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A GENERAL FIXED POINT THEOREM FOR SEVERAL MAPPINGS IN G-METRIC SPACES

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Dedicated to the memory of Professor Gabriel Puiu

Abstract. In this paper we improve, extend and generalize the main results from [17], in G - metric spaces for mappings satisfying implicit relations.

1. INTRODUCTION

In [4], [5], Dhage introduced a new class of generalized metric space called D-metric space. Mustafa and Sims. Mustafa and Sims [8], [9] proved that most of the claims concerning the fundamental topological structure of D - metric spaces are incorrect and introduced appropriate notion of generalized metric space, named G - metric space. In fact, Mustafa and other authors [3], [7] - [14], [19] studied many fixed point results for self mappings in G - metric spaces under certain conditions. In [1] and [7], some fixed point results for two mappings satisfying a form of compatibility are proved. Quite recently, Karayilan and Telci [6] proved some fixed point theorems for two mappings satisfying two contractive (extensive) conditions in G - metric spaces. In [17], Popa and Puiu proved a fixed point theorem for several mappings in metric spaces which generalize the main results from [19]. In [15], [16], Popa initiated the study of fixed points for mappings satisfying an implicit relation.

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In this paper we improve, extend and generalize the corresponding results from [17] in G - metric spaces for mappings satisfying implicit relations.

2. PRELIMINARIES

Definition 2.1. *Let X be a nonempty set and $G : X^3 \to \mathbb{R}$ be a function satisfying the following properties:*

$(G_1) : G(x, y, z) = 0$ *if* $x = y = z$,

$(G_2) : 0 < G(x, x, y)$ *for all* $x, y \in X$ *with* $x \neq y$,

$(G_3) : G(x, x, y) \leq G(x, y, z)$ *for all* $x, y, z \in X$ *with* $z \neq y$,

$(G_4) : G(x, y, z) = G(x, z, y) = G(y, z, x) = ...$ *(symmetry in all three variables),*

$(G_5) : G(x, y, z) \leq G(x, a, a) + G(a, y, z)$ *for all* $x, y, z, a \in X$ *(rectangle inequality).*

Then, the function G is called a G - metric on X and the pair (X, G) is called a G - metric space.

Note that $G(x, y, z) = 0$, then $x = y = z$.

Definition 2.2 ([9])**.** *Let (X, G) be a G - metric space. A sequence (x_n) in X is said to be:*

a) *G - convergent if for $\varepsilon > 0$, there exists an $x \in X$ and $k \in \mathbb{N}$ such that for all $m, n \geq k$, $G(x, x_n, x_m) < \varepsilon$.*

b) *G - Cauchy sequence if for $\varepsilon > 0$, there exists $k \in \mathbb{N}$ such that for $n, m, p \geq k$, $G(x_n, x_m, x_p) < \varepsilon$, that is $G(x_n, x_m, x_p) \to 0$ as $n, m, p \to \infty$.*

A G - metric space is said to be G - complete if any G - Cauchy sequence is G - convergent.

Lemma 2.1 ([9])**.** *Let (X, G) be a G - metric space. Then the following properties are equivalent:*

1) (x_n) *is* G *- convergent to* x*;*

2) $G(x_n, x_n, x) \to 0$ *as* $n \to \infty$*;*

3) $G(x_n, x, x) \to 0$ *as* $n \to \infty$*;*

4) $G(x_n, x_m, x) \to 0$ *as* $n, m \to \infty$*.*

Lemma 2.2 ([9])**.** *If (X, G) is a G - metric space, then the following properties are equivalent:*

1) *The sequence (x_n) is G - Cauchy;*

2) *For every $\varepsilon > 0$, there exists $k \in \mathbb{N}$ such that $G(x_n, x_m, x_m) < \varepsilon$ for all $n, m > k$.*

Definition 2.3 ([9]). *Let (X, G) and (X', G') be two G - metric spaces. A function $f : (X, G) \to (X', G')$ is said to be G - continuous at a point $a \in X$ if for $\varepsilon > 0$, there exists $\delta > 0$ such that for all $x, y \in X$ and $G(a, x, y) < \delta$, then $G'(f(a), f(x), f(y)) < \varepsilon$.*

A function f is G - continuous if it is G - continuous at each $a \in X$.

