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MATHEMATICS AND ART

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"The essence of the spirit lies in its freedom" (G. W. F. Hegel)
"The essence of mathematics lies in its freedom" (G. Cantor)
"Art means freedom" (Gustav Klimt)

Abstract. The present study focuses on the analysis, under various aspects, of the relation between mathematics, literature (mainly poetry) and the plastic arts (mainly painting). The first part presents the similarity of the creative act in mathematics and in art, from the point of view of several remarkable creators, both mathematicians and artists. The second part emphasizes the relation between mathematics and poetry exemplified in the vision of some great mathematicians and poets. The work of the outstanding mathematician-poet Dan Barbilian-Ion Barbu represents the main focus. The third part outlines the connection between mathematics and painting, illustrated in the works and ideas of some representative personalities of the world culture.

1. INTRODUCTION: MATHEMATICAL CREATION, ARTISTIC CREATION

The foregoing mottos reveal to us an extremely important fact: the philosopher, the mathematician and the painter, all three of them embodying first-rate personalities, use the same term-*freedom*-to define the essence of their domain of creation. This *unifying* word carries important significance in the present study. The study engages in examining and presenting different significant and interesting aspects concerning the relation between mathematics and art. Regretfully, due to the limited space offered to the article, a lot of other important aspects, deserving attention and investigation, are not to be mentioned here. Some of these points of view are to be included in the next study, still limited by the impossibility of encompassing all the important aspects, worthy of being specified.

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There should be mentioned the fact that the word *art* is to be used as a generic term for various branches of knowledge: literature (mainly poetry), music, plastic arts (painting, sculpture and engraving), architecture, etc.

Certainly, between mathematics and art there exist essential differences which any educated person can take notice of. No one will ever confuse a theorem of mathematics with a lied by Schubert or with a painting by Van Gogh. Nevertheless, despite all the obvious distinctions, a strong and surprising relation can be established between mathematics and art, the determining factor of this connection being, in the opinion of the Romanian mathematician and philosopher Imre Tóth, the one that "in the phenomenology of the spirit, art and mathematics have been competing to achieve the same glorious aim: awakening the spirit to the self-awareness of the creative act" ([11])

It is indeed true that the creative act is more strikingly present in art, but for high-rate mathematicians and logicians the creative act is all the same visible in mathematics too. Furthermore, the mathematical creation and the artistic creation represent for these personalities similar spiritual activities. In his uniquely exuberant style, Imre Tóth states the same principle in his magnificent study "*Achilles*": "the process of creation develops within the spiritual world, initially in a spontaneous way. In all fields where noetic objects are created and especially in mathematics, the spirit engaged in the permanent and uninterrupted act of creation/genesis, propagates and proliferates as a luxuriant vegetation, continuously populating its space with its own creations. The mathematician himself, or the artist, reveals oneself as a blind force of Nature, as Nature itself, which through uninhibited unconsciousness creates and is created, gives birth and is given birth to, without being aware of this". It is often said that intuition and fantasy hold the first place in art, whereas in mathematics, the logic and rigour hold the foreground, investing the mathematical universe with iciness, a state rather alien to the human being for whom art is much more intimate. Does this really happen? Those who know better say that things are much more complex in reality, that they are not so rigidly separated, being rather strongly related.

Let us take into consideration the ideas of the great American mathematician Marston Morse (1892-1977) who noticed that "the first main connection between mathematics and art lies in the fact that discovery in mathematics is not an issue of logic, but rather the result of some miraculous powers that nobody can understand and where the unconscious recognition of beauty should play an important part. Out of an infinity of patterns, a mathematician chooses a sample, for beauty's sake, and he roots it, without

anyone being aware of this. Then, the logic of words and of shapes places the pattern in an accessible position. Only now it is possible to communicate it to others. The first pattern stamps itself in one's memory". (quoted from [5]) There is no doubt that Marston Morse, as a great creator himself, knew very well what he was talking about.

There will be plenty of opportunities to convince ourselves that fantasy and intuition, logic and rigour represent attributes which can be equally attached to a mathematician and an artist. Marston More is again fully credible when he asserts that "mathematics is the sister and the necessary auxiliary branch of arts and is touched by madness and genius". This context brings him closer to the English philosopher and mathematician Alfred North Whitehead (Bertrand Russell's co-operator in writing the monumental three-volume work "*Principia Mathematica*", Cambridge, 1910-1913), in whose opinion "mathematics is a divine madness of the human spirit" Their paths come into contact with poetry by means of Plato: "poetry is a divine madness". It seems as if the word "madness" has been abused, but there should be no doubt that these mathematicians perceive mathematical madness as a creative fantasy (apparently unbounded, but still limited by the principle of non-contradiction, the one granting it the great chance of veracity) which allows the mathematician, worthy of this work, to create an abstract and autonomous world, belonging entirely to the universe of thinking (expression of his genius) and which, precisely due to its high degree of generality and abstracting, enables (him or others) to apply it to the world of matter and sense.

