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# COMPUTER-BASED ITERATIVE AND INCREMENTAL TEACHING OF MATHEMATICS

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**Abstract.** Mathematics is the subject that may legitimate a certain affinity between the humans and the dimension of gods. The high level of abstraction that mathematics exhibits may turn down the most genuine intentions of understanding and using the truths that are based on sophisticated paradigms regarding the organization and interpretation of numbers. In this paper, we propose a first attempt to answer the question: "Is it possible to use the computer in order to attenuate the difficulties one may encounter when learning mathematics?"

## 1. COMPUTER-ASSISTED LEARNING - WHICH WAY?

The academic environment and industry both developed a passion for *computer-assisted learning*. This passion is derived from a broader goal of the nowadays human society, the fulfillment of which may seriously impact the development of those abilities that are requested by the business and industry environments through the lifelong learning process.

The accelerated pace of technological changes, doubled by the unprecedented complexity growth of the storage and query systems together with the communication systems, have radically changed the learning equation in, virtually, all knowledge fields.

The creation and explosive diversification of IT services has radically changed the attitude towards the equation of learning, briefly described in Figure 1.

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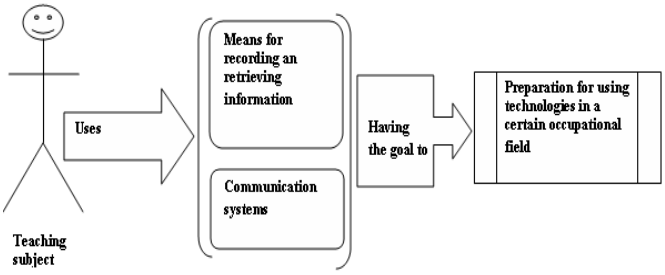


Figure 1. Teaching, everywhere and always oriented on the means of recording, retrieving and communicating.

The computer, in his desktop or networked flavour, has become the tool that allows for the teaching scenarios to be re-designed. Either the computer is a simple information dataset or it performs activities that traditionally belong to a human instructor, the computer tends to convert the learning process from a group activity into an individual one.

The subject of the learning process competes with himself at the highest possible extent. Nevertheless, this may be the aspect that should be taken into account when designing new generations of computer-assisted teaching systems, mainly the natural human tendency to attend group teaching activities. Nevertheless, there are also individuals that exhibit a higher learning performance when they are isolated from the rest of the learning community. The experts that study computer-assisted teaching systems should take into account both hypostases in which teaching activities may occur.

While it is obvious that a computer is a reliable partner for anyone who wants to be properly informed, it is questionable whether a computer can be turned into a reliable learning tool in the future, following all the required dimensions: personality, motive abilities, sensibility, and creativity.

The above considerations may exhibit certain stability provided information technologies didn't continue their attack towards human life environments. It is not excluded for such an attack to produce important changes in the structure of educational systems. Distance education is an example of a mutation requested by the convergent action of two factors: the growing need for a superior instruction in the information society, together with a tighter dependency of the environment where people build their professional carriers. The future in education will involve new technologies, mobility, and pragmatism. Probably, these will be the guidelines that will structure the educational system of the future. The absence of the human teacher in many moments of the teaching process has to be compensated by a series of technological and methodological innovations. All these innovations

are meant to turn the learning process, in its key moments, from a group sport into an individual one.

Following, we'll present a few ideas that any future researches should consider when designing teaching software systems. It follows immediately these ideas can be used even in the case of teaching mathematics, which is still the irrefutable queen of sciences.

## 2. VISUALLY-ORIENTED COMPUTER ASSISTED LEARNING PARADIGMS

At the beginning, we highlight the importance of the visual orientation in any computer assisted learning system. This importance is proved by scientific studies that deal with accessibility problems of educational processes. A good instructor is appreciated considering the easiness he exercises in order to transmit a certain easy-to-understand message. More precisely, it is preferred that teacher who supplies messages that are at the same time easy to decrypt and rigorous, making use of proper didactic methods and technologies.