Lemma 2.3 ([9]). *Let (X, G) and (X', G') be two G - metric spaces. A function $f : (X, G) \to (X', G')$ is G - continuous at a point $x \in X$ if and only if it is G - sequentially continuous, that is, whenever (x_n) is G - convergent to x we have $f(x_n)$ is G' - convergent to $f(x)$.*

Lemma 2.4 ([9]). *Let (X, G) be a G - metric space, then the function $G(x, y, z)$ is jointly continuous in all three of its variables.*

The following theorem is proved in [17].

Theorem 2.1. *Let (X, d) be a complete metric space and $P_1, P_2, ..., P_k : (X, d) \to (X, d)$ k mappings such that*

$$d(P_i x, P_{i+1} y) \leq a d(x, y) + b(d(x, P_i x) + d(y, P_{i+1} y)) + c(d(x, P_{i+1} y) + d(y, P_i x))$$

for all $x, y \in X$, where $P_{k+1} = P_1$ and $a + 2b + 2c < 1$.

Then $P_1, P_2, ..., P_k$ have a unique common fixed point.

3. IMPLICIT RELATIONS

Definition 3.1. *Let $\mathfrak{F}_G$ be the set of all continuous functions $F(t_1, ..., t_6) : \mathbb{R}_+^6 \to \mathbb{R}$ such that*

(F_1) : *F is nonincreasing in variable t_5,*

(F_2) : *There exists $h_1 \in [0, 1)$ such that for all $u, v \geq 0$, $F(u, v, v, u, u + v, 0) \leq 0$ implies $u \leq h_1 v$,*

(F_3) : *There exists $h_2 \in [0, 1)$ such that for all $t, t' > 0$, $F(t, t, 0, 0, t, t') \leq 0$ implies $t \leq h_2 t'$.*

Example 3.1. *$F(t_1, ..., t_6) = t_1 - a t_2 - b t_3 - c t_4 - d t_5 - e t_6$, where $a, b, c, d, e \geq 0$ and $0 < a + b + c + 2d + e < 1$.*

(F_1) : Obviously.

(F_2) : Let $u, v \geq 0$ be and $F(u, v, v, u, u+v, 0) = u - au - bv - cu - d(u + v) \leq 0$ which implies $u \leq h_1 v$, where $0 \leq h_1 = \dfrac{a + b + d}{1 - (c + d)} < 1$.

(F_3) : Let $t, t' > 0$ be and $F(t, t, 0, 0, t, t') = t - at - dt - et' \leq 0$, then $t \leq h_2 t'$, where $0 \leq h_2 = \dfrac{e}{1 - (a + d)} < 1$.

Example 3.2. *$F(t_1,...,t_6) = t_1 - k\max\{t_2,t_3,t_4,t_5,t_6\}$ where $k \in \left[0,\frac{1}{2}\right)$.*

(F_1) : Obviously.
(F_2) : Let $u,v \geq 0$ be and $F(u,v,v,u,u+v,0) = u - k\max\{u,v,u+v\} \leq 0$. Hence $u \leq h_1 v$, where $0 \leq k_1 = \dfrac{k}{1-k} < 1$.
(F_3) : Let $t,t' > 0$ be and $F(t,t,0,0,t,t') = t - k\max\{t,t'\}$. If $t > t'$, then $t(1-k) \leq 0$, a contradiction. Hence $t \leq t'$, which implies $t \leq h_2 t'$, where $0 \leq h_2 = k < 1$.

