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INTUITIONISTIC FUZZY GO-CONNECTEDNESS BETWEEN INTUITIONISTIC FUZZY SETS

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Abstract. The aim of this paper is to introduce and discuss the concept of intuitionistic fuzzy GO-connectedness between intuitionistic fuzzy sets in intuitionistic fuzzy topological spaces.

1. Introduction

After the introduction of fuzzy sets by Zadeh [24] in 1965 and fuzzy topology by Chang [5] in 1967, several researches were conducted on the generalizations of the notions of fuzzy sets and fuzzy topology. The concept of intuitionistic fuzzy set was introduced by Atanassov [1,2] as a generalization of fuzzy set. In the last 25 years various concepts of fuzzy mathematics have been extended for intuitionistic fuzzy sets. In 1997 Coker [6] introduced the concept of intuitionistic fuzzy topological spaces.

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Recently many fuzzy topological concepts such as fuzzy compactness [8], fuzzy connectedness [22,23], fuzzy separation axioms [4,12,14], fuzzy nets and filters [13] , fuzzy metric spaces [21], fuzzy continuity [10] fuzzy multifunction's [16] and fuzzy g-closed sets[19] have been generalized for intuitionistic fuzzy topological spaces.

Connectedness is one of the basic notions in topology. The concept of "connectedness between sets" was first introduced by Kuratowski [11] in general topology. A space X is said to connected between subset A and B iff there is no closed-open set F in X such that A $\subseteq$ F and $A \cap F = \emptyset$ [Kuratowski 1968, p142]. Since then various weak and strong form of connectedness between sets such as s-connectedness between sets [9], p-connectedness between sets [17], GO-connectedness between sets [25] have been introduced and studied in general topology. In 1993 Thakur and Malviya [18] extended the notions of connectedness between sets in Fuzzy topology. Recently Thakur and Thakur [20] extended the concepts of connectedness between sets in intuitionistic fuzzy topology. In the present paper we introduced and study the concepts of intuitionistic fuzzy GO-connectedness between intuitionistic fuzzy sets in intuitionistic fuzzy topological spaces.

2. Preliminaries

Definition 2.1[1]: Let X be a non empty fixed set. An intuitionistic fuzzy set A is an object having the form

A= {<x, μ_A(x), γ_A(x) >: x$\in$X }

Where the functions μ_A:X$\rightarrow$ [0,1] and γ_A: X$\rightarrow$[0,1] denote the degree of membership (namely μ_A(x)) and the degree of non membership (namely γ_A(x)) of each element x$\in$X to the set A, respectively, and 0 $\leq \mu_A$(x)+ γ_A(x) $\leq$ 1 for each element x$\in$ X.

Definition 2.2 [1]: Let X be a nonempty set and the intuitionistic fuzzy sets A and B be in the form A = $\{<x, \mu_A(x), \gamma_A(x)> : x \in$ X$\}$, B = $\{<x, \mu_B(x), \gamma_B(x)> : x \in$ X$\}$ And let {Ai : $i \in$ J} be an arbitrary family of intuitionistic fuzzy sets in X. Then:

(a) A $\subseteq$ B if $\forall x \in X$ [$\mu_A(x) \leq \mu_B(x)$ and $\gamma_A(x) \geq \gamma_B(x)$];

(b) A = B if A $\subseteq$ B and B $\subseteq$ A;

(c) $A^c = \{<x, \gamma_A(x), \mu_A(x)> : x \in X\}$;

(d) $\cap_{Ai} = \{<x, \wedge\mu_{Ai}(x), \vee\gamma_{Ai}(x)> : x \in X\}$;

(e) $\cup_{Ai} = \{<x, \vee\mu_{Ai}(x), \wedge\gamma_{Ai}(x)> : x \in X\}$;

(f) $\tilde{0} = \{<x, 0, 1> : x \in X\}$ and $\tilde{I} = \{<x, 1, 0> : x \in X\}$;

Definition 2.3 [6]: Two intuitionistic fuzzy sets A and B of X are said to be q-coincident (AqB for short) if and only if there exists an element x∈X such that μ_A(x) > $\gamma_B(x)$ or $\gamma_A(x) < \mu_B$(x).

