**"Vasile Alecsandri" University of Bacău**
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# ANOTHER GENERAL DECOMPOSITION THEOREM OF CLOSED FUNCTIONS

TAKASHI NOIRI AND VALERIU POPA

**Abstract.** We introduce a new function $f : (X, \tau) \to (Y, m_Y)$ called a $gm$-closed function, where $(X, \tau)$ is a topological space and $(Y, m_Y)$ is an $m$-space. This function enables us to unify certain kind of modifications of closed functions.

## 1. Introduction

Semi-open sets, preopen sets, $\alpha$-open sets, $b$-open sets and $\beta$-open sets play an important role in the research of generalizations of closed functions in topological spaces. By utilizing these sets, many authors introduced and studied various types of modifications of closed functions. The notion of generalized closed (briefly $g$-closed) sets in topological spaces is introduced by Levine [15]. After that, the notions of $gs$-closed sets [7], $gp$-closed sets [24], $\alpha g$-closed sets [16], $gsp$-closed sets [10] (or $g\beta$-closed sets) are introduced and investigated. In [27], [28] and [25], the present authors introduced and studied the notions of $m$-structures, $m$-spaces and $m$-closed functions. In [23], the first author of the present paper introduced the notion of generalized $m$-closed sets which unifies the notions of $g$-closed sets, $gs$-closed sets, $gp$-closed sets, $\alpha g$-closed sets and $gsp$-closed sets.

The purpose of this paper is to obtain a new general decomposition of $m$-closed functions by utilizing generalized $m$-closed sets. Contra closed functions due to Baker [6] are useful in obtaining a sufficient condition for a $gm$-closed function to be $m$-closed.

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## 2. Preliminaries

Let $(X, \tau)$ be a topological space and $A$ a subset of $X$. The closure of $A$ and the interior of $A$ are denoted by $\mathrm{Cl}(A)$ and $\mathrm{Int}(A)$, respectively. We recall some generalized open sets in topological spaces.

**Definition 2.1.** Let $(X, \tau)$ be a topological space. A subset $A$ of $X$ is said to be

(1) *$\alpha$-open* [21] if $A \subset \mathrm{Int}(\mathrm{Cl}(\mathrm{Int}(A)))$,
(2) *semi-open* [14] if $A \subset \mathrm{Cl}(\mathrm{Int}(A))$,
(3) *preopen* [19] if $A \subset \mathrm{Int}(\mathrm{Cl}(A))$,
(4) *$\beta$-open* [1] or *semi-preopen* [3] if $A \subset \mathrm{Cl}(\mathrm{Int}(\mathrm{Cl}(A)))$,
(5) *$\gamma$-open* [12] or *b-open* [4] if $A \subset \mathrm{Int}(\mathrm{Cl}(A)) \cup \mathrm{Cl}(\mathrm{Int}(A))$.

The family of all semi-open (resp. preopen, $\alpha$-open, $\gamma$-open, $\beta$-open) sets of $(X, \tau)$ is denoted by $\mathrm{SO}(X)$ (resp. $\mathrm{PO}(X)$, $\alpha(X)$, $\gamma(X)$ or $\mathrm{BO}(X)$, $\beta(X)$ or $\mathrm{SPO}(X)$).

**Definition 2.2.** Let $(X, \tau)$ be a topological space. A subset $A$ of $X$ is said to be *$\alpha$-closed* [20] (resp. *semi-closed* [9], *preclosed* [19], *$\beta$-closed* [1] or *semi-preclosed* [3], *$\gamma$-closed* [12] or *b-closed* [4]) if the complement of $A$ is $\alpha$-open (resp. semi-open, preopen, $\beta$-open, $\gamma$-open).

**Definition 2.3.** Let $(X, \tau)$ be a topological space and $A$ a subset of $X$. The intersection of all $\alpha$-closed (resp. semi-closed, preclosed, $\beta$-closed, $\gamma$-closed) sets of $X$ containing $A$ is called the *$\alpha$-closure* [20] (resp. *semi-closure* [9], *preclosure* [11], *$\beta$-closure* [2] or *semi-preclosure* [3], *$\gamma$-closure* [12] or *b-closure* [4]) of $A$ and is denoted by $\alpha\mathrm{Cl}(A)$ (resp. $\mathrm{sCl}(A)$, $\mathrm{pCl}(A)$, ${}_{\beta}\mathrm{Cl}(A)$ or $\mathrm{spCl}(A)$), $\mathrm{Cl}_{\gamma}(A)$ or $\mathrm{bCl}(A)$).

