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Scientific Studies and Research
Series Mathematics and Informatics
Vol. 19 (2009), No. 1, 145 - 154

MINIMAL STRUCTURES, PUNCTUALLY m-OPEN FUNCTIONS AND BITOPOLOGICAL SPACES

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Abstract. By using m-open functions from a topological space into an m-space, we establish the unified theory for several weak forms of open functions between bitopological spaces.

1. Introduction

Semi-open sets, preopen sets, α-open sets, β-open sets and δ-open sets play an important role in the researching of generalizations of open functions in topological spaces and bitopological spaces. By using these sets, several authors introduced and studied various types of modifications of open functions in topological spaces and bitopological spaces. Maheshwari and Prasad [9] and Bose [1] introduced the concepts of semi-open sets and semi-open functions in bitopological spaces. Jelić [2], [4], Kar and Bhattacharyya [5] and Khedr et al. [6] introduced and studied the concepts of preopen sets and preopen functions in bitopological spaces. The notions of α-open sets and α-open functions in bitopological spaces were studied in [3], [11] and [7].

Keywords and phrases: m-structure, m-open set, (i, j)-m-open function, bitopological space.
(2000)Mathematics Subject Classification: 54A05, 54C10; 54E55.

Recently, in [13] and [14] the present authors introduced the notions of minimal structures, m-spaces and m-continuity. Quite recently, in [12], they have introduced the *global* notion of m-open functions.

In the present paper, we introduce the notion of punctual m-open functions. We obtain some characterizations of punctual m-open functions and characterize the set of all points at which a function is not m-open. In the last part, punctually m-open functions in bitopological spaces is introduced and investigated.

2. Preliminaries

Let (X, τ) be a topological space and A a subset of X. The closure of A and the interior of A are denoted by $\mathrm{Cl}(A)$ and $\mathrm{Int}(A)$, respectively. Throughout the present paper, (X, τ) and (Y, σ) always denote topological spaces and (X, τ_1, τ_2) and (Y, σ_1, σ_2) denote bitopological spaces. The closure of A and the interior of A with respect to τ_i are denoted by $i\mathrm{Cl}(A)$ and $i\mathrm{Int}(A)$, respectively, for $i = 1, 2$.

Definition 2.1. A subfamily m_X of the power set $\mathcal{P}(X)$ of a nonempty set X is called a *minimal structure* (or briefly *m-structure*) [13], [14] on X if $\emptyset \in m_X$ and $X \in m_X$.

By (X, m_X) (or briefly (X, m)), we denote a nonempty set X with a minimal structure m_X on X and call it an *m-space*. Each member of m_X is said to be *m_X-open* (or briefly *m-open*) and the complement of an m_X-open set is said to be *m_X-closed* (or briefly *m-closed*).

Definition 2.2. Let X be a nonempty set and m_X an m-structure on X. For a subset A of X, the *m_X-closure* of A and the *m_X-interior* of A are defined in [10] as follows:

(1) $m_X\text{-}\mathrm{Cl}(A) = \cap\{F : A \subset F, X - F \in m_X\}$,

(2) $m_X\text{-}\mathrm{Int}(A) = \cup\{U : U \subset A, U \in m_X\}$.

Lemma 2.1. (Maki et al. [10]). *Let (X, m_X) be an m-space. For subsets A and B of X, the following properties hold:*

(1) $m_X\text{-}\mathrm{Cl}(X - A) = X - m_X\text{-}\mathrm{Int}(A)$ *and* $m_X\text{-}\mathrm{Int}(X - A) = X - m_X\text{-}\mathrm{Cl}(A)$*,*

(2) If $(X - A) \in m_X$*, then* $m_X\text{-}\mathrm{Cl}(A) = A$ *and if* $A \in m_X$*, then* $m_X\text{-}\mathrm{Int}(A) = A$*,*

(3) $m_X\text{-}\mathrm{Cl}(\emptyset) = \emptyset$, $m_X\text{-}\mathrm{Cl}(X) = X$, $m_X\text{-}\mathrm{Int}(\emptyset) = \emptyset$ *and* $m_X\text{-}\mathrm{Int}(X) = X$*,*

(4) If $A \subset B$, then m_X-Cl$(A) \subset m_X$-Cl(B) and m_X-Int$(A) \subset m_X$-Int(B),
(5) $A \subset m_X$-Cl(A) and m_X-Int$(A) \subset A$,
(6) m_X-Cl$(m_X$-Cl$(A)) = m_X$-Cl(A) and m_X-Int$(m_X$-Int$(A)) = m_X$-Int(A).

