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ON THE VEHICLE ROUTING PROBLEM WITH TIME WINDOWS AND MULTIPLE USE OF VEHICLES

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Abstract. Due to the energy crisis and to the new types of demands from customers, nowadays transportation and logistics environment require computational and simulation methods in order to reduce costs and improve performances. In this paper, the solutions for vehicle routing problems with time windows and multiple use of vehicles are reviewed. Also, competitiveness of Ant Colony Optimization algorithms, when compared with other metaheuristic techniques or exact algorithms for this problem, are considered.

1. INTRODUCTION

Vehicle routing problems arise in real-life: distribution of products, courier services, postal traffic and catalog ordering of goods from a remote retailer, public transport, urban solid waste collection. In all these cases, a set of vehicles must be available to serve a set of transportation requests. Each request specifies one or more clients or locations that have to be visited by the same vehicle and various constraints that restrict the way to visit them. Under all constraints, the design of a set of routes to serve the customers must be done to minimize the total distance traveled.

Vehicle Routing Problem (VRP) falls under a broader category of transportation problems, which also includes fleet management, facility location, traffic assignment, air traffic control. Cutting costs and reducing pollution in today's competitive environment is essential. Except for the costs of purchased goods, transportation represents, on the average, a higher percentage of logistics costs than any other logistics activity [6].

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VRP was introduced by Dantzig and Ramser in *Management Science*, 1959 and is known to be NP-complete. Its basic form is the following: m identical vehicles serve n customers with known demands from a single depot. Let Q be the capacity of a single vehicle, and L the time limit for a single trip of a vehicle. A trip is a sequence of customers starting from and finishing at the depot. The assumptions are:

- each vehicle leaves the depot, visits at least one customer and returns to the depot;
- each customer must be served;
- each customer is served by only one vehicle;
- for a vehicle, the total amount of demand of the customers on its trip cannot exceed Q, while the duration of the trip must not be longer than L.

But in real-life practice, various conditions appear and this formulation of VRP does not cover them: there may be different types of vehicles; some customers ask for goods to be delivered, others for goods to be picked up, while others ask for both services; there may be time windows for service and/or for the depot; there may be more than one depot or a vehicle can serve only a subset of customers due to time/space limitations.

Vehicle Routing Problem with Time Windows (VRPTW) introduces hard time windows: given an interval $[a_i, b_i]$, the vehicle servicing customer i is bound to arrive before b_i. It may arrive before a_i, but then the service cannot start until the time window begins. Once the service has begun, a given time s_i (service time) must elapse before the vehicle may leave that node.

There are a lot of exact algorithms that have been developed for VRPTW: Fisher and Jaikumar (1981), Kolen et al. (1987), Desrochers et al. (1992), Fisher et al. (1997), Kohl and Madsen (1997), and Kohl et al. (1997). These algorithms can solve instances up to one hundred nodes, provided that the time windows are very tight. But when larger instances or wider time windows are tackled, these algorithms are not effective and heuristics are needed. All of the latest algorithms in the literature are metaheuristic. This issue will be addressed with respect to a variant of VRPTW, namely the VRPTW and Multiple Use of Vehicles.

2. THE VEHICLE ROUTING PROBLEM WITH TIME WINDOWS AND MULTIPLE USE OF VEHICLES

The condition classically assumed in VRPs, that a vehicle can perform only one route, is very limitative. Nowadays, every company wants to use its resources as efficient as possible, so by allowing a vehicle to perform more trips, less vehicles and drivers could be used while more customers could be served within a given period. This situation happens for example when the capacity of the vehicle is relatively small. In this case, frequent returns to the depot are required to load or unload the vehicle. For example, in the home delivery of perishable goods, like food, routes are of short duration and must be combined to form a complete workday [1].

The Vehicle Routing Problem with Multiple Use of Vehicles is a variant of the classical vehicle routing problem that has been intensively studied in the last two decades. It arises when each vehicle performs several routes during a planning horizon. Taillard, Laporte and Gendrau (1996) addressed the problem with the name above, while others introduced various names: "multi-trips" for Brandão and Mercer (1998), "multiple trips" for Petch and Salhi (2004). Recent results come from Olivera and Viera (2007), Salhi and Petch (2007), Alonso, Alvarez and Beasly (2008).