**Definition 2.4.** Let $(X, \tau)$ be a topological space and $A$ a subset of $X$. The union of all $\alpha$-open (resp. semi-open, preopen, $\beta$-open, $\gamma$-open) sets of $X$ contained in $A$ is called the *$\alpha$-interior* (resp. *semi-interior*, *preinterior*, *$\beta$-interior* or *semi-preinterior*, *$\gamma$-interior* or *b-interior*) of $A$ and is denoted by $\alpha\mathrm{Int}(A)$ (resp. $\mathrm{sInt}(A)$, $\mathrm{pInt}(A)$, ${}_{\beta}\mathrm{Int}(A)$ or $\mathrm{spInt}(A)$, $\mathrm{Int}_{\gamma}(A)$ or $\mathrm{bInt}(A)$).

**Definition 2.5.** Let $(X, \tau)$ be a topological space. A subset $A$ of $X$ is said to be

(1) *g-closed* [15] if $\mathrm{Cl}(A) \subset U$ whenever $A \subset U$ and $U \in \tau$,
(2) *$\alpha g$-closed* [16] if $\alpha\mathrm{Cl}(A) \subset U$ whenever $A \subset U$ and $U \in \tau$,
(3) *gs-closed* [7] if $\mathrm{sCl}(A) \subset U$ whenever $A \subset U$ and $U \in \tau$,
(4) *gp-closed* [24] if $\mathrm{pCl}(A) \subset U$ whenever $A \subset U$ and $U \in \tau$,
(5) *gsp-closed* or *$g\beta$-closed* [10] if $\mathrm{spCl}(A) \subset U$ whenever $A \subset U$ and

$U \in \tau$,

(6) *gb-closed* or *$\gamma g$-closed* [12] if $\mathrm{Cl}_\gamma(A) \subset U$ whenever $A \subset U$ and $U \in \tau$.

**Definition 2.6.** A subset $A$ of a topological space $(X, \tau)$ is called a *LC-set* [8] (resp. *B-set* [30], *A-7-set* [31], *$\eta$-set* [26], *BC-set* [13], *BT-set*) if $A = U \cap V$, where $U \in \tau$ and $V$ is closed (resp. semi-closed, preclosed, $\alpha$-closed, $b$-closed, $\beta$-closed) in $(X, \tau)$.

Throughout the present paper, $(X, \tau)$ and $(Y, \sigma)$ always denote topological spaces and $f : (X, \tau) \to (Y, \sigma)$ presents a function.

**Definition 2.7.** A function $f : (X, \tau) \to (Y, \sigma)$ is said to be *closed* (resp. *semi-closed* [22], *preclosed* [11], *$\alpha$-closed* [19], *b-closed* [4], *$\beta$-closed* [1]) if $f(A)$ is closed (resp. semi-closed, preclosed, $\alpha$-closed, $b$-closed, $\beta$-closed) in $(Y, \sigma)$ for each closed set $A$ in $(X, \tau)$.

**Definition 2.8.** A function $f : (X, \tau) \to (Y, \sigma)$ is said to be *g-closed* [18] (resp. *gs-closed*, *gp-closed*, *$\alpha g$-closed*, *gb-closed*, *$g\beta$-closed*) if $f(A)$ is $g$-closed (resp. $gs$-closed, $gp$-closed, $\alpha g$-closed, $gb$-closed, $g\beta$-closed) in $(Y, \sigma)$ for each closed set $A$ in $(X, \tau)$.

**Definition 2.9.** A function $f : (X, \tau) \to (Y, \sigma)$ is said to be *LC-closed* (resp. *B-closed*, *A-7-closed*, *$\eta$-closed*, *BC-closed*, *BT-closed*) if $f(A)$ is a $LC$-set (resp. $B$-set, $A$-7-set, $\eta$-set, $BC$-set, $BT$-set) in $(Y, \sigma)$ for each closed set $A$ in $(X, \tau)$.

## 3. Minimal structures

**Definition 3.1.** Let $X$ be a nonempty set and $\mathcal{P}(X)$ the power set of $X$. A subfamily $m_X$ of $\mathcal{P}(X)$ is called a *minimal structure* (briefly *m-structure*) on $X$ [27] if $\emptyset \in m_X$ and $X \in m_X$.

By $(X, m_X)$, we denote a nonempty set $X$ with a minimal structure $m_X$ on $X$ and call it an $m$-space. Each member of $m_X$ is said to be *$m_X$-open* and the complement of an $m_X$-open set is said to be *$m_X$-closed*.

**Remark 3.1.** Let $(X, \tau)$ be a topological space. Then, the families $\tau$, $\mathrm{SO}(X)$, $\mathrm{PO}(X)$, $\alpha(X)$, $\beta(X)$, and $\gamma(X)$ are all $m$-structures on $X$.