Example 3.3. *$F(t_1,...,t_6) = t_1 - k\max\left\{t_2,t_3,t_4,\frac{1}{2}(t_5+t_6)\right\}$ where $k \in [0,1)$.*

(F_1) : Obviously.
(F_2) : Let $u,v \geq 0$ be and

$$F(u,v,v,u,u+v,0) = u - k\max\left\{u,v,\frac{1}{2}(u+v)\right\} \leq 0.$$

If $u > v$, then $u(1-k) \leq 0$, a contradiction. Hence $u \leq v$ which implies $u \leq h_1 v$, where $0 \leq h_1 = k < 1$.
(F_3) : Let $t,t' > 0$ be and $F(t,t,0,0,t,t') = t - k\max\left\{t,\frac{1}{2}(t+t')\right\} \leq 0$. If $t > t'$, then $t(1-k) \leq 0$, a contradiction. Hence $t \leq t'$,which implies $t \leq h_2 t'$, where $0 \leq h_2 = k < 1$.

Example 3.4. *$F(t_1,...,t_6) = t_1^2 - t_1(at_2 + bt_3 + ct_4) - dt_5t_6$, where $a,b,c,d \geq 0$ and $0 < a+b+c+d < 1$.*

(F_1) : Obviously.
(F_2) : Let $u,v \geq 0$ be and $F(u,v,v,u,u+v,0) = u^2 - u(av + bv + cu) \leq 0$. If $u > 0$, then $u - av - bv - cu \leq 0$ which implies $u \leq h_1 v$, where $0 \leq h_1 = \dfrac{a+b}{1-c} < 1$. If $u = 0$, then $u \leq h_1 v$.
(F_3) : Let $t,t' > 0$ be and $F(t,t,0,0,t,t') = t^2 - at^2 - ctt' \leq 0$, which implies $t \leq h_2 t'$, where $0 \leq h_2 = \dfrac{c}{1-a} < 1$.

Example 3.5. *$F(t_1,...,t_6) = t_1 - k\max\left\{t_2,\frac{t_3+t_4}{2},\frac{t_5+t_6}{2}\right\}$ where $k \in [0,1)$.*

(F_1) : Obviously.

(F_2) : Let $u, v \geq 0$ be and $F(u, v, v, u, u+v, 0) = u - k \max\left\{v, \dfrac{u+v}{2}\right\}$ ≤ 0. If $u > v$, then $u(1-k) \leq 0$, a contradiction, hence $u \leq v$ which implies $u \leq h_1 v$, where $0 \leq h_1 = k < 1$.

(F_3) : Let $t, t' > 0$ be and $F(t, t, 0, 0, t, t') = t - k \max\left\{t, \dfrac{t+t'}{2}\right\} \leq 0$. If $t > t'$, then $t(1-k) \leq 0$, a contradiction. Hence, $t \leq t'$, which implies $t \leq h_2 t'$, $0 \leq h_2 = k < 1$.

Example 3.6. $F(t_1, ..., t_6) = t_1^3 - c\dfrac{t_3^2 t_4^2 + t_5^2 t_6^2}{1 + t_2 + t_3 + t_4}$, *where* $c \in [0, 1)$.

(F_1) : Obviously.

(F_2) : Let $u, v \geq 0$ be and $F(u, v, v, u, u+v, 0) = u^3 - c\dfrac{u^2 v^2}{1 + 2v + u} \leq 0$. If $u > 0$, then $u \leq cv \cdot \dfrac{v}{1 + 2v + u} \leq cv$. Hence, $u \leq h_1 v$, where $0 \leq h_1 = c < 1$. If $u = 0$, then $u \leq h_1 v$.

(F_3) : Let $t, t' > 0$ be and $F(t, t, 0, 0, t, t') = t^3 - c\dfrac{t^2 t'^2}{1+t} \leq 0$ which implies $t^2 - c\dfrac{t}{1+t} \cdot t'^2 \leq c \cdot t'^2$, which implies $t \leq h_2 t'$, where $0 \leq h_2 = c < 1$.

Example 3.7. $F(t_1, ..., t_6) = t_1^2 - a t_2^2 - c\dfrac{t_5 t_6}{1 + t_3^2 + t_4^2}$, *where* $a, c \geq 0$ *and* $0 < a + c < 1$.