3. PROBLEM FORMULATION

We consider a fixed-size fleet of vehicles (each of capacity Q) denoted by set V, that delivers goods from a depot to a set of customer nodes $N = \{1, 2, ..., n\}$ in a complete, directed graph with arc set A. A distance d_{ij} and a travel time t_{ij} are associated with every arc $(i, j)\in A$. Each customer $i\in N$ is characterized by a revenue g_i, a demand q_i, a service or dwell time s_i and a time window $[a_i, b_i]$, where a_i is the earliest time to begin service and b_i is the latest time. The working day of each vehicle is made of a sequence of routes where each route starts and ends at the depot (some of these routes might be empty). These routes are denoted by set K. We can assume that the routes served by any vehicle are numbered in increasing order, that is, a vehicle serves route l after route k if and only if $l > k$.

The depot is denoted by 0 or n+1 (depending if it is the initial or terminal node of an arc), with $s_0 = s_{n+1}= 0$, $q_0 = q_{n+1}= 0$, $a_0 = a_{n+1}= 0$, $b_0 = b_{n+1}= \infty$. The symbol N^+ is used for $N\cup\{0, n+1\}$ and A^+ for $A\cup\{(0, n+1)\}$, where (0, n+1) is a presumptive arc with distance $d_{0,n+1}$=0 and travel time $t_{0,n+1}$=0. Every customer in a route must be served before the given deadline associated with that route. The latter is defined by adding a constant t_{max} to the route start time. Also, a setup time σ^k for loading the vehicle is associated with each route $k\in K$.

In order to express the constraints, the following variables will be used:

- x_{ij}^k indicates whether or not arc (i, j) appears in route k, $\forall k\in K$, $\forall (i, j) \in A^+$ (when $x_{0,n+1}^k = 1$, route k is empty)

- y_i^k indicates whether or not customer i is served by route k, $\forall k \in K$, $\forall i \in N$
- t_i^k indicates when service starts at customer i if it is served by route k. When customer i it is not served by route k, this value is meaningless. For each $k \in K$, t_0^k and t_{n+1}^k are the moments of time when route starts and ends at the depot
- $\forall k,l \in K$ with $k<l$, binary variable z_{kl} indicates whether or not route l immediately follows route k in the workday of one of the vehicles.

Considering the objective to minimize the difference between the total distance traveled and the total revenue earned from served customers, weighted by a parameter α, the problem can be formulated as

$$\min \sum_{k \in K} \sum_{(i,j) \in A} d_{ij} x_{ij}^k - \alpha \sum_{k \in K} \sum_{i \in N} g_i y_i^k$$

with M an arbitrary large constant and subject to:

$\sum_{j \in N^+} x_{ij}^k = y_i^k, i \in N, k \in K$ and $\sum_{k \in K} y_i^k \le 1, i \in N$ (every customer should be visited only once)

$\sum_{i \in N^+} x_{ih}^k - \sum_{j \in N^+} x_{hj}^k = 0, h \in N, k \in K$, $\sum_{i \in N^+} x_{0i}^k = 1, k \in K$ and $\sum_{i \in N^+} x_{i(n+1)}^k = 1, k \in K$ (flow conservation constraints that describe individual routes)

$\sum_{i \in N} q_i y_i^k \le Q, k \in K$ (total demand on a route cannot exceed the capacity of a vehicle)

$t_i^k + s_i + t_{ij} - M(1 - x_{ij}^k) \le t_j^k, (i,j) \in A, k \in K$, $a_i y_i^k \le t_i^k \le b_i y_i^k, i \in K, k \in K$, $t_0^k \ge \sigma^k, k \in K$ and $t_i^k \le t_0^k + t_{\max}, i \in N, k \in K$ (this last four relations ensure the feasibility of the time schedule)

$\sigma^k = \beta \sum_{i \in N} s_i y_i^k, k \in K$ (the vehicle loading time for a route is the sum of the service times of all customers in the route, multiplied by β)

$t_0^l + M(1 - z_{kl}) \ge t_{n+1}^k + \sigma_l, k,l \in K, k < l$ and $\sum_{k \in K} \sum_{l \in K | l > k} z_{kl} \ge |K| - |V|, k,l \in K$

(ensure the proper route sequencing within the workdays, for the individual vehicles; there are at most $|V|$ workdays)

$x_{ij}^k \in \{0,1\}, i,j \in A, k \in K$

$y_j^k \in \{0,1\}, i \in N, k \in K$.