**Definition 3.2.** Let $X$ be a nonempty set and $m_X$ an $m$-structure on $X$. For a subset $A$ of $X$, the *$m_X$-closure* of $A$ and the *$m_X$-interior* of $A$ are defined in [17] as follows:

(1) $\mathrm{mCl}(A) = \cap\{F : A \subset F, X - F \in m_X\}$,

(2) $\mathrm{mInt}(A) = \cup\{U : U \subset A, U \in m_X\}$.

**Remark 3.2.** Let $(X, \tau)$ be a topological space and $A$ a subset of $X$. If $m_X = \tau$ (resp. SO$(X)$, PO$(X)$, $\alpha(X)$, $\beta(X)$, $\gamma(X)$), then we have

(1) $\mathrm{mCl}(A) = \mathrm{Cl}(A)$ (resp. $\mathrm{sCl}(A)$, $\mathrm{pCl}(A)$, $\alpha\mathrm{Cl}(A)$, ${}_\beta\mathrm{Cl}(A)$, $\mathrm{Cl}_\gamma(A)$),

(2) $\mathrm{mInt}(A) = \mathrm{Int}(A)$ (resp. $\mathrm{sInt}(A)$, $\mathrm{pInt}(A)$, $\alpha\mathrm{Int}(A)$, ${}_\beta\mathrm{Int}(A)$, $\mathrm{Int}_\gamma(A)$) .

**Lemma 3.1.** (Maki et al. [17]). *Let $X$ be a nonempty set and $m_X$ a minimal structure on $X$. For subsets $A$ and $B$ of $X$, the following properties hold:*

*(1)* $\mathrm{mCl}(X - A) = X - \mathrm{mInt}(A)$ *and* $\mathrm{mInt}(X - A) = X - \mathrm{mCl}(A)$,

*(2) If* $(X - A) \in m_X$, *then* $\mathrm{mCl}(A) = A$ *and if* $A \in m_X$, *then* $\mathrm{mInt}(A) = A$,

*(3)* $\mathrm{mCl}(\emptyset) = \emptyset$, $\mathrm{mCl}(X) = X$, $\mathrm{mInt}(\emptyset) = \emptyset$ *and* $\mathrm{mInt}(X) = X$,

*(4) If* $A \subset B$, *then* $\mathrm{mCl}(A) \subset \mathrm{mCl}(B)$ *and* $\mathrm{mInt}(A) \subset \mathrm{mInt}(B)$,

*(5)* $A \subset \mathrm{mCl}(A)$ *and* $\mathrm{mInt}(A) \subset A$,

*(6)* $\mathrm{mCl}(\mathrm{mCl}(A)) = \mathrm{mCl}(A)$ *and* $\mathrm{mInt}(\mathrm{mInt}(A)) = \mathrm{mInt}(A)$.

**Definition 3.3.** A minimal structure $m_X$ on a nonempty set $X$ is said to have *property* $\mathcal{B}$ [17] if the union of any family of subsets belong to $m_X$ belongs to $m_X$.

**Remark 3.3.** Let $(X, \tau)$ be a topological space. Then, the families $\tau$, SO$(X)$, PO$(X)$, $\alpha(X)$, $\beta(X)$, and $\gamma(X)$ are all $m$-structures with property $\mathcal{B}$.

**Lemma 3.2.** (Popa and Noiri [29]). *Let $X$ be a nonempty set and $m_X$ a minimal structure on $X$ satisfying property $\mathcal{B}$. For a subset $A$ of $X$, the following properties hold:*

*(1)* $A \in m_X$ *if and only if* $\mathrm{mInt}(A) = A$,

*(2) $A$ is $m_X$-closed if and only if* $\mathrm{mCl}(A) = A$,

*(3)* $\mathrm{mInt}(A) \in m_X$ *and* $\mathrm{mCl}(A)$ *is $m_X$-closed.*

## 4. Generalized $m$-closed sets

**Definition 4.1.** Let $(X, \tau)$ be a topological space and $m_X$ an $m$-structure on $X$. A subset $A$ of $X$ is said to be *generalized $m$-closed* (briefly *gm-closed*) [23] if $\mathrm{mCl}(A) \subset U$ whenever $A \subset U$ and $U \in \tau$. The complement of a $gm$-closed set is said to be *gm-open*.

**Remark 4.1.** Let $(X, \tau)$ be a topological space and $m_X$ an $m$-structure on $X$. We put $m_X = \tau$ (resp. SO$(X)$, PO$(X)$, $\alpha(X)$, $\beta(X)$, $\gamma(X)$). Then, a $gm$-closed set is a $g$-closed (resp. $gs$-closed, $gp$-closed, $\alpha g$-closed, $gsp$-closed, $\gamma g$-closed) set.