Lemma 2.1 [6]: For any two intuitionistic fuzzy sets A and B of X, ¬ (AqB) if and only if A ⊂ B^c

Definition 2.4:[6]**:** An intuitionistic fuzzy topology on a nonempty set X is a family $\Im$ of intuitionistic fuzzy sets in X, satisfying the following axioms:

(T1) $\tilde{o}$, $\tilde{I}$ $\in\Im$.

(T2) $G_1 \cap G_2 \in \Im$ for any G_1, $G_2 \in \Im$.

(T3) $\cup G_i \in \Im$for any arbitrary family { G_i: i ∈J}⊆ $\Im$.

In this case the pair (X,$\Im$) is called an intuitionistic fuzzy topological space.

Each intuitionistic fuzzy set in $\Im$ is known as an intuitionistic fuzzy open set in X.

The complement A^c of an intuitionistic fuzzy open set A is called an intuitionistic fuzzy closed set in X.

Definition 2.5[6]: Let (X,$\Im$) be an intuitionistic fuzzy topological space and

A = <x, $\mu_A(x)$, $\gamma_A(x)$ > be an intuitionistic fuzzy set in X. Then the fuzzy interior and fuzzy closure of A are defined by :

cl(A) = ∩{K : K is an intuitionistic fuzzy closed set in X and A ⊆K},

int(A) = ∪{G : G is an intuitionistic fuzzy open set in X and $G \subseteq K$}.

Lemma 2.2 [6]: For any intuitionistic fuzzy set A in (X,$\Im$) we have:

(a) A is an intuitionistic fuzzy closed set in X ⇔ cl(A) = A,

(b) A is an intuitionistic fuzzy open set in X ⇔ int(A) = A.

(c) cl(A^c) = $(int(A))^c$;

(d) int(A^c) = $(cl(A))^c$

Definition 2.6[19]: An intuitionistic fuzzy set A of an intuitionistic fuzzy topological space (X,$\Im$) is called :

(1) intuitionistic fuzzy g-closed if cl(A) ⊆ O whenever A ⊆ O and O is intuitionistic fuzzy open.
(2) intuitionistic fuzzy g-open if its complement A^c is intuitionistic fuzzy g-closed.

Remark 2.1[**19**]: Every intuitionistic fuzzy closed (resp. intuitionistic fuzzy open) set is intuitionistic fuzzy g-closed (resp. intuitionistic fuzzy g-open) but the converse may not be true.

Definition 2.7[**19**]**:** Let (X,$\Im$) be an intuitionistic fuzzy topological space and A be an intuitionistic fuzzy set in X. Then the g-interior and g-closure of A are defined as follows:

gcl(A) = $\cap\{K : K$ is an intuitionistic fuzzy g-closed set in X and $A \subseteq$K$\}$,

gint(A) = $\cup\{G : G$ is an intuitionistic fuzzy g-open set in X and $G \subseteq K\}$.

Definition 2.8 [**19**]**:** An intuitionistic fuzzy topological space is said to be intuitionistic fuzzy GO-connected if no non-empty intuitionistic fuzzy set is both intuitionistic fuzzy g- open and intuitionistic fuzzy g-closed.

Definition 2.9[**20**]: An intuitionistic fuzzy topological space (X, $\Im$) is said to be intuitionistic fuzzy connected between intuitionistic fuzzy sets A and B if there is no intuitionistic fuzzy closed open set F in X such that A $\subset$ F and $\neg$ (FqB).