The unavoidable trade off we pay to rigour during any modeling process is generally compensated by the creation of models that feature a clear to understand and efficient structure. The structure is, in general, the way we assure ourselves an adequate framework to promote rigour in the process of developing our own models. The description of any model's structure can be conducted using a highly formalized language (outstanding advantages regarding the rigour with which that model is described, but fewer persons capable of understanding it), or making use of a language that exploits the natural human tendency towards intuitive approaches.

The importance granted to visual in any explanatory endeavour may simplify the understanding of the message in question. The representations that are excessively formalized are intended for minds that are able to handle abstractions properly. Visual representations are intended for minds that feel at ease with moderate abstraction levels. Owing to his structure, a human being is prone to prefer concrete objects to abstractions. Nevertheless, the elaboration of visual-only models is questionable. It is like we accepted science can be made only with comics.

The visual component has to be properly mixed with formalization items.

Beyond the battle between formal and visual, there are also a few other aspects that is worth to be mentioned when operating with visual, such as:

- The suggestion power of visual models is higher than that of abstract formalisms or that of any text on which obvious structural constraints are exercised.

- The specification of certain standards for elaborating visual models is a preoccupation in accordance with the efforts that specialists in HCI (Human-Computer Interaction) perform.
- The capacity of visual models to keep awake attention is sensibly higher than that of formal models.
- Imagination is highly stimulated by visual models.
- Complexity is easier accessible with visual models, etc.

A suggestive statement that may justify the importance granted to visual may be the following: the usage of visual approaches for the man-computer communication may turn this dialogue process into a smoother, more efficient and diverse process.

## 3. THE SPIRAL MODEL

We won't perform a special pledge for the utilization of the spiral model as a pattern in the computer-assisted learning.

Iterativity is generally used as a mean for progressively add additional knowledge in a certain field of knowledge, or to correct the omissions that any consumer product may exhibit during its lifecycle.

What we have in mind when we insist on using a spiral model when designing and using an educational software is the necessity to make use of a proper procedure for progressively master the complexity of a certain knowledge field. Obviously, this requirement raises for the instructor a special problem for properly structuring the knowledge base and for the designer of the educational system serious problems regarding the structure and design of the system, if one wishes to intelligently simulate iterativity [1].

As a consequence, the creators of educational software should value an important philosophical truth: knowledge related to human beings by identifying and gathering of details (sensation), continues with the formation of concepts (perception) and is concluded by the capacity of a high level thinking process (reasoning). These are three research subjects of equal importance for those who want to turn computers into intelligent learning partners for humans. The spiral has been discovered long ago and consistently applied in the mathematical study, as a paradigm. The art of efficiently applying the spiral for the educational systems creation has to be pushed

[1] Man's mind in action resembles to that of an eagle over the chase territories. A high altitude flight is equivalent to the exploration of high level abstractions. A low level raid for catching the prey is equivalent to the exploration of operational concepts. Attacking the prey at the ground level may be perceived as the effort of the one that tries to decipher the meaning of details.

farther, considering the problem of the human instructor flexibility rests an open one. The amount of information used in mathematics, its diversity and logical dependency, the strongly differentiated practical requirements, all these are worrying reasons for someone who wants to elaborate punctual teaching scenarios or template scenarios for learning mathematics, regardless of the intellectual level of the teaching subject.

Finally, the logic of the spiral is employed by a human in the process of learning/modeling in order to understand/use the abstraction on several levels, as a procedure of looking for stability landmarks in the logic of knowledge/producing the systems/explanatory models. These are additional reasons in order to focus our attention on the importance of the analysis for the educational software systems engineering.

## 4. METHODIC INCREMENTALITY

In the software industry, incrementality is the capacity of some development models to guarantee customer's satisfaction, through displaying a clear image of the project's progress, relative to the assumed goals. Making use of incrementality, in the software industry, we turn our beneficiary into an ally, a partner, and a constant sponsor. Is it possible to make use of incrementality to optimize a learning exercise? In order to answer to this question, we provide a practical definition for an educational system.