Lemma 2.2. (Popa and Noiri [13]). *Let (X, m_X) be an m-space and A a subset of X. Then $x \in m_X$-Cl(A) if and only if $U \cap A \neq \emptyset$ for every $U \in m_X$ containing x.*

Definition 2.3. A minimal structure m_X on a nonempty set X is said to have *property $\mathcal{B}$* [10] if the union of any family of subsets belonging to m_X belongs to m_X.

Lemma 2.3. (Popa and Noiri [15]). *Let (X, m_X) be an m-space and m_X satisfy property $\mathcal{B}$. Then for a subset A of X, the following properties hold:*
(1) $A \in m_X$ if and only if m_X-Int$(A) = A$,
(2) A is m-closed if and only if m_X-Cl$(A) = A$,
(3) m_X-Int$(A) \in m_X$ and m_X-Cl(A) is m_X-closed.

3. Punctual m-open functions

Definition 3.1. Let (Y, m_Y) be an m-space. A function $f : (X, \tau) \to (Y, m_Y)$ is said to be *m-open* at $x \in X$ if for each open set U containing x, there exists $V \in m_Y$ containing $f(x)$ such that $V \subset f(U)$. If f is m-open at each point $x \in X$, then f is said to be *m-open*.

Theorem 3.1. *A function $f : (X, \tau) \to (Y, m_Y)$ is m-open at $x \in X$ if and only if for each open set U containing x, $x \in f^{-1}(m_Y$-Int$(f(U)))$.*

Proof. *Necessity.* Let U be an open set containing x. Then, there exists $V \in m_Y$ such that $f(x) \in V \subset f(U)$ and hence $f(x) \in m_Y$-Int$(f(U))$. Therefore, we obtain that $x \in f^{-1}(m_Y$-Int$(f(U)))$.

Sufficiency. Suppose that $x \in f^{-1}(m_Y$-Int$(f(U)))$ for each open set U containing x. Then $f(x) \in m_Y$-Int$(f(U))$. Hence there exists $V \in m_Y$ containing $f(x)$ such that $V \subset f(U)$. Therefore, f is m-open at x.

Theorem 3.2. *A function $f : (X, \tau) \to (Y, m_Y)$ is m-open if and only if m_Y-Int$(f(U)) = f(U)$ for each open set U of X.*

Proof. *Necessity.* Let U be an open set of X and $x \in U$. Then, by Theorem 3.1 we have $x \in f^{-1}(m_Y\text{-Int}(f(U)))$. Hence $f(x) \in m_Y\text{-Int}(f(U))$. Therefore, $f(U) \subset m_Y\text{-Int}(f(U))$ and by Lemma 3.1 $f(U) = m_Y\text{-Int}(f(U))$.

Sufficiency. Let $x \in X$ and U be an open set of X containing x. Then we have $f(x) \in f(U) = m_Y\text{-Int}(f(U))$. Therefore $x \in f^{-1}(m_Y\text{-Int}(f(U))$. By Theorem 3.1, f is m-open at x.

Remark 3.1. (a) The characterization of Theorem 3.2 is used in [12] as the definition of m-open functions.

(b) If m_Y has property $\mathcal{B}$, by Lemma 2.3 we obtain that a function $f : (X, \tau) \to (Y, m_Y)$ is m-open if and only if $f(U)$ is m_Y-open for each open set U of X.

(c) Let (Y, σ) be a topological space. If $m_Y = \mathrm{SO}(Y)$ (resp. $\mathrm{PO}(Y)$, $\alpha(Y)$, $\beta(Y)$), we obtain the definition of semi-open (resp. preopen, α-open, β-open) function, where $\mathrm{SO}(Y)$ (resp. $\mathrm{PO}(Y)$, $\alpha(Y)$, $\beta(Y)$) is the family of all semi-open (resp. preopen, α-open, β-open) sets of Y.