(F_1) : Obviously.

(F_2) : Let $u, v \geq 0$ be and $F(u, v, v, u, u+v, 0) = u^2 - a v^2 \leq 0$, which implies $u \leq h_1 v$, where $0 \leq h_1 = \sqrt{a} < 1$.

(F_3) : Let $t, t' > 0$ be and $F(t, t, 0, 0, t, t') = t^2(1-a) - c t t' \leq 0$ which implies $t \leq h_2 t'$, where $0 \leq h_2 = \dfrac{c}{1-a} < 1$.

Example 3.8. $F(t_1, ..., t_6) = t_1 - a t_2 - b t_3 - c \max\{2t_4, t_5 + t_6\}$, *where* $a, b, c \geq 0$ *and* $a + b + 2c < 1$.

(F_1) : Obviously.

(F_2) : Let $u, v \geq 0$ be and $F(u, v, v, u, u+v, 0) = u - av - c \max\{2u, u+v\} \leq 0$. If $u > v$ then $u(1 - (a + b + 2c)) \leq 0$, a contradiction. Hence $u \leq v$, which implies $u \leq h_1 v$, where $0 \leq h_1 = \dfrac{a+b+c}{1-c} < 1$.

(F_3) : Let $t, t' > 0$ be and $F(t, t, 0, 0, t, t') = t - at - c(t + t') \leq 0$, which implies $t \leq h_2 t'$, where $0 \leq h_2 = \dfrac{c}{1-(a+c)} < 1$.

Example 3.9. *$F(t_1, ..., t_6) = t_1 - at_2 - bt_3 - c\max\{t_4 + t_5, 2t_6\}$, where $a, b, c \geq 0$ and $a + b + 3c < 1$.*

(F_1) : Obviously.

(F_2) : Let $u, v \geq 0$ be and $F(u, v, v, u, u + v, 0) = u - av - bv - c(2u + v) \leq 0$, which implies $u \leq h_1 v$, where $0 \leq h_1 = \dfrac{a+b+c}{1-2c} < 1$.

(F_3) : Let $t, t' > 0$ be and $F(t, t, 0, 0, t, t') = t - at - c\max\{t, 2t'\} \leq 0$. If $t > 2t'$, then $t(1 - (a + c)) \leq 0$, a contradiction. Hence, $t \leq h_2 t'$, where $0 \leq h_2 = \dfrac{2c}{1-a} < 1$.

Example 3.10. *$F(t_1, ..., t_6) = t_1 - c\max\{t_2, t_3, \sqrt{t_4 t_6}, \sqrt{t_5 t_6}\}$, where $c \in (0, 1)$.*

(F_1) : Obviously.

(F_2) : Let $u, v \geq 0$ be and $F(u, v, v, u, u + v, 0) = u - cv \leq 0$ and $u \leq h_1 v$, where $0 \leq h_1 = c < 1$.

(F_3) : Let $t, t' > 0$ be and $F(t, t, 0, 0, t, t') = t - c\max\{t, \sqrt{tt'}\} \leq 0$. If $t > t'$, then $t(1 - c) \leq 0$, a contradiction. Hence, $t \leq t'$ which implies $t \leq h_2 t'$, where $0 \leq h_2 = c < 1$.

4. MAIN RESULT

Theorem 4.1. *Let (X, G) be a G - metric space and $P_i : (X, G) \to (X, G), i = 1, 2, ..., k$ such that*

$$F(G(P_i x, P_{i+1} y, P_{i+1} y), G(x, y, y), G(x, P_i x, P_i x), G(y, P_{i+1} y, P_{i+1} y), G(x, P_{i+1} y, P_{i+1} y), G(y, P_i x, P_i x)) \leq 0 \tag{4.1}$$

for i=1,2,...,k, where $P_{k+1} = P_1$ for all $x, y \in X$ and F satisfying (F_3). Then $P_1, ..., P_k$ have at most a unique common fixed point.