The statement that the fantasy of a mathematician of genius can compete with that of a Salvador Dali (evidently without alluding to his "pathology") should be of no surprise! This fact is attested by the Cantorian ordinal numbers, the non-Euclidean geometries, the non-standard universes and by so many other mathematical entities challenging the general intuition (e.g. Peano's curve which "fills" a square, Möbius' band, a notable example of a surface with one side, Mandelbrot's fractions with fractional dimensions, Hawking's imaginary time, etc.) In a well-known work ("*Creation poétique et création scientifique*", Paris, 1980) the great French biologist Jean Bernard clearly distinguishes between the two types of creation, grounding his assertion on the distinction between the two terms: *discovery* and *invention*, regarding the first as a term specific to science and the second as being specific to art. We feel inclined to give more credit to the great French mathematician Jacques Hadamard (1865-1963) and to his major, famous work ("*The Psychology of Invention in the Mathematical Field*", New York, 1954), as he believes that the two terms are tightly connected and that, furthermore,

from a psychological point of view, there is no distinction between them. Hadamard also holds our attention when (starting from a letter addressed by Mozart to his sister, Nanerl, wherein the great Austrian composer is analyzing his entire creative act) he reaches the conclusion that the process of creation is identical and passes through similar stages, both in mathematics and in art. These stages are the following (they will be mentioned without being analysed): the conscious preparation, the incubation, *the illumination*, the completion (a conscious and rational stage of verifying and giving the finishing touches to a theory). Another representative French mathematician, Henri Poincaré (1854-1912) comes to the same conclusion, emphasizing the moment of *illumination*, as a moment of finding unexpectedly the solution to a certain problem, offering as an example his own personal discovery, when contrary to all expectations he formulated the fundamental idea of the theory of Fuchsian functions: "One morning, strolling by the cliff, it suddenly *flashed* across my mind the idea that the arithmetical transformations of the indefinite quadrangular figures are identical with those of the non-Euclidean geometry". The same happens with the creative act in art, as Mozart, Lamartine and Paul Valéry confess to us.

It is of great interest and importance that both Paul Valéry and Poincaré, a poet and a mathematician, both of them highly praised and appreciated, consider that the act of creation depends on two fundamental operations: *combinations* and *selections*, the essential one being the latter, as it "defines" the genius. These aspects and many other points of view, to be highlighted along the whole length of the article, are thoroughly studied by the academician Solomon Marcus, in different works ([5], [6], [7], [8]), wherein the author is constantly concerned with integrating mathematics within the universal culture, by means of establishing significant connections between mathematics and other domains of the mind: literature, arts, philosophy, etc. Another interesting, but less surprising aspect, is that mathematicians give proof of a truly literary talent whenever trying to render their conceptual universe as an accessible one to those lacking expert knowledge of mathematics. A first illuminating example is Imre Tóth's work: "*Achilles. The Elleatic Paradoxes in the Phenomenology of the Spirit"* which is valued as a threefold masterpiece: mathematical, philosophical and literary.

There is great pleasure and excitement, too, in mentioning the excellent books on mathematics of the memorable teacher Florica Câmpan (The *Alex. Ioan Cuza* University of Iaşi) which, besides providing a large amount of information and scientific analysis, captivate readers with their literary beauty. And, since reference is made to the literary beauty of some

mathematically-oriented texts, the next fragment belonging to the famous Swiss mathematician and philosopher, Ferdinand Gonseth is worthy of being quoted: "In its essence, mathematics is nothing more than an assembly of schematic procedures and visions of our spirit, a conscious reply of the subconscious activity implementing in us an image of the world and a great deal of norms determining our actions and reactions. It is not an edifice of a fixed foundation, of an absolute stability, but it is rather an aerial construction miraculously holding its structure: the most audacious and unimaginable adventure of the spirit". *("Les fondements de mathématiques"*, Paris, 1926). Within the same category of interesting books dealing with mathematics, there can also be included Alfred Rényi's book "*Dialogues about Mathematics*", Oskar Becker's book "*The Splendor and Limits of Mathematical Thinking*", Norbert Wiener's book "*I'm a Mathematician*", Godefrey Herold Hardy's book" *The Apology of Mathematics*" and Henri Poincaré's books: "*Science and Hypothesis*", "*The Value of Science*", "*Science and Method*" and the list is far from being exhausted.

Significantly, these authors are engaged in analyzing the similarity between mathematics and art, at the level of creation and at the level of aesthetic value. Norbert Wiener admits that, unlike to the widely accessible arts- music and painting- mathematics, despite its many similarities to these arts, is accessible only to a limed, specialized area of audience. He, therefore, concludes: "Someone, unless he is a mathematician, will hardly accept that mathematics possesses a cultural and aesthetic fascination, that it represents something connected to beauty, vigor and inspiration". (quoted from [9]) The idea of the aesthetic value of mathematics is similarly pointed out by G.H.Hardy: "The paradigms of the mathematician, as those of the painter or of the poet, should convey the idea of beauty. The ideas, the colours and the words should be harmoniously assembled. Beauty is the first test: the ugly mathematics holds no durable place in the world". ([9])

In "*Science and Methodology*", Poincaré pays tribute to the beauty of mathematics, which he places next to the beauty of artistic works: "It seems surprising that sensibility should be invoked side by side to mathematical demonstrations which, apparently, appeal only to intelligence. But this involves forgetting the feeling of mathematical beauty, the harmony of numbers and forms, the elegance of geometry. It is a matter of a true aesthetic feeling experienced by all the veritable mathematicians. Sensibility is actually here…. Face-to-face with his own work, the geometer discovers a type of pleasure similar and of the same extent to that of the artist".