Theorem 3.3. *For a function $f : (X, \tau) \to (Y, m_Y)$, the following properties are equivalent:*

(1) f is m-open at x;

(2) If $x \in \mathrm{Int}(A)$ for $A \in \mathcal{P}(X)$, then $x \in f^{-1}(m_Y\text{-Int}(f(A)))$;

(3) $x \in \mathrm{Int}(f^{-1}(B))$ for $B \in \mathcal{P}(Y)$, then $x \in f^{-1}(m_Y\text{-Int}(B))$;

(4) If $x \in f^{-1}(m_Y\text{-Cl}(B))$ for $B \in \mathcal{P}(Y)$, then $x \in \mathrm{Cl}(f^{-1}(B))$.

Proof. (1) $\Rightarrow$ (2): Let $A \in \mathcal{P}(X)$ and $x \in \mathrm{Int}(A)$. Then, there exists an open set U such that $x \in U \subset A$ and hence $f(x) \in f(U) \subset f(A)$. Since f is m-open at x, by Theorem 3.1 $x \in f^{-1}(m_Y\text{-Int}(f(U))) \subset f^{-1}(m_Y\text{-Int}(f(A)))$.

(2) $\Rightarrow$ (3): Let $B \in \mathcal{P}(Y)$ and $x \in \mathrm{Int}(f^{-1}(B))$. Then, $f(x) \in m_Y\text{-Int}(f(f^{-1}(B))) \subset m_Y\text{-Int}(B)$. Therefore, we have $x \in f^{-1}(m_Y\text{-Int}(B))$.

(3) $\Rightarrow$ (4): Let $B \in \mathcal{P}(Y)$ and $x \notin \mathrm{Cl}(f^{-1}(B))$. Then $x \in X - \mathrm{Cl}(f^{-1}(B)) = \mathrm{Int}(X - f^{-1}(B)) = \mathrm{Int}(f^{-1}(Y - B))$. By (3) we have $x \in f^{-1}(m_Y\text{-Int}(Y - B)) = X - f^{-1}(m_Y\text{-Cl}(B))$. Therefore, $x \notin f^{-1}(m_Y\text{-Cl}(B))$.

(4) $\Rightarrow$ (1): Let U be an open set of X containing x and $B = Y - f(U)$. Since $\mathrm{Cl}(f^{-1}(B)) = \mathrm{Cl}(f^{-1}(Y - f(U))) = \mathrm{Cl}(X - f^{-1}(f(U))) \subset X - \mathrm{Int}(U) = X - U$ and $x \in U$, we obtain that $x \notin \mathrm{Cl}(f^{-1}(B))$. By (4), we have $x \notin f^{-1}(m_Y\text{-Cl}(B)) = f^{-1}(m_Y\text{-Cl}(Y - f(U))) = X - f^{-1}(m_Y\text{-Int}(f(U)))$.

Therefore, $x \in f^{-1}(m_Y\text{-Int}(f(U)))$. By Theorem 3.1, f is m-open at x.

For a function $f : (X, \tau) \to (Y, m_Y)$, we denote

$$D^0(f) = \{x \in X\colon f \text{ is not } m\text{-open at } x\}.$$

Theorem 3.4. *For a function $f : (X, \tau) \to (Y, m_Y)$, the following properties hold:*

$D^0(f) = \cup_{U \in \tau}\{U - f^{-1}(m_Y\text{-Int}(f(U)))\}$
$= \cup_{A \in \mathcal{P}(X)}\{\text{Int}(A) - f^{-1}(m_Y\text{-Int}(f(A)))\}$
$= \cup_{B \in \mathcal{P}(Y)}\{\text{Int}(f^{-1}(B)) - f^{-1}(m_Y\text{-Int}(B))\}$
$= \cup_{B \in \mathcal{P}(Y)}\{f^{-1}(m_Y\text{-Cl}(B)) - \text{Cl}(f^{-1}(B))\}$.