Proof. Suppose that $P_1, ..., P_k$ have two fixed points u and v. Then by (4.1) we have successively

$$F(G(P_i u, P_{i+1} v, P_{i+1} v), G(u, v, v), G(u, P_i u, P_i u), G(v, P_{i+1} v, P_{i+1} v), G(u, P_{i+1} v, P_{i+1} v), G(v, P_i u, P_i u)) \leq 0,$$

$$F(G(u, v, v), G(u, v, v), 0, 0, G(u, v, v), G(v, u, u)) \leq 0$$

By (F_3) we obtain

$$G(u, v, v) \leq h_2 G(v, u, u). \tag{4.2}$$

Similarly, for $x = v$ and $y = u$ we obtain

$$G(v, u, u) \leq h_2 G(u, v, v). \tag{4.3}$$

Therefore, $G(u, v, v)(1 - h_2^2) \leq 0$ which implies $u = v$. □

Theorem 4.2. *Let (X, G) be a G - complete metric space and $P_i : (X, G) \to (X, G)$, i=1,2,...,k satisfying inequality (4.1) for all $i = 1, ..., k$, $P_{k+1} = P_1$ and $F \in \mathfrak{F}_G$. Then $P_1, P_2, ..., P_k$ have a unique common fixed point.*

Proof. Let x be an arbitrary point in X. We define a sequence (x_n) such that

$$\begin{aligned} x_1 &= P_1 x, x_2 = P_2 x_1, ..., x_k = P_k x_{k-1}, \\ x_{k+1} &= P_1 x_k, x_{k+2} = P_2 x_{k+1}, ..., x_{2k} = P_k x_{2k-1}, \\ & ... \\ x_{nk+1} &= P_1 x_{nk}, x_{nk+2} = P_2 x_{nk+1}, ..., x_{(n+1)k} = P_k x_{(n+1)k-1}, \\ & ... \end{aligned}$$

By (4.1) we have successively

$$F(G(P_1 x, P_2 x_1, P_2 x_1), G(x, x_1, x_1), G(x, P_1 x, P_1 x), \\ G(x_1, P_2 x_1, P_2 x_1), G(x, P_2 x_1, P_2 x_1), G(x_1, P_1 x, P_1 x)) \leq 0,$$

$$F(G(x_1, x_2, x_2), G(x, x_1, x_1), G(x, x_1, x_1), \\ G(x_1, x_2, x_2), G(x, x_2, x_2), 0) \leq 0.$$

By (G_5) we have

$$G(x, x_2, x_2) \leq G(x, x_1, x_1) + G(x_1, x_2, x_2). \tag{4.4}$$

By (F_1) and (4.4) we obtain

$$F(G(x_1, x_2, x_2), G(x, x_1, x_1), G(x, x_1, x_1), \\ G(x_1, x_2, x_2), G(x, x_1, x_1) + G(x_1, x_2, x_2), 0) \leq 0.$$

By (F_2) we obtain

$$G(x_1, x_2, x_2) \leq h_1 G(x, x_1, x_1). \tag{4.5}$$

Similarly, we obtain

$$G(x_{k-1}, x_k, x_k) \leq h_1^{k-1} G(x, x_1, x_1).$$

By (4.2) we have successively

$$F(G(P_k x_{k-1}, P_1 x_k, P_1 x_k), G(x_{k-1}, x_k, x_k), G(x_{k-1}, P_{k-1} x_{k-1}, P_{k-1} x_{k-1}), \\ G(x_k, P_k x_k, P_k x_k), G(x_{k-1}, P_1 x_k, P_1 x_k), G(x_k, x_{k+1}, x_{k+1})) \leq 0.$$

By (G_5)

$$G(x_{k-1},x_{k+1},x_{k+1}) \leq G(x_{k-1},x_k,x_k)+G(x_k,x_{k+1},x_{k+1}).$$

By (F_1) we obtain

$$\begin{array}{c} F(G(x_k,x_{k+1},x_{k+1}),G(x_{k-1},x_k,x_k),G(x_{k-1},x_k,x_k),\\ G(x_k,x_{k+1},x_{k+1}),G(x_{k-1},x_k,x_k)+G(x_k,x_{k+1},x_{k+1}),0) \leq 0. \end{array}$$