It's obvious that all these great mathematicians refer to a profoundly humane type of mathematics, dispelling, by virtue of authority, the deformed character of inhuman coldness, which is ascribed to the mathematical universe by some commentators totally caught in ignorance. A lot of mathematicians and logicians have been interested in studying the nature of the mathematical notions and of the mathematical reasoning. Aiming to find the secret of this type of argumentation and consequently of the mathematical creation, the renowned French logician Ed. Goblot exposes his conclusion in "*Traité de Logique*" (Paris, Armand Colin, 1937): "The spirit is indeed guiding its operations. Out of all these operations, the spirit *chooses* freely the ones considered suitable for its aims, and this *freedom* gives full scope to a quasi-unlimited manifestation of *logical invention*. *This art is as free* (our emphasis) and as creative as that of the painter, of the musician or of the poet; literally, it is a poetic art *poietike téhne*". (quoted from [4]) In "*Science and Hypothesis*", Poincaré emphasizes the same idea of creative *freedom* defining mathematics. Analysing the role of *hypotheses (axioms)* which "limit themselves to definitions or disguised conventions", he adds the following: "The latter are frequently found in mathematics and adjacent sciences. These sciences derive their rigour exactly from this source; this type of conventions is the outcome of *the free activity* of the spirit, which in this domain is *totally unrestricted*. The spirit is free to make its own assertions, because it holds the power to decide". The same *unifying* word we were mentioning at the beginning of the study appears in Abraham Fraenkel's exclamation:"*Freedom*! This word alone sums up the act of giving rise to the modern mathematics".

2. MATHEMATICS AND POETRY

Some of the most representative mathematicians render evident a seemingly surprising aspect of the relation between mathematics and poetry: "certainly, a mathematician that is not in the least a poet stands no chance of ever becoming a fully accomplished mathematician". This statement belonging to Karl Weierstrass, with reference to Abel and Jacobi's works, are selected from one of Weierstrass' letters addressed to Sophia Kowalevskaia, a brilliant Russian mathematician and an extremely beautiful woman, whom Dostoievski, Weierstrass and Bunsen were all infatuated with. The Swedish mathematician Gösta Mittag-Leffler makes the following addition: "Weierstrass' phrase that the real mathematician is a poet may seem particularly strange to the large audience. Still, this phrase holds true. It does not only presuppose that the mathematician and the poet need imagination and intuition. All sciences are in need of these, but nevertheless mathematics

requires them out of the ordinary. The best ever written works of Abel are genuinely logical poems, where the beauty of form reveals the depth of thinking". (quoted from [4]) It is understandable why Weierstrass was constantly encouraging his students to study Abel's works. It is interesting to notice that even Mihai Eminescu, the one to become the most celebrated Romanian poet, was a remarkable presence among them. Leopold Kronecker joined this community of ideas: "Are not mathematicians veritable and innate poets? Indeed they are, just that their representations ought to be demonstrated".

The conclusion of the affinity between mathematics and art, under the sign of creative *freedom*, belongs to Abraham Fraenkel: "complying with no servitude, the mathematician is a *free* and intrepid creator. He creates in total *freedom*, guided purely by his *artist*'s intuition". According to what has been aforementioned, the connection between mathematics and art seems to be tackled mainly by mathematicians. The study also offers us the possibility to explore the opinions of some great artists and poets and to notice with great satisfaction that the relation between mathematics and art, i.e. the value of mathematics, are competently analyzed by artists too, some of them devoting exceptional hymns to mathematics.

The connection mathematics-art materializes itself within the scope of actual achievements: the artistic interests of great mathematicians can be offered as an example, still without their artistic activity reaching the high level of mathematical creation. William Hamilton (1805-1865), an Irish mathematician of genius, the author of the theory of quaternions and (in association with Lagrange) of the theory of analytical mechanics, created poetry, as well, stimulated by his poetic visionary nature. He befriended with highly distinguished poets of his time: Wordsworth and Coleridge. His culture was vast: he was conversant with the Greek culture, the classical languages (Hebrew, Greek and Latin) and the modern ones in use, and he was acquainted with Homer's and Milton's works, as well. Still within the area of British culture, it is worth mentioning the famous companions, the two brilliant English mathematicians, co-workers and friends, James Sylvester (1814-1897) and Arthur Cayley (1821-1895). The first was a poet and a fervent amateur of music, the latter was interested in painting and architecture. In order to earn his living, Cayley was also forced to work as a jurist and as a notary. In other parts of the continent, the German mathematician Leopold Kronecker (1823-1891) distinguishes himself as an excellent pianist and truly valuable economist, and the two illustrious mathematicians from Transylvania, Farkas and János Bolyai (1775-1865, respectively 1802-1860) are to be noticed as

scholars of encyclopedical formation. Farkas was a geometer of merit and a great dramatist, writing a play inspired from Greek mythology. Is it possible to describe the astral beauty of geometry much better than Farkas Boylai already did it?: "It is Jacob's ladder climbed by the spirit towards the magnific celestial spheres with wings of flame carrying it to all the galaxies, beyond the violently burning sun". His son, János, one of the brilliant authors of the non-Euclidean geometry and forerunner to semiotics and cybernetics, was a composer and theoretician of music and an eminent violinist giving concerts in Târgu Mureş. The connection between mathematics and literature tends to emphasise the importance of the second term, without referring to the idea of genius in either of the two.