Proof. Let $x \in D^0(f)$. Then, by Theorem 3.1, there exists an open set U_0 containing x such that $x \notin f^{-1}(m_Y\text{-Int}(f(U_0)))$. Hence $x \in U_0 \cap (X - f^{-1}(m_Y\text{-Int}(f(U_0)))) = U_0 - f^{-1}(m_Y\text{-Int}(f(U_0))) \subset \cup_{U \in \tau}\{U - f^{-1}(m_Y\text{-Int}(f(U)))\}$.

Conversely, let $x \in \cup_{U \in \tau}\{U - f^{-1}(m_Y\text{-Int}(f(U)))\}$. Then there exists $U_0 \in \tau$ such that $x \in U_0 - f^{-1}(m_Y\text{-Int}(f(U_0)))$. Therefore, by Theorem 3.1 $x \in D^0(f)$.

For the second equation, let $x \in D^0(f)$. Then, by Theorem 3.3, there exists $A_1 \in \mathcal{P}\ (X)$ such that $x \in \text{Int}(A_1)$ and $x \notin f^{-1}(m_Y\text{-Int}(f(A_1)))$. Therefore, $x \in \text{Int}(A_1) - f^{-1}(m_Y\text{-Int}(f(A_1))) \subset \cup_{A \in \mathcal{P}(X)}\{\text{Int}(A) - f^{-1}(m_Y\text{-Int}(f(A)))\}$.

Conversely, $x \in \cup_{B \in \mathcal{P}(Y)}\{\text{Int}(f^{-1}(B)) - f^{-1}(m_Y\text{-Int}(B))\}$. Then there exists $A_1 \in \mathcal{P}(X)$ such that $x \in \text{Int}(A_1) - f^{-1}(m_Y\text{-Int}(f(A)))$. By Theorem 3.3, $x \in D^0(f)$. The other equations are silimarly proved.

4. Minimal structures on bitopological spaces

First, we shall recall some definitions of weak forms of open sets in a bitopological space.

Definition 4.1. A subset A of a bitopological space (X, τ_1, τ_2) is said to be

(1) (i, j)-*semi-open* [9] if $A \subset j\text{Cl}(i\text{Int}(A))$, where $i \neq j$, i, $j = 1$, 2,
(2) (i, j)-*preopen* [2] if $A \subset i\text{Int}(j\text{Cl}(A))$, where $i \neq j$, i, $j = 1$, 2,
(3) (i, j)-*α-open* [3] if $A \subset i\text{Int}(j\text{Cl}(i\text{Int}(A)))$, where $i \neq j$, i, $j = 1$, 2,
(4) (i, j)-*semi-preopen* [6] if there exists an (i, j)-preopen set U such that $U \subset A \subset j\text{Cl}(U)$, where $i \neq j$, i, $j = 1$, 2.

The family of (i,j)-semi-open (resp. (i,j)-preopen, (i,j)-α-open, (i,j)-semi-preopen) sets of (X,τ_1,τ_2) is denoted by $(i,j)\mathrm{SO}(X)$ (resp. $(i,j)\mathrm{PO}(X)$, $(i,j)\alpha(X)$, $(i,j)\mathrm{SPO}(X)$).

Remark 4.1. Let (X,τ_1,τ_2) be a bitopological space and A a subset of X. Then $(i,j)\mathrm{SO}(X)$, $(i,j)\mathrm{PO}(X)$, $(i,j)\alpha(X)$ and $(i,j)\mathrm{SPO}(X)$ are all m-structures on X. Hence, if $m_{ij} = (i,j)\mathrm{SO}(X)$ (resp. $(i,j)\mathrm{PO}(X)$, $(i,j)\alpha(X)$, $(i,j)\mathrm{SPO}(X)$), then we have

(1) m_{ij}-Cl$(A) = (i,j)$-sCl(A) [9] (resp. (i,j)-pCl(A) [6], (i,j)-αCl(A) [11], (i,j)-spCl(A) [6]),

(2) m_{ij}-Int$(A) = (i,j)$-sInt(A) (resp. (i,j)-pInt(A), (i,j)-αInt(A), (i,j)-spInt(A)).

Remark 4.2. Let (X,τ_1,τ_2) be a bitopological space.