**Definition 1**. *We call educational system a set of resources that facilitate the accomplishment of some educational projects.*

**Definition 2**. *We call educational project the activity of realizing a product whose functionality can be integrated in the dynamics of an educational system. A product of this kind is called educational product.*

The beneficiary grants to an educational product the exact amount of attention his immediate and future interests require. It is already a truism to say the one that makes progresses during the learning process has the strongest reasons to multiply the educational efforts. Therefore, the subject of the teaching process is interested to use a learning system featured by incremental capabilities. The educational system must avoid the spirals that suggest to the subject the feeling of "déjà vu".

The indirect beneficiary (the actual or future employer of the subject) is also interested in the incrementality of the educational system, as it is not possible for him to accept the validity of the old adage "repetitio mater studiorum", considered in a flattered environment. We can go even further by saying that both types of beneficiaries are interested, including for the knowledge system to acquire certain self-incrementing capabilities.

## 5. CONSTRUCTIVE EQUILIBRIUM BETWEEN GENERAL AND PARTICULAR

The discourse of this paper involves the correct understanding of the natural relation between general and particular in the learning process. Although partially amendable, this relation can be mapped on the relation between theory and applications. The general belongs mainly to the interest domain of theory, while the particular belongs to the structure of applications. The bridges established between theory and applications are obvious and they require an adequate effort in order for the experimental or theoretical study to be accomplished. There are voices that promote favoring the practical approaches as opposed to theoretical studies, inviting the experts to master all the applications in a certain field. What is the use of knowing a lot of chemistry when someone is an oenologist, considering all that he needs in order to perform the job is located in his mouth [2 ] ? Why should a computer scientist learn a lot of mathematics, when he will most likely never use the "Lebesgue measure" in his carrier?

The truth is that the equilibrium established between theory and practice has to be reconsidered. There are jobs for which operational abilities make the measure of a skilled employee. Nevertheless, there are also jobs that require a mixture between theoretical and practical abilities to be established. Finally, there are some almost natural exceptions that need superior theoretical and analytical abilities in detriment of theoretical ones3. Are the statistics this taxonomy implies enough for the kind of world we are moving to? We are not able to offer an answer featured by a three-decimal accuracy. Nevertheless, it is obvious the fact that the extension of the abilities of the informational society will determine, in the long run, some changes regarding the dosage of these types of abilities. During the Middle Age, physical power was considered the supreme virtue, as it was opening all the doors in a world dominated by endless wars. The spiritual power was sometimes complementing the physical power but, nevertheless, it was not able to replace it entirely.

In the informational society, the human being is obligated to cultivate his ability to manage information and its derivatives. The headquarter of these abilities shifts entirely to the brain, and muscular power will become a completely decorative element. As a consequence, we are entitled to postulate that the future of the educational systems not only needs to solve problems that are typical for IT systems but, it also has to be able to increase the role of

[2] The assertion is, of course, questionable.

[3] Mathematicians and philosophers are often called "individuals that keep their heads high above in the clouds".

the abstraction power, in order for the subjects involved to be able to deal with the enormous amount of data that waits to be processed, like cherries wait to be picked up on branches.

The school of future, considered on the general-particular dimension, will have to learn those that cross its door step, the art to select and look for the relevant information, together with the ability to add a greater value to this information inside the school and outside its normal boundaries.

## 6. A PROPER TRIBUTE TO THE SOUND FILE

At the moment when it will have to play the role of electronic teacher, getting the computer off its traditional speechless state will have to be a priority for the designers of intelligent educational systems. This is because:

- The sound smoothes the way of knowledge to the optimal recording formula. The self-reading is ideal when someone is self-taught. The sonorous intervention of the teacher has two immediate advantages: the gathering of the subject's attention and the possibility to manipulate this attention through the insertion of emotional substance in the discourse. The text can also transmit some emotional substance, but sound is the safest way to highlight the emotional substance of a discourse.
- As researchers say about flowers they like classic music, the decoration of a lesson's background with an appropriate sound file may enforce the comfort that the subject of the teaching process needs.
- An appropriate sound file may also stimulate creativity.
- Finally, designing an educational software that is able to process sound in an integrative manner, may increase its overall attractiveness.