Charles Dodgson (1832-1898) was a teacher of algebra at Oxford, wrote a study about the linear systems and had a major contribution to logistics, publishing textbooks of algebra, geometry and trigonometry. But all of these aspects do not consolidate Dodgson' reputation in the history of mathematics, as this name would not even be uttered nowadays, if it had not been for his particularly remarkable talent. Dodgson published, under the pen name Lewis Caroll, those wonderful "stories" which have been fascinating so many generations, all around the world (both children and grown-ups): "*Alice's Adventures in Wonderland*" and "*Through the looking glass*". Beyond the world of fairy-tales, Alice's adventures are fully marked by "mental puzzles, dialogues with a mathematical substratum, logically exploiting the weaknesses of the English language, and here and then there exist echoes of the non-Euclidean geometry". ([10]) It is, therefore, a highly appropriate moment to mention the points of view of other artists and poets about mathematics and its similarity to art.

Regarded, either as a swindler or as a man of genius, the French poet Isidore Ducasse, known as count of Lautréamont (born in Montevideo-1846, died in Paris-1870) held a great interest in and passion for mathematics. He believed "*poetry is, in the true sense of the word, geometry*". "*Les Chants de Maldoror*", his famous, but quaint work, revealing an abundant non-Euclidean content and an abstruse reality, similar to the visionary universe of Bolyai and Lobacevski (see I.Tóth "*Achilles*") comprises a lot of interesting reflections on mathematics, the science he truly venerated: "Arithmetic! Algebra! Geometry! Majestic Trinity! Indestructible shining triangle! He who has no knowledge of you lacks his thinking! Yet, the one who knows and appraises you can be deprived of all worldly things. The world embodies for him only moral illusions and fantasies; you, on the other hand, concise geometry are rigorously connecting your steady propositions and through the constant value

of your unquestionable laws, the powerful reflection of that supreme truth, visibly imprinted within the order of the universe, can shine as quick as lightning…because the Almighty has totally revealed himself and his attributes in this memorable triangle and has determined, out of the depths of chaos, the genesis of your treasured theorems and of your imposing splendors. My spirit kneels down to you and his veneration glorifies your divine image as it equally praises the Almighty".

Such veneration equals and completes the one expressed by the great German romanticist Novalis (1772-1801, by his real name Friedrich Leopold von Hardenberg) who has been accumulating wide scientific and general knowledge during his university studies at Jena, Leipzig and at Wittenberg (1790-1794), mathematics holding a special position among his cultural studies. NOVALIS' "*Hymn to Mathematics*" gives expression to an unequalled enthusiasm and to a rare understanding of mathematics: "Mathematics is the life of Gods. All their emissaries must be mathematicians. *Pure mathematics is religion*. One can reach mathematics only through *theophany*. *Mathematics is poetry*. The poetic philosopher finds himself in the position of the *Absolute Creator*. The triangle and the sum of its angles are the result of such an act of creation. *The mathematician is, therefore, a poetic philosopher contemplating the mind as a distinct universe*". The last sentence of this fragment leads us to the conclusion that Novalis has very well understood that the mathematician rather than describing the outer material world, he aims at creating a *free*, *autonomous* world thoroughly belonging to the universe of thinking; this aspect is clearly stated by Novalis when pointing out that the language of poetry is an "*autonomous language*", similarly to "*mathematical formulas*" that "*make up a self-representative world, playing only with themselves*", furthermore considering that "*algebra*" and "structure" symbolize the intellectual features of poetry. Novalis, and immediately after him, the great American poet Edgar Allan Poe (1809-1849) introduces and theorizes in poetical theory the concept of "*calculation*". Edgar Poe refers to the affinity between poetical matters and "the rigorous logic of a mathematical problem", considering that "*any poem is a theorem, and its verses are its demonstration*".

The French poet Charles Baudelaire (1821-1849), considered by T.S. Elliot "the greatest illustration of modern poetry", takes over and expands the concept of "*calculation*", categorically stating that "all that is beautiful and noble is the result of reason and *calculation*". To Baudelaire the *metaphor* equals "*the mathematical preciseness/exactness*", and *the style* can be placed next to the "*wonders of mathematics*". "*Les Fleurs du Mal*", Baudelaire's chef

d'oeuvre, is the "the book with the most rigorous architecture in European poetry/lyricism", its author considering that poetry is contiguous with music and mathematics (see Hugo Friedrich: "*The Structure of Modern Lyricism*", Hamburg, 1956; Bucharest, Universal Literature Printing House, 1969).