(a) Let $m_{ij} = (i,j)\mathrm{SO}(X)$ (resp. $(i,j)\alpha(X)$). Then, by Lemma 2.1 we obtain the result established in Theorem 13 of [9] and Theorem 1.13 of [8] (resp. Theorem 3.6 of [11]).

(b) Let $m_{ij} = (i,j)\mathrm{SO}(X)$ (resp. $(i,j)\mathrm{PO}(X)$, $(i,j)\alpha(X)$, $(i,j)\mathrm{SPO}(X)$). Then, by Lemma 2.2 we obtain the result established in Theorem 1.15 of [8] (resp. Theorem 3.5 of [6], Theorem 3.5 of [11], Theorem 3.5 of [6]).

Remark 4.3. Let (X,τ_1,τ_2) be a bitopological space.

(a) It follows from Theorem 2 of [9] (resp. Theorem 4.2 of [5] or Theorem 3.2 of [6], Theorem 3.2 of [11], Theorem 3.2 of [6]) that $(i,j)\mathrm{SO}(X)$ (resp. $(i,j)\mathrm{PO}(X)$, $(i,j)\alpha(X)$, $(i,j)\mathrm{SPO}(X)$) is an m-structure on X satisfying property $\mathcal{B}$.

(b) Let $m_{ij} = (i,j)\mathrm{SO}(X)$ (resp. $(i,j)\mathrm{PO}(X)$, $(i,j)\alpha(X)$, $(i,j)\mathrm{SPO}(X)$). Then, by Lemma 2.3 we obtain the result established in Theorem 1.13 of [8] (resp. Theorem 3.5 of [6], Theorem 3.6 of [11], Theorem 3.6 of [6]).

5. Punctual m-open functions and bitopological spaces

Definition 5.1. A function $f : (X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ is said to be *(i,j)-semi-open* [1] (resp. *(i,j)-preopen* [5], *(i, j)-α-open* [7], *(i, j)-semi-preopen*) if for each τ_i-open set U of X, $f(U)$ is (i,j)-semi-open (resp. (i,j)-preopen, (i,j)-α-open, (i,j)-semi-preopen) in Y.

Definition 5.2. Let (Y,σ_1,σ_2) be a bitopological space and m_{ij} an m-structure on Y determined by σ_1 and σ_2. A function $f : (X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ is said to be *(i,j)-m-open at $x \in X$* if $f : (X,\tau_i) \to (Y,m_{ij})$

is m-open at x. The function f is said to be *(i, j)-m-open* if it is (i, j)-m-open at each $x \in X$.

Remark 5.1. (a) By Definition 5.2, it follows that a function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is (i, j)-m-open at x if and only if for each τ_i-open set U containing x, there exists an m_{ij}-open set V of Y such that $f(x) \in V$ and $V \subset f(U)$.

(b) Let $m_{ij} = (i, j)\mathrm{SO}(X)$ (resp. $(i, j)\mathrm{PO}(X)$, $(i, j)\alpha(X)$, $(i, j)\mathrm{SPO}(X)$). If $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is (i, j)-m-open at $x \in X$, then f is said to be (i, j)-semi-open (resp. (i, j)-preopen, (i, j)-α-open, (i, j)-semi-preopen) at x.

(c) By Remark 4.3(a), $(i, j)\mathrm{SO}(Y)$, $(i, j)\mathrm{PO}(Y)$, $(i, j)\alpha(Y)$ and $(i, j)\mathrm{SPO}(Y)$ are all m-structures on Y satisfying property $(\mathcal{B})$. Therefore, it follows from Theorem 3.2 that $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is (i, j)-semi-open (resp. (i, j)-preopen, (i, j)-α-open, (i, j)-semi-preopen) if $f : (X, \tau_i) \to (Y, m_{ij})$ is m-open, where $m_{ij} = (i, j)\mathrm{SO}(Y)$ (resp. $(i, j)\mathrm{PO}(Y)$, $(i, j)\alpha(Y)$, $(i, j)\mathrm{SPO}(Y)$).

By Definition 5.2 and Theorems 3.1 and 3.3, we obtain the following theorems.