3. Intuitionistic Fuzzy GO-Connectedness between Intuitionistic Fuzzy Sets

Definition 3.1: An intuitionistic fuzzy topological space (X, $\Im$) is said to be intuitionistic fuzzy GO-connected between intuitionistic fuzzy sets A and B if there is no intuitionistic fuzzy g-closed g-open set F in X such that A $\subset$ F and $\neg$ (FqB).

Theorem 3.1: If an intuitionistic fuzzy topological space (X,$\Im$) is intuitionistic fuzzy GO-connected between intuitionistic fuzzy sets A and B, then it is intuitionistic fuzzy connected between A and B.

Prof : If (X, $\Im$) is not intuitionistic fuzzy connected between A and B , then there exists an intuitionistic fuzzy closed open set F in X such that A$\subset$ F and $\neg$ (FqB). Then by Remark 2.1, F is an intuitionistic fuzzy g-closed g-open set in X such that A$\subset$ F and $\neg$ (FqB). Hence (X, $\Im$) is not intuitionistic fuzzy GO-connected between A and B, which contradicts our hypothesis.

Remark 3.1: The converse of Theorem 3.1 may be false, as the following example shows :

Example 3.1 : Let X = { a ,b } and U ={ < a , 0.5, 0.4 > ,< b , 0.6 ,0.4 > }

A= { <a, 0.2, 0.7 > ,<b,0.3,0.6>} and B = { < a, 0.5, 0.4 > ,< b, o.4, o.5 > } be intuitionistic fuzzy sets on X .let $\Im$ = { $\tilde{0}$, $\tilde{I}$,U } be an intuitionistic fuzzy topology on X. Then (X,$\Im$) is intuitionistic fuzzy connected between A and B but it is not intuitionistic fuzzy GO connected between A and B.

Theorem 3.2: An intuitionistic fuzzy topological space (X, $\Im$) is intuitionistic fuzzy GO-connected between intuitionistic fuzzy sets A and B if and only if there is no intuitionistic fuzzy g-closed g-open set F in X such that A $\subset$ F $\subset$ B^c.

Proof: Necessity: Let (X, $\Im$) is intuitionistic fuzzy GO-connected between intuitionistic fuzzy sets A and B. Suppose on the contrary , that F is an intuitionistic fuzzy g-closed g-open set in X such that A $\subset$ F $\subset$ B^c. Now F $\subset$ B^c which implies that $\neg$ (FqB). Therefore F is an intuitionistic fuzzy g-closed g-open set in X such that A $\subset$ F and $\neg$ (FqB). Hence (X,$\Im$) is not intuitionistic fuzzy GO-connected between intuitionistic fuzzy sets A and B, which is a contradiction.

Sufficiency: Suppose on the contrary, that (X,$\Im$) is not intuitionistic fuzzy GO-connected between intuitionistic fuzzy sets A and B. Then there exists an intuitionistic fuzzy g-closed g-open set F in X such that A$\subset$ F and $\neg$ (FqB). Now, $\neg$ (FqB) which implies that F $\subset$ B^c. Therefore F is an intuitionistic fuzzy g-closed g-open set in X such that A $\subset$ F $\subset$ B^c, which contradicts our assumption.

Theorem 3.3: If an intuitionistic fuzzy topological space(X,$\Im$) is intuitionistic fuzzy GO-connected between intuitionistic fuzzy sets A and B, then A and B are non- empty.

Proof: If the intuitionistic fuzzy set A is empty, then A is an intuitionistic fuzzy g-closed g-open set in X and A $\subset$ B. Now we claim that $\neg$ (AqB). If AqB, then there exists an element x$\in$ X such that $\mu_A(x) > \Upsilon_B$(x) or Υ_A(x) < μ_B (x) .But μ_A (x) = $\tilde{0}$ and Υ_A(x) = $\tilde{I}$ for all x$\in$ X. Therefore no point x$\in$X for which μ_A(x) >Υ_B (x) or Υ_A (x) < μ_B(x), which is a contradiction. Hence $\neg$ (AqB) and (X,$\Im$) is not intuitionistic fuzzy GO-connected between intuitionistic fuzzy set A and intuitionistic fuzzy set B.