4. SOLUTIONS FOR VRP WITH MULTIPLE USE OF VEHICLES

Fisher [4] proposed the following classification for vehicle routing algorithms:

- Simple heuristics, based on local search and sweep (developed between 1960 and 1970)
- Mathematical programming based heuristics, that approximate VRP with generalized assignments and set partitioning problems
- Exact optimization (K-tree, Lagrangean Relaxation) and artificial intelligence methods: Simulated Annealing, Tabu Search, Ant Colony Optimization and Genetic Algorithms.

4.1. The first study that includes the multi-trip idea is due to Fleischmann (1990) and was referred by Taillard, Laporte and Gendreau [11]. Their solution is a population based algorithm using tabu search and bin packing approaches and even allows vehicles to perform trips longer than the given time limit, paying some penalty.

4.2. Brandão and Mercer [2] based their work on the operations of a real life company, for VRP with multiple use of vehicles and a list of supplementary assumptions: vehicle have different capacities, customers are accessed considering some restrictive rules, unloading times of vehicles are allowed, additional vehicles can be hired if the fleet of the company cannot satisfy all demands, drivers use real map data and the objective function of the problem takes into consideration transportation costs related to fuel, salaries, maintenance. The algorithm of Brandão and Mercer uses two techniques:

a. Nearest neighbor and insertion procedure. This step creates a set of trips (called layer) such that each trip is associated to a different vehicle. The insertion procedure tries to include more customers to the trips of the layer. After any layer is obtained, the procedures are reapplied to improve it by adding unserved customers to the existing trips.

b. Tabu search algorithm, with two moves: insertion move places a customer into a trip (if the neighborhood condition is satisfied and the capacity of the vehicle is not violated) or creates a new trip, while a swap changes two customers between two trips.

4.3. Petch and Salhi [8] show that by enabling multi-trips, important savings in all transportation costs can be obtained. Their solution is composed of a multi-phase construction heuristic, a combination of the algorithms presented at 4.1. and 4.2.:

a. Phase 1 implies construction of VRP solutions with Yellow's savings algorithm, improvement with 2-opt and 3-opt exchange refining and elimination of repeated solutions.

b. Generation of VRPM solution from the one obtained at phase 1, using bin packing algorithm.

c. Implementation of a tour partition approach, in order to obtain many trips. Available trips are partitioned into smaller, feasible trips using geographical route codification.

4.4. Olivera and Viera [7] give a solution based on the adaptive memory procedure of Rochat and Taillard and a mathematical programming model based on a set covering formulation of the problem.

4.5. Salhi and Petch [9] use a genetic algorithm approach. The chromosomes are defined by a sector of a circle, where the solution for customers in a sector composes the encoding. Small VRP sub-problems are solved with savings heuristic, while bin packing heuristics is used to obtain a complete set of vehicle trips.

4.6. Azi, Gendreau and Potvin [1] use search and branching strategies to develop an exact branch-and-price algorithm for solving the vehicle routing problem with time windows and multiple use of vehicles. An elementary shortest path algorithm is exploited to solve the pricing subproblems. The authors point that the algorithm is limited by problem size and some characteristics of a given problem and they clearly state that a heuristic approach remains a viable alternative for large instances.

Other solutions for various forms of VRPM as well as real life examples are listed in [10].

5. ANT COLONY OPTIMIZATION HEURISTIC APPLIED TO SOLVE VEHICLE ROUTING PROBLEM WITH TIME WINDOWS AND MULTIPLE USE OF VEHICLES

Metaheuristic algorithms such as simulated annealing (SA), tabu search (TS), genetic algorithms (GA), and ant colony optimization (ACO) have been used to solve the VRPTW in order to escape local optima and enlarge the search.