Theorem 5.1. *Let (Y, σ_1, σ_2) be a bitopological space and m_{ij} an m-structure on Y determined by σ_1 and σ_2. A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is (i, j)-m-open at $x \in X$ if and only if $x \in f^{-1}(m_{ij}\text{-}\mathrm{Int}(f(U)))$ for every τ_i-open set U containing x.*

Theorem 5.2. *Let (Y, σ_1, σ_2) be a bitopological space and m_{ij} an m-structure on Y determined by σ_1 and σ_2. For a function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:*

(1) f is (i, j)-m-open at $x \in X$;

(2) If $x \in i\mathrm{Int}(A)$ for $A \in \mathcal{P}(X)$, then $x \in f^{-1}(m_{ij}\text{-}\mathrm{Int}(f(A))$;

(3) If $x \in i\mathrm{Int}(f^{-1}(B))$ for $B \in \mathcal{P}(Y)$, then $x \in f^{-1}(m_{ij}\text{-}\mathrm{Int}(B))$;

(4) If $x \in f^{-1}(m_{ij}\text{-}\mathrm{Cl}(B))$ for $B \in \mathcal{P}(Y)$, then $x \in i\mathrm{Cl}(f^{-1}(B))$.

For example, put $m_{ij} = (i, j)\mathrm{SO}(Y)$, then we obtain the following characterizations:

Corollary 5.1. *For a function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:*

(1) f is (i, j)-semi-open at $x \in X$;

(2) If $x \in i\mathrm{Int}(A)$ for $A \in \mathcal{P}(X)$, then $x \in f^{-1}((i, j)\text{-}\mathrm{sInt}(f(A))$;

(3) If $x \in i\mathrm{Int}(f^{-1}(B))$ for $B \in \mathcal{P}(Y)$, then $x \in f^{-1}((i, j)\text{-}\mathrm{sInt}(B))$;

(4) If $x \in f^{-1}((i, j)\text{-}\mathrm{sCl}(B))$ for $B \in \mathcal{P}(Y)$, then $x \in i\mathrm{Cl}(f^{-1}(B))$.

For a function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, we denote

$$D^0_{ij}(f) = \{x \in X\text{: } f \textit{ is not (i, j)-m-open at } x\},$$

then by Definition 5.2 and Theorem 3.4 we obtain the following theorem:

Theorem 5.3. *For a function $f : (X, \tau) \to (Y, m_Y)$, where m_{ij} is a minimal structure on Y determined by σ_1 and σ_2, the following properties hold:*

$D^0_{ij}(f) = \cup_{U \in \tau_i}\{U - f^{-1}(m_{ij}\text{-Int}(f(U)))\}$
$= \cup_{A \in \mathcal{P}(X)}\{i\text{Int}(A) - f^{-1}(m_{ij}\text{-Int}(f(A)))\}$
$= \cup_{B \in \mathcal{P}(Y)}\{i\text{Int}(f^{-1}(B)) - f^{-1}(m_{ij}\text{-Int}(B))\}$
$= \cup_{B \in \mathcal{P}(Y)}\{f^{-1}(m_{ij}\text{-Cl}(B)) - i\text{Cl}(f^{-1}(B))\}$.

For example, if we put

$m_{ij} = (i, j)\text{PO}(Y)$ and $D^{P0}_{ij}(f) = \{x \in X\text{: } f \textit{ is not (i, j)-preopen at } x\}$,

then we obtain the following properties:

Corollary 5.2. *For a function $f : (X, \tau) \to (Y, m_Y)$, the following properties hold:*

$D^{PO}_{ij}(f) = \cup_{U \in \tau_i}\{U - f^{-1}((i, j)\text{-pInt}(f(U)))\}$
$= \cup_{A \in \mathcal{P}(X)}\{i\text{Int}(A) - f^{-1}((i, j)\text{-pInt}(f(A)))\}$
$= \cup_{B \in \mathcal{P}(Y)}\{i\text{Int}(f^{-1}(B)) - f^{-1}((i, j)\text{-pInt}(B))\}$
$= \cup_{B \in \mathcal{P}(Y)}\{f^{-1}((i, j)\text{-pCl}(B)) - i\text{Cl}(f^{-1}(B))\}$.

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