Theorem 3.4: If an intuitionistic fuzzy topological space (X,$\Im$) is intuitionistic fuzzy GO-connected between intuitionistic fuzzy sets

A and B and A$\subset$ A_1 and B$\subset$ B_1, then (X,$\Im$) is intuitionistic fuzzy GO-connected between A_1 and B_1.

Proof: Suppose (X,$\Im$) is not intuitionistic fuzzy GO-connected between intuitionistic fuzzy sets A_1 and B_1. Then there is an intuitionistic fuzzy g-closed g-open set F in X such that $A_1 \subset$ F and $\neg$ (FqB1). Clearly, A $\subset$ F. Now we claim that $\neg$ (FqB). If FqB ,then there exists a point x$\in$ X such that μ_F(x) $>$ Υ_B(x) or Υ_F(x) $<$ μ_B (x). Without loss of generality suppose a point x$\in$ X such that μ_F(x) $>$ Υ_B(x). Now B$\subset$ B_1,Υ_B(x) $\geq$ Υ_{B1}(x). And so μ_F(x) $>$ Υ_{B1}(x) and FqB1, a contradiction. Consequently (X,$\Im$) is not intuitionistic fuzzy GO-connected between intuitionistic fuzzy sets A and B.

Theorem 3.5: An intuitionistic fuzzy topological space (X,$\Im$) is intuitionistic fuzzy GO-connected between intuitionistic fuzzy sets A and B if and only if it is intuitionistic fuzzy Go- connected between gcl(A) and gcl(B).

Proof: Necessity: Follows from Theorem 3.4, because A$\subset$ gcl(A) and B $\subset$ gcl(B).

Sufficiency: Suppose (X,$\Im$) is not intuitionistic fuzzy GO-connected between intuitionistic fuzzy sets A and B. Then there is an intuitionistic fuzzy g-closed g-open set F of X such that A$\subset$F and $\neg$ (FqB). Since F is intuitionistic fuzzy g-closed and A$\subset$F, gcl(A)$\subset$F. Now, $\neg$ (FqB) which implies that F$\subset$ B^c . Therefore F = gint F $\subset$ gint (B^c)= $(gcl(B))^c$. Hence (Fqgcl(B)) and X is not intuitionistic fuzzy GO-connected between gcl(A) and gcl(B).

Theorem 3.6: Let (X,$\Im$) be an intuitionistic fuzzy topological space and A and B be two intuitionistic fuzzy sets in X. If AqB then (X,$\Im$) is intuitionistic fuzzy GO-connected between A and B.

Proof: If F is any intuitionistic fuzzy g-closed g-open set of X such that A$\subset$F, then

AqB hence FqB.

Remark 3.2: The converse of Theorem 3.6 may not be true, as the following example shows

Example 3.2 Let X = {a, b} and U = {<a, 0.2, 0.6 >, < b, 0.3, 0.5> }, A = { <a, 0.4, 0.3>, < b, 0.3, 0.6 > } and B = { <a, 0.2, 0.5>, <b, 0.5, 0.4> } be intuitionistic fuzzy sets on X. Let $\Im = \{\tilde{0}, \tilde{I}, U\}$ be an intuitionistic fuzzy topology on X. Then (X,$\Im$) is intuitionistic fuzzy GO-connected between intuitionistic fuzzy sets A and B but $\neg$ (AqB).

Theorem 3.7: An intuitionistic fuzzy topological space (X,$\Im$) is intuitionistic fuzzy GO-connected if and only if it is intuitionistic fuzzy GO-connected between every pair of its non- empty intuitionistic fuzzy sets.