Other first-class poets experience the same reverberation within the intimacy of mathematics, which is actually situated very close to poetry and music. The French poet Stephane Mallarmé (1842-1898) takes over Baudelaire's concept of *mathematics* and creates a wonderful "poetry of the space". Paul Claudel (1862-1955), acquiring his poetical formation in the literary circle of Mallarmé, believes that "*music is the soul of geometry*". The Belgian poet Maurice Maeterlinck (1862-1949), awarded The Nobel Prize in Literature (1911), the famous author of the play "*Pelléas and Mélisande*" (1892), with Claude Debussy creating its musical version, wrote many interesting works about mathematics. Strongly guided by the conviction that "mathematics precedes our thinking, our ability to imagine and perceive", Maeterlinck meditates on whether "mathematics is not a magic instrument which, as in fairy-tales, becomes the owner of the hand believed to be the owner, determining it to perform, without its knowledge, miracles?" He gives the following answer: "obviously, mathematics enables us to envisage what is inside of us. Mathematics translates what we can not still verbalize, what we can not still reflect on…and when entrance to a superior space, a non-Euclidean one consisting of more than three dimensions is granted, mathematics pleads/upholds that this space is really within us, waiting there from the beginning of the world. Mathematics can be seen as one of the most curious instruments of investigation, an unexpected performer of the latent man". Supporting such a view about mathematics, Maeterlinck shows oneself as a fervent defender of the non-Euclidean geometry whose legitimacy he acclaims: "from a mathematical and geometrical point of view, all ideas about space, regardless of its dimensions, can be inferred and justified in a totally logical manner". This attitude emerged in a period when the non-Euclidean geometries still had many opponents among mathematicians and philosophers. According to his aforementioned ideas, Maeterlinck seems to be completing Edgar Poe, in the endeavour of subscribing his *poetical reform* to the logical value of mathematics: "these terms/ conditions inspired my thinking as an inevitable result of an argumentation as logic as the one setting up whatever Euclidean demonstration". It is a real comfort for the mathematicians to take notice of the fact that the most representative poets in the world faithfully adhere to a similar vision. This sense of delight reaches its climax when mentioning Paul Valéry (1871-1945), the poet and the theoretician describing

poetry as true "*mathematics*", being a highly competent expert, irretrievably fond of "the most beautiful of all sciences". The great French poet particularly appraised the high value of the Romanian writer and scholar Matila Ghyka (1881-1965) one of the promoters of *mathematical aesthetics*, next to another Romanian scholar, Pius Servien (Şerban) Coculescu (1903-1959) and to the great American mathematician Georg D. Birkhoff (1884-1944). Paul Valéry really grasped the essence of mathematics which is confirmed by many excerpts dedicated to mathematics in his theoretical writings. For example, in completion to Cantor's statement about *the freedom of mathematics* (see the second motto of the study) the French poet states the following: "The *freedom* of its manner of acting, the sense of self-awareness acquired through its own process of development seem to be keeping away mathematics from reality, placing it in a world of games, futile difficulties and gracefulness, but they have nevertheless invested it with a marvelous flexibility".

Valéry considers "the theorem to be art related" and the geometricians to be worthy of appreciation because "one can get knowledge of excellent instances of their curious striving for this rigorous kind of beauty", which brings him closer to the great mathematician Herman Weyl: "I have constantly been feeling a need of merging the truth and the beauty. But, every time I had to choose between them I always chose the beauty". Valéry's reflective thoughts upon geometry, axiomatic, logic and semiotics are challenging indeed, but his vision about Greek geometry remains truly valuable in time: "Greece founded geometry. It represented a foolish project: *we are still arguing* about a *possible act of madness*. How was this bizarre type of creation brought into being? Think that neither the Egyptians, nor the Chinese, the Chaldeans or the Indians have ever reached an answer to this. Think that it is a fascinating adventure, a benefit ten thousand times more valuable and more poetic than that of the Golden Fleece". (quoted from [9])

Face-to face with such a profound admiration, there comes as no surprise the fact that a first-rate mathematician as Alfred Renyi regards Greek mathematics as the greatest achievement of the Hellenic spirit and another, equally important mathematician, G.H. Hardy, expresses his opinion in the following manner: "Immortality comes only with mathematics. Aeschylus will certainly sink into oblivion as the major part of his work has already faded away, while an Euclidean theorem and its demonstration will never be forgotten, gaining permanence. A Diophantine equation is as rigorous and as irresolute (according to Gödel's incompleteness theorem) as when it was carried out in writing for the first time. This miracle arises in no other human

structure or endeavour. First and foremost we are primates endowed with skills of calculating. "

Great mathematicians and poets have been mentioned so far, each with his own manifestation of creative genius in one single area of interest. The history of world culture records two exceptional situations of personalities equally endowed with mathematical and poetical genius. Chronologically speaking, the first to be mentioned is Omar Ibrahim El Khayam (1048-1123) the greatest poet of medieval Persia and, at the same time, the most representative mathematician of the East. As far as mathematics is concerned, Omar Khayam has managed to outdistance with a few centuries the Western European mathematicians, and his famous work "*Rubayate*" asserted himself among the great poets of the world. Despite all these, between mathematics and poetry, standing as two types of individually marked creations, there can not yet be established a certain connection. The connection, all the more one of mutual determination, "the wedding" between mathematics and poetry, enabling the two first-rank spheres of the mind to be an integral part of the same area of interests, shall be achieved in centuries to come, in that extremely interesting panorama of the Romanian culture from the first part of the 20th century. This glorious moment was brilliantly accomplished in the world culture by that "exceptional figure known as Dan Barbilian in mathematics and as Ion Barbu in poetry" as it appears in the words of the great literary critic and historian, George Călinescu.