Ant Colony Optimization (ACO) is used as a memory-based meta-heuristic, designed to simulate the capability of ants to determine shortest paths between their nests and food sources. In ACO, a colony of artificial ants cooperates in finding good solutions for difficult discrete optimization problems. ACO algorithms maintain a finite-size colony of artificial ants that searches collectively for good-quality solutions to the optimization problem under consideration. Each ant builds a solution, or just a component of a solution, starting from an initial state, selected according to some problem-dependent criteria. While building its own solution, each ant collects information on the problem characteristics and on its own performance and uses this information to modify the representation of the problem, as seen by the other ants ([5]). Ants can act concurrently and independently, showing cooperative behavior. They do not use direct communication: the exchange information among ants is done through stigmergy. To search for a feasible solution, ants use a step by step constructive approach. The complexity of each ant is such that even a single ant is able to find a solution (that is not necessarily very good). High-quality solutions are found only as the emergent result of global cooperation among all the agents of the colony, concurrently building different solutions.

Ant Systems (AS) are particular cases of ACO algorithms – in fact, were the first examples of such algorithms in the literature. AS and ACO raised a lot of interest in the scientific community. There are now available a wide range of implementations of them, applied to different problems such as: scheduling, vehicle routing, network routing, resources allocation, etc.

Bullnheimer, Hartl and Strauss (1998) offered the first application of an Ant System (AS) for VRP. Renaud, Boctor and Laporte (2002) developed an AS algorithm based on a simultaneous route construction mechanism. Reimann, Doerner and Hartl (2004) proposed an approach called D-ants. But the most efficient ACO algorithm for the VRPTW is MACS-VRPTW by Gambardella, Taillard, and Agazzi [5]. The central idea of the MACS-VRPTW algorithm is to use two colonies (MACS stands for Multi Ant Colony System): one colony, named ACS-VEI, minimizes the number of vehicles and the other one, named ACS-TIME, the time. The two colonies are completely independent, since each one has its own pheromone trail, but they collaborate by sharing a variable that describes the best current solution. Every ant in the colony tries to build a feasible solution to the problem.

All studies approaching VRP by ACO show that in order to obtain good solutions it is necessary to use local search improvement strategies in the algorithm. The most important can be classified as follows:

a. Insertion procedures: nearest insertion, cheapest insertion, farthest insertion, quick insertion or convex hull insertion algorithms build a solution by determining the least expensive insertion of a node into a route.

b. Improvement procedures: 2-opt, 3-opt, Or-opt examine all the routes that are neighboring to a given route, modifying it step by step and maintaining feasibility of the solution. This method requires long computational time, but we think it is especially appropriated for VRPTW with multiple use of vehicles, because can allow nodes to switch between the routes associated to a vehicle within a working day.

In [12], the authors present an interesting case study: they apply the ACO algorithm with MAX-MIN and rank-based ant system for airport ground service, that can be formulated as vehicle routing problem with tight time windows, short travel time and re-used vehicles. The use of some policies to deal with non-homogeneous fleet of vehicles makes the algorithm suitable for real-life situations. As an original element, an efficient heuristic called earliest due date first (EDD) is incorporated in order to improve the performance of ACO. Experiments with a similar ACO are made in [13].

It is also important to notice that researchers also attempted hybrid approaches, that proved to have a high potential to provide good solutions at low computational time [11]. Experimentations show that some algorithms are successful in finding solutions within 1% of known optimal solutions. The use of multiple ant colonies is found to provide a competitive solution technique for larger problems.

6. CONCLUSIONS AND FUTURE WORK

VRPTW with multiple use of vehicles is a highly complex problem, having a significant practical importance in nowadays context. There are few studies that approached this VRP variant. Among them, ACO heuristic is practical and offers - when compared to other techniques – high-quality solutions.

Our future work will focus on comparing the efficiency of various improvement procedures on a multiple colony system designed for this problem.

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