\alpha(fx, gy)] / 2\}$,

whenever the right hand side is positive.

If f and g are R-weakly commuting mappings of type (Ag), then f and g have a unique common fixed point.

Proof of Theorem 3.8 is similar to that of Theorem 3.5 except for obvious modifications and hence omitted.

3.9 Corollary (R.P. Pant [7, Theorem 3]): *Let f and g be two non-compatible self mappings defined on a metric space (X, d) satisfying the conditions that*

(1) $\lim_{n\to\infty} ffx_n = ft$ and $\lim_{n\to\infty} gfx_n = gt$, *whenever* $\{x_n\}$ *is a sequence in X such that* $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = t$ *for some t in X and*

(2) $d(fx, fy) < \max\{d(gx, gy), d(fx, gx), d(fy, gy), [d(fy, gx) + d(fx, gy)] / 2\}$,

whenever right hand side is positive. If f and g are R-weakly commuting mappings of type (Ag), then f and g have a unique common fixed point.

Proof: Let Λ be a singleton and use Theorem 3.8.

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