Dan Barbilian (1895-1961) is one of the most important Romanian mathematicians (perhaps the greatest), and as a poet (writing under the pen name of Ion Barbu) he is ranked among the first five great poets of Romanian literature. Professor Solomon Marcus fairly highlights *the unity* between Ion Barbu's activity and that of Dan Barbilian: "Ion Barbu left ineffaceable traces on Dan Barbilian and viceversa: Ion Barbu's work is incomprehensible without grasping the essence of Dan Barbilian's thinking and work"([7]) This stands as a fundamental assumption in the exegesis of the work of *the mathematician-poet*, the most appropriate phrase comprising the uniqueness defining the inseparable bond of the two names: Dan Barbilian-Ion Barbu. This exegesis can only be effected by people possessing strong mathematical qualification, additionally holding vast knowledge of the world and mastery of an adequate critical apparatus. Two talented, distinguished researchers managed to carry out this task, publishing two referential books in the Romanian culture. In 1968 the physicist Basarab Niolescu published the work "*Ion Barbu. The cosmology of Secondary Game*" (Pro Literature Publishing House), and the mathematician Mandics Gyorgy published in 1984 the work

entitled "*Ion Barbu. Restrained Gesture*" (Bucureşti, Mihai Eminescu Publishing House). Both of these works are grounded on the assumption of a profound unity between mathematics and poetry, which is pointed out, by the first author, in the axiomatic structure of "*Secondary Game*" (the volume of poetry published by Ion Barbu in 1930) and in the favourable synthesis of the scientific language with the lyrical one, whereby the mathematical terms acquire unexpected poetical connotations; the latter author aims at demonstrating, little by little, that the poetical universe of "*Secondary Game*" has a similar structure with that of a Kleinean geometry, identifying the main set of transformations and the invariants concerning this fundamental set, emphasizing that "*Secondary Game*" poetically corresponds to the "Erlangen Program", the famous lecture delivered in 1872 by Felix Klein at the University of Erlangen, whereby the German mathematician provided a *unifying* definition for different types of geometries and paved the way for the progress of mathematics, with a highly extensive and powerful cultural impact. The mathematician-poet Barbilian-Barbu considers himself "a representative of the Erlangen Program, of that ideological trend which, in dealing with the extension of consequences and the reversal of ideas, can very well be compared to the *Speech of Method* or to the *Reform* itself. The pre-Erlangen narrow specialization or the opaque technicality is to be substituted by an enlightened eclecticism. It thoroughly analyses every single theory, without losing sight of the homogeneity and unity of the whole structure. Thus, the major mathematical research acquires an organization and orientation similar to the poetical function which, adjusting by means of metaphor disjunctive elements, displays a structure identical to the sensitive universe. Similarly, by means of its axiomatic or theoretical foundation, mathematics assimilates various doctrines and serves the instructive purpose, the one set up by the unifying moral universe of concepts". (Dan Barbilian: "*The Autobiography of the Scientist*", 1940). The mathematician-poet considers the relation geometry-poetry as being essential in understanding the latter, as from his point of view "there is, somewhere, in the high ranking sphere of geometry, an illuminated place where it meets poetry. We are Einstein's contemporaries competing with Euclid in representing abstract universes, since we must inevitably compete with the demiurge in imagining attainable worlds. For him the oneiric dream becomes a new source of inspiration. As with geometry, poetry is to be understood as a certain representational system symbolizing possible forms of existence". In order to create an universe totally pertaining to thinking (as is the case with pure mathematics), poetry must evade the easiness of "idle poetry" (with a