It is natural to enforce, even in this case, that an efficient usage of the sound file in educational software systems is, on one side, a problem of interdisciplinary research (psychology, didactics, sociology, man-computer interaction) and, on the other side, a problem of design and implementation under the sign of some extraordinary constraints that concern quality assurance.

## 7. ACKNOWLEDGING THE IMPORTANCE OF APPLICATIONS

It is another truism that has to be considered in the future in the process of realizing educational software systems. The bidirectional way that is bounded at the two edges by "theory" and "applications" has to be used in both directions in order to ensure the efficiency of the learning process, in general, but especially when speaking about mathematics. The beauty of the theoretical abstractions gains consistency through some proper applications. Accepting the challenges that the confrontation with applications involves is meaningful

and clear, provided the usage of theory is conducted in a methodic manner. Let us also enforce the fact that the educational software will meet the expectations of the applications oriented learning process subject, only if the teacher exercises a proper work in order to configure an appropriate set of applications. This should be characterized by diversity regarding the problematic involved and the degree of difficulty, and structured in such a way that the subject of the learning process will deal with it in an iterative and incremental manner.

## 8. FAST AND RELIABLE DOCUMENTING SOURCES

All technical pre-requisites that are necessary to solve the problem of fast and reliable documenting sources are accomplished. In the same way a teacher is considered a good one if he resembles to an encyclopedia, in a similar way an educational software has to be able to acquire supplementary and complementary information, based on the core of its knowledge base.

The main reason that suggests the possibility to offer such capabilities is simple: the truly curious human subject feels the need to perceive and check a knowledge system looking at it from various angles. The difficulties that have to be surpassed are clear: the integration of documenting abilities, in such a way to avoid clogging the interface, and the specification of the documenting system such that to mention all the already existent possibilities, but also to allow for some other documenting opportunities to be considered.

From an algorithmic point of view, when speaking about the problematic of documentation, we'll necessarily reach the need to use some intelligent, reliable and fast search engines, in order to get quick answers to the problems that practically justify the necessity of the documenting system's existence.

## 9. SUPPORT FOR ASSESSMENT

A learning process without assessment is meaningless, whatever the environment in which it is conducted. In practice, the evaluation of a learning system's results can be accomplished punctually and/or globally. We may establish the individual level of the subjects involved or, we may evaluate, based on appropriate synthetic indicators, the overall performance of the learning system. Individual assessment is needed by every subject in order to gather as accurate as possible information about the personal preparation level. Individual assessment is also requested by the teaching system as a mean for certifying the preparation level of every subject.

The main problem that arises when speaking about evaluation is hidden in the question: "How can evaluation be done?" The answer to this question is complicated because:

The liberality of evaluation is a goal that is difficult to reach.

- The assurance of equal chances in the process of evaluation is a constraint with a negative impact on the evaluation strategy design.
- The reasons that guide the evaluation activity may change, both in terms of composition and structure.
- The correctness of the assessment procedure is always prone to improvements.
- There may always occur communication problems that are prone to alter the initial intentions of the assessment procedure, etc.

We believe that we have just dealt with a small part of the problematic that accompanies the assessment activities. As a consequence, it is natural to mention that simulating the assessment activities is a permanent challenge from a methodological perspective, with appropriate consequences from a technological perspective. The simulation of a multiple choice test, regardless its complexity, is not a problem any more, both technologically and methodologically.

The simulation of an exam that is centered towards the assessment of creativity is, in turn, a problem that waits for more and more refined solutions, as it depends on technologies and paradigms that still doesn't exist or are under development.

It is possible for the dilemmas related to evaluation to be partially clarified at the moment when its role will be rethought, considering the commandments that the information society will provide to the researchers who, under the pressure of the fast rhythm of change are forced to provide answers in which the insertion of speculation is reduced to a minimum, in order to contribute to the implementation of systems with rigorously quantified operational value.

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