Proof: Necessity: Let A, B be any pair of intuitionistic fuzzy subsets of X. Suppose (X,$\Im$) is not intuitionistic fuzzy GO-connected between intuitionistic fuzzy sets A and B. Then there exists an intuitionistic fuzzy g-closed g-open set F of X such that A$\subset$ F and $\neg$ (FqB). Since intuitionistic fuzzy sets A and B are non- empty, it follows that F is a non- empty proper intuitionistic fuzzy g-closed g-open set of X. Hence (X,$\Im$) is not intuitionistic fuzzy GO-connected.

Sufficiency: Suppose (X,$\Im$) is not intuitionistic fuzzy GO-connected. Then there exists a non-empty proper intuitionistic fuzzy g-closed g-open set F of X. Consequently X is not intuitionistic fuzzy GO-connected between F and F^c, a contradiction.

Remark 3.3: If a fuzzy topological space (X,$\Im$) is intuitionistic fuzzy GO-connected between a pair of its intuitionistic fuzzy subsets it is not necessarily that (X,$\Im$) is intuitionistic fuzzy GO-connected between every pair of its intuitionistic fuzzy subsets and so is not necessarily intuitionistic fuzzy GO-connected, as the following example shows

Example 3.3: Let X = {a, b} and U = {< a, 0.5, 0.4>, < b, 0.6, 0.4 >}, A = {< a, 0.4, 0.3 >, < b, 0.6, 0.4 > } , B = { < a, 0.5, 0.2 >, < b, 0.4, 0.4 >}

C = < a, 0.2 , 0.7 >, < b, 0.3, 0.6>}and D = {< a, 0.5, 0.4>, < b, 0.4, 0.5 > } be intuitionistic fuzzy sets on X. Let $\Im = \{\tilde{0}, \tilde{I}, U\}$ be an intuitionistic fuzzy topology on X. Then (X,$\Im$) is intuitionistic fuzzy connected between intuitionistic fuzzy sets A and B but it is not intuitionistic fuzzy connected between intuitionistic fuzzy sets C and D. Also (X,$\Im$) is not intuitionistic fuzzy GO-connected.

Theorem 3.8: Let (Y,$\Im_Y$) be a subspace of a intuitionistic fuzzy topological space (X,$\Im$) and A, B be intuitionistic fuzzy subsets of Y. If (Y,$\Im_Y$) is intuitionistic fuzzy GO-connected between A and B then so is (X,$\Im$).

Proof: Suppose, on the contrary, that (X,$\Im$) is not intuitionistic fuzzy GO- connected between intuitionistic fuzzy sets A and B. Then there exists an intuitionistic fuzzy g-closed g-open set F of X such that A⊂F and ¬ (FqB).

Put F_Y = F∩Y. Then F_Y is intuitionistic fuzzy g-closed g-open set in Y such that A⊂ F_Y and ¬ (F_Y qB). Hence (Y,$\Im_Y$) is not intuitionistic fuzzy GO connected between A and B, a contradiction.

Theorem 3.9 Let (Y,$\Im$Y) be an intuitionistic fuzzy closed open subspace of a intuitionistic fuzzy topological space (X,$\Im$) and A,B be intuitionistic fuzzy subsets of Y. If (X,$\Im$) is intuitionistic fuzzy GO-connected between intuitionistic fuzzy sets A and B, then so is (Y,$\Im$Y).

Proof: If (Y,$\Im_Y$) is not intuitionistic fuzzy GO-connected between intuitionistic fuzzy sets A and B, then there exists an intuitionistic fuzzy g-closed g-open set F of Y such that A⊂F and ¬ (FqB). Since Y is intuitionistic fuzzy closed open in X, F is an intuitionistic fuzzy g-closed g-open set in X. Hence X cannot be intuitionistic fuzzy GO-connected between intuitionistic fuzzy sets A and B, a contradiction.