complete absence of thinking), since "the line we are devoted to, proves to be an intricate type of *freedom*: the world is purified to the point of reflecting only the image of our spirit. *A typical act of narcissism*. As the absolute: a pure direction, a sign of the mind. But the true clock of poetry must strike as closer as possible to this sign"([1]). Barbu-Barbilian's aesthetics is of great interest as a creation in itself. The connections he establishes between great mathematicians and poets of the world are instructive and all the more surprising. In a conference he held in 1947 dedicated to the French poet Jean Moréas (another conference, held too in 1947, was dedicated to Rimbaud) the mathematician-poet brings to our attention remarkable associations: "Rimbaud is admired for his *heuristic* genius, a new world and way of feeling associated with the poetical field. He evokes the venturesome experiences taking place at the border of the spirit: the great achievements of Galois or of Niels Abel". Another impressive fragment: "When it comes to Moréas (especially in "*Stanzas*") it is not a matter of invention, but of purification and of reduction ….(he) ceaselessly evokes Felix Klein's formalism or rather Hilbert's logical purism. Indeed, Hilbert's work *Die Grundlagen Der Geometrie* (*The Bases of Geometry*)-which is frequently referred to as "the new Euclid"- published in 1899, the year when the first "*Stanzas*" appeared (Jean Moréas' volume of poetry), will always hang in balance and be compared with Moréas' eternal book. Is not "*Stanzas*" to be considered "the bases" and the reflection of the purest poetry? Thanks to Hilbert's effort, geometry rediscovers Euclid, with the help of Moréas, poetry returns to Alcaeus. But this revival is strengthened by the complete force of our thinking". Furthermore, Barbilian-Barbu associated Edgar Poe's poetical reform with Augustin Cauchy's mathematical reform: "in the same way as Poe, Cauchy becomes estranged from the purist concerns. He can not imagine an axiomatic foundation and restriction to his area of knowledge. His critical work takes into consideration the methodological conditions of reasoning, the rigour and effectiveness of the algorithms". ([1])

The French poet Jean Arthur Rimbaud grants him the opportunity to draw a comparison between the poetical method and the scientific one. Starting from the premise that the subject of Rimbaud's poetry is (from "*Illumination*") is the "cosmic nothingness", the embryonic existences, the germs, the nubile landscapes, the limbs, the mathematician-poet considers that "choosing as a field of poetical operations the essential elements of a nature rounded off through the addition of an ideal type of existence, Rimbaud proves once again his science-oriented mind", comparing him with "the geometrician instinctively heading, during his investigations, towards the

singular points of particular curves in order to infer from them truths informing him precisely about the norm". ([1]) Rimbaud studied mathematics, topography, geology, etc. and through his inspired verse "he restores to science its sacred nature" ([1]) (Barbilian-Barbu).

3. MATHEMATICS AND PAINTING

The relation mathematics-poetry reaches its climax with the work of Dan Barbilian-Ion Barbu. There is a corresponding moment in the relation mathematics-painting illustrated by the great painter-geometrician Piero Della Francesca (1410?-1492), one of the most representative personalities of Italian Renaissance, rightly considered a masterpiece creator by the well-known art historian, Fred Bérence. Called by Luca Pacioli "the monarch of painting" and "the monarch of mathematics, Francesca achieved through his work of art and through his studies of geometry ("*De Prosspetiva Pingendi*" and "*De Quinque Corporibus Regularibus*") an excellent synthesis between painting and geometry, the idea demanded by an artistic type of thinking, that of a necessary connection between painting and geometry, being perfected by the painter and geometrician, PAMPHILE, the master of Apelles (4^{th} century B.C.), the greatest painter of Greek antiquity.

This idea has also engaged the particular attention of other important personalities, representatives of the Italian Renaissance such as Leon Batista Alberti (1404?-1472), Andrea del Verrochio (1435-1488) and his talented disciple Leonardo da Vinci (1452-1519) or representatives of the German Renaissance such as Albrecht Durer (1471-1528). Worthy of being noticed is the fact that Luca Pacioli, Francesca's disciple in mathematics, starts his encyclopeadical work "*Summa de arithmetica, geometria et proportionalita*" (Venice, 1494) with a chapter dedicated to the painters-mathematicians, comprising exceptional names of the Renaissance: first of all, Piero della Francesca and his brilliant disciple, Melozzo da Forli, then the Bellinis, Mantegna, Luca da Cortona (Signorelli), Perugio, Botticelli, Filippino Lippi, Ghirlandaio. During Renaissance the plastic arts were strongly related to music, through the musical connections, more precisely through *the theory of musical consonances* which enabled Leon Batista Alberti, a personality as vast as Verrocchio and Leonardo da Vinci, to expand *the mathematical theory of music* initiated by Pythagoras (a future study will deal thoroughly with this matter) and continued by Plato in the dialogue of "*Timaios*", in order to describe a geometrically structured Universe owing its stability and beauty precisely to this structure. An extremely important fact is that the great artists of Renaissance (all over the world) have strictly complied, within their works

of art, with the *musical/mathematical connections* imposed by Alberti: diapente (2/3), diatessaron (3/4), diapason (1/2), double diapente (4/6/9), double diatessaron (9/12/16), ets. ([2]), the artists being convinced that these mathematical connections contribute to enhancing the artistic beauty.