By (F_2) we obtain

$$G(x_k,x_{k+1},x_{k+1}) \leq h_1 G(x_{k-1},x_k,x_k) \leq h_1^k G(x,x_1,x_1).$$

By induction we obtain

$$G(x_n,x_{n+1},x_{n+1}) \leq h_1^n G(x,x_1,x_1).$$

Moreover, for all $m,n \in \mathbb{N}$, $m>n$ we have repeated use the rectangular inequality that

$$\begin{array}{rcl} G(x_n,x_m,x_m) & \leq & G(x_n,x_{n+1},x_{n+1})+G(x_{n+1},x_{n+2},x_{n+2})+...+\\ & & +G(x_{m-1},x_m,x_m)\\ & \leq & (h_1^n+h_1^{n+1}+...+h_1^m)G(x,x_1,x_1)\\ & \leq & \dfrac{h_1^n}{1-h_1}G(x,x_1,x_1) \end{array}$$

and

$$\lim_{m,n\to\infty} G(x_n,x_m,x_m)=0.$$

Hence (x_n) is a G - Cauchy sequence in (X,G). Since (X,G) is a G - complete metric space, there exist $u \in X$ such that $\lim_{n\to\infty} x_n = u$. We prove that $P_s u = u$ for $s=1,2,...,k$.

By (4.1) we obtain

$$\begin{array}{c} F(G(P_s x_{nk+s},P_{s+1}u,P_{s+1}u),G(x_{nk+s},u,u),\\ G(x_{nk+s},P_s x_{nk+s},P_s x_{nk+s}),G(u,P_{s+1}u,P_{s+1}u),\\ G(x_{nk+s},P_{s+1}u,P_{s+1}u),G(x_{nk+s},P_s x_{nk+s},P_s x_{nk+s})) \leq 0, \end{array}$$

$$\begin{array}{c} F(G(x_{nk+s+1},P_{s+1}u,P_{s+1}u),G(x_{nk+s},u,u),\\ G(x_{nk+s},x_{nk+s+1},x_{nk+s+1}),G(u,P_{s+1}u,P_{s+1}u),\\ G(x_{nk+s},P_{s+1}u,P_{s+1}u),G(u,x_{nk+s+1},x_{nk+s+1})) \leq 0. \end{array}$$

Letting n tend to infinity we obtain

$$F(G(u,P_{s+1}u,P_{s+1}u),0,0,G(u,P_{s+1}u,P_{s+1}u),G(u,P_{s+1}u,P_{s+1}u),0) \leq 0.$$

By (F_2) we obtain $G(u,P_{s+1}u,P_{s+1}u)=0$, which implis that $u = P_{s+1}u$ and u is a common fixed point for $P_2,...,P_k$ and $P_{k+1}=P_1$.

By Theorem 4.1 u is the unique common fixed point for $P_1,P_2,...,P_k$. □

Corollary 4.1. *Let (X, G) be a G - metric complete space and let $P_1, P_2, ..., P_k : (X, G) \to (X, G)$ such that*

$$\begin{aligned}F(G(P_1x, P_2y, P_2y), G(x, y, y),\\ G(x, P_1x, P_1x), G(y, P_2y, P_2y),\\ G(x, P_2y, P_2y), G(y, P_1x, P_1x) \leq 0\end{aligned} \tag{4.6}$$

$$\begin{aligned}F(G(P_2x, P_1y, P_1y), G(x, y, y),\\ G(x, P_2x, P_2x), G(y, P_1y, P_1y),\\ G(x, P_1y, P_1y), G(y, P_2x, P_2x) \leq 0\end{aligned} \tag{4.7}$$

for all $x, y \in X$ and $F \in \mathfrak{F}_G$. Then P_1 and P_2 have a unique common fixed point.

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