References

[1] K. Atanassov, **Intuitionistic Fuzzy Sets**, In VII ITKR's Session, (V. Sgurev, Ed.) Sofia, Bulgaria, 1983

[2] K. Atanassov and S. Stoeva., **Intuitionistic Fuzzy Sets**, In Polish Symposium on Interval and Fuzzy Mathematics , Poznan, 1983, 23-26

[3] K. Atanassov, **Intuitionistic Fuzzy Sets**. Fuzzy Sets and Systems, 20 1986, 87-96.

[4] Bayhan Sadik, **On Separation Axioms in Intuitionistic Topological Spaces**.
Intern. Jour. Math. Math. Sci. 27102001, 621-630.

[5] C.L. Chang, **Fuzzy Topological Spaces**, J. Math. Anal. Appl. 241968, 182-190.

[6] D. Coker, **An Introduction to Intuitionistic Fuzzy Topological Spaces**, Fuzzy Sets and Systems 881997, 81-89.

[7] D. Coker and M. Demirci, **On Intuitionistic Fuzzy Points. Notes on Intuitionistic Fuzzy Sets** 211995, 78-83

[8] D. Coker and A. Es. Hayder , **On Fuzzy Compactness in Intuitionistic Fuzzy Topological Spaces** ,The Journal of Fuzzy Mathematics (3-4}(1995), 899-909.

[9] K.K. Dubey . and O.S. Panwar , **Some properties of s-connectedness between sets and s-connected mapping**. Indian J. pure Math..154 1984, 343-354.

[10] H. Gurcay, D. Coker and A. Es. Hayder, **On Fuzzy Continuity in Intuitionistic Fuzzy Topological Spaces**. The Journal of Fuzzy Mathematics 521997,365-378.
[11] K. Kuratowski . **Topology** Vol. II (transl.), Academic Press New York (**??**).
[12] F. G. Lupianez **Separation axioms in intuitionistic fuzzy topological spaces**, International Journal of Pure and Applied Mathematics 1712004 ,29-34
[13] F. G. Lupianez **Nets and filters in intuitionistic fuzzy topological spaces**, Information Sciences 1762006, 2396-2404
[14] F. G. Lupianez **Hausdorffness in intuitionistic fuzzy topological Spaces**, The Journal of Fuzzy Mathematics 1232004,521-525
[15] S. N. Maheshwari , S. S. Thakur and R. Malviya, **Connected Between Fuzzy Sets**, The Journal of Fuzzy Mathematics,14 1993,757-759
[16] O. Ozbakir and D. Coker, **Fuzzy Multifunction's in Intuitionistic Fuzzy Topological Spaces Notes on Intuitionistic Fuzzy Sets** 531999.
[17] S.S.Thakur and P. Paik **p-connectedness between sets**, Jour of Sci. Res. B.H.U. 371 1987 59-63.
[18] S.S.Thakur and R. Malviya. **Fuzzy connectedness between fuzzy sets**, The Journal of Fuzzy Mathematics Vol.11993 4. 757-759.
[19] S. S. Thakur and Rekha Chaturvedi **Generalized closed sets in intuitionistic fuzzy topology**, The Journal of Fuzzy Mathematics 1632008, 559-572
[20] S. S. Thakur and Mahima Thakur, **Intuitionistic fuzzy set Connected mappings** . The Journal of Fuzzy Mathematics (Accepted)
[21] B. Tripathy, Intuitionistic Fuzzy Metric Spaces, Notes on Intuitionistic Fuzzy Sets 521999
[22] N. Turanli and D. Coker, **Fuzzy Connectedness in Intuitionistic Fuzzy Topological Spaces**. Fuzzy Sets And Systems 116 3 2000 ,369-375.
[23] Yong Chan Kim and S.E.Abbas , **Connectedness in intuitionistic fuzzy topological spaces**, Commun.Koren.Math.Soc.201 2005,117-134.
[24] L. A. Zadeh, **Fuzzy Sets**, Information and Control, 181965,338-353.
[25] I. Zorlutuna ,M. Kucuk and Y.Kucuk , **Slightly Generalized Continuous Functions** , Kochi J. Math. 32008, 99-108.

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