The particular representation of "*The School of Athens*", one of Raphael's masterpieces is very significant in this respect, as the *diagram of the musical proportions* (Pythagoras being accompanied by two of his disciples) is enframed by the allegorical figures of Harmony and Geometry, Plato with "Timaios" in his hands occupying the central position, embodying thus the philosophical-geometrical synthesis of the beauty. ([2]) Much later, in the 19th century, the renowned French painter, Paul Cézanne (1839-1906), one of the masters of modern art, was following the pattern of a similar tradition, as geometry played a major part in his work, corresponding to his conception that " nature should be analyzed through cylinder, sphere and cone". It is really amazing to discover that Cézanne detected similarities of conception and of method between his art and "The Erlangen Program", Felix Klein's famous 1872 lecture. When Cézanne addressed Klein, emphasizing "that I was immediately surprised by the multiple associations that can be drawn between your interests and mine", that "our methods are similar", that "we are both guided by reflecting upon structure and by interrelating the reflections" or that "what you labeled group of transformation is an invariant independent of its object", and "the transformation leaves the geometrical aspects of the object unchanged or, as you specified, its inherent properties", he managed to relate art and mathematics, proving an amazing understanding of a major mathematical work in a period when few mathematicians have succeeded in grasping its essence.

Carrying on further describing and providing significant examples of the relation mathematics-art, it seems quite adequate to complete the previous observations with the interesting approach of the famous French poet, Guillaume Apollinaire (1880-1918)-an excellent art critic and passionate supporter of mathematics- which was communicated with the occasion of inaugurating the cubist art exhibition in Paris, 1912, entitled "*The Golden Section*" and taking place under the patronage of Henri Poincaré. Apollinaire considers that "geometry is the essence of art, as geometry has always been the rule of painting, of art". Comparing the artists with the modern scholars surpassing the three-dimensional space frame, Appollinaire notices that artists, too, have annexed the *fourth dimension* which "was actually the concrete expression of a large number of young artists, who, in meditating upon scientific writings, were expecting the emergence of a sublime art".

Further on, special consideration will be credited to an important Dutch artist, the painter and engraver Maurits C. Escher (1898-1972, whose exemplary co-operation with the illustrious mathematician H.S.M. Coxeter points out the remarkable results ensuing from such a refined co-operation between an artist and a mathematician, particularly when both of them are aiming to reach a high level of performance in their field of creation and are capable of a profound mutual communication. Regarding the relation with mathematics, Escher artistically transposes the sets of transformations belonging to the Euclidean geometry and to the non-Euclidean, hyperbolical geometry (Bolyai's and Lobacevski's geometry), being concerned with setting up a distinctive order "constantly discovering new relations between objects, as the ambiguity of the convex and concave, allowing the simultaneous ascent and descent, in the same direction, on a scale ([6]), through a constant process of duplication and metamorphosis of non-existence into existence, of upward into downward, of inward into outward" ([11])

All of these aspects reveal the deep perplexity of the human spirit overwhelmed by the ecstatic contemplation of its inner beauties. Escher considered the hyperbolical geometry as the most adequate means and background for attaining this supreme state. Escher's "*Limit circles*" is the best exemplification of this type of geometry, representing, in fact, an artistic correspondent of the famous geometrical *patterns* of Bolyai and Lobacevski, giving credit for this to Klein and Poincaré. Escher also traces remarkable artistic values in Möbius' famous surface, holding an extremely important place in differential geometry. Escher's creation enables a very intersting extension of the relation mathematics-art, which is furthermore strenghtened by Gödel's limitating theorems of incompleteness (or of impossibility) (1931), the climax of 20^{th} century mathematical logic, standing as a major scientific breakthrough with a great impact on the entire contemporary culture.

Professor Solomon Marcus refers to a memorable event, concerning the publication of a book focusing on the affiliation of important personalities in the world culture : "An impressive book (Douglas R. Hofstadter: "*Gödel, Escher, Bach: an eternal golden braid*", Basic Books, New York, 1970) minutely analyses the manner in which J.S. Bach's art of fugue, glimpsed in "Alice's Adventures in Wonderland" of the writer-mathematician Lewis Caroll and in various works of M. C. Escher, is infused with the recurrent and self-referential thinking of modern mathematical logic, referring mainly to Kurt Gödel's attainments". ([7]) Professor Marcus takes notice of another aspect of the remarkable cultural value of Gödel's theorems: "Following the immense efforts of Bertrand Russell and David Hilbert, two of the

mathematicians embarking upon the task of effacing from the mathematical language the hazard of narcissism, Kurt Gödel pointed the imminence of this trap, as it actually represents the fate, the essence of mathematics. But the attempt to avoid it is as inevitable and essential as the trap itself". ([6])

The "essence of mathematics", consisting in "the absolute freedom of its language", which inevitably brings it closer to "narcissism", relates it to the "the difficult freedom" of Ion Barbu's poetry: "purifying the world to the point of reflecting only the image of our spirit. *A typical act of narcissism*". The mathematician – poet Dan Barbilian - Ion Barbuasserted his poetical creed four years before the publication of Gödel's theorems. Doubtlessly, the relation mathematics-art is quite complex and inexhaustible. Without exhausting the topic, the study emphasises certain important aspects of the problem, in the event of a future article tackling other relevant issues. The study began by revealing the importance of the concept of freedom. The same term appears in the concluding fragment of Nicolas Bourbaki (the well-known pseudonym of the group of French mathematicians, reformers of the modern mathematics), outlining the *absolute freedom* of the mathematician in choosing his axioms: "This freedom related to the absence of any connection to the real, inevitably determines us to think about modern art; actually we can hold forth that, from a certain point of view, current mathematics is more of an art, than of a science".

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