**Definition 4.2.** Let $(X, \tau)$ be a topological space and $m_X$ an $m$-structure on $X$. A subset $A$ of $X$ is called an *mlc-set* if $A = U \cap V$, where $U \in \tau$ and $V$ is $m_X$-closed.

**Remark 4.2.** Let $(X, \tau)$ be a topological space. If $m_X = \tau$ (resp. $\mathrm{SO}(X)$, $\mathrm{PO}(X)$, $\alpha(X)$, $\mathrm{BO}(X)$, $\beta(X)$), then an *mlc*-set is a *LC*-set (resp. a *B*-set, an *A*-7-set, an $\eta$-set, a *BC*-set, a *BT*-set).

**Theorem 4.1.** *Let $(X, \tau)$ be a topological space and $m_X$ an $m$-structure on $X$ with property $\mathcal{B}$. Then a subset $A$ of $X$ is $m_X$-closed if and only if $A$ is gm-closed and an mlc-set.*

**Proof**. *Necessity.* Suppose that $A$ is $m_X$-closed. Let $U \in \tau$ and $A \subset U$. Since $A$ is $m_X$-closed, by Lemma 3.2 $A = \mathrm{mCl}(A)$ and hence $\mathrm{mCl}(A) \subset U$. This shows that $A$ is *gm*-closed. Because $A = X \cap A$, where $X \in \tau$ and $A$ is $m_X$-closed, $A$ is an *mlc*-set.

*Sufficiency.* Suppose that $A$ is *gm*-closed and an *mlc*-set. Since $A$ is an *mlc*-set, $A = U \cap V$, where $U \in \tau$ and $V$ is $m_X$-closed. Since $A \subset U$ and $A$ is *gm*-closed, $\mathrm{mCl}(A) \subset U$. By lemmas 3.1 and 3.2, $\mathrm{mCl}(A) \subset \mathrm{mCl}(V) = V$. Hence $\mathrm{mCl}(A) \subset U \cap V = A$. By Lemma 3.1, $\mathrm{mCl}(A) = A$ and by Lemma 3.2 $A$ is $m_X$-closed.

**Corollary 4.1.** *Let $(X, \tau)$ be a topological space and $A$ a subset of $X$. Then,*

*(1) $A$ is closed if and only if it is g-closed and a LC-set* [26],
*(2) $A$ is semi-closed if and only if it is gs-closed and a B-set,*
*(3) $A$ is preclosed if and only if it is gp-closed and A-7-set,*
*(4) $A$ is $\alpha$-closed if and only if it is $\alpha g$-closed and an $\eta$-set,*
*(5) $A$ is $\beta$-closed if and only if it is $g\beta$-closed and a BT-set,*
*(6) $A$ is b-closed if and only if it is gb-closed and a BC-set* [13].

**Proof**. This is an immediate consequence of Theorem 4.1.

## 5. A decomposition of $m$-closed functions

**Definition 5.1.** Let $(X, \tau)$ be a topological space and $(Y, m_Y)$ an $m$-space. A function $f : (X, \tau) \to (Y, m_Y)$ is said to be *m-closed* [25] if $f(F)$ is $m_Y$-closed in $(Y, m_Y)$ for each closed set $F$ of $(X, \tau)$.

**Remark 5.1.** Let $f : (X, \tau) \to (Y, \sigma)$ be a function and $m_Y = \sigma$ (resp. $\mathrm{SO}(Y)$, $\mathrm{PO}(Y)$, $\alpha(Y)$, $\beta(Y)$, $\mathrm{BO}(Y)$). If $f$ is $m$-closed, then $f$ is closed (resp. semi-closed, preclosed, $\alpha$-closed, $\beta$-closed, $b$-closed).

**Definition 5.2.** Let $(Y, \sigma)$ be a topological space and $m_Y$ an $m$-structure on $Y$. A function $f : (X, \tau) \to (Y, m_Y)$ is said to be *gm-closed* if $f(F)$ is *gm*-closed for each closed set $F$ of $(X, \tau)$.

**Remark 5.2.** Let $f : (X, \tau) \to (Y, \sigma)$ be a function and $m_Y = \sigma$ (resp. SO($Y$), PO($Y$), $\alpha(Y)$, $\beta(Y)$, BO($Y$)). If $f$ is $gm$-closed, then $f$ is $g$-closed (resp. $gs$-closed, $gp$-closed, $\alpha g$-closed, $g\beta$-closed, $gb$-closed).

**Definition 5.3.** Let $(Y, \sigma)$ be a topological space and $m_Y$ an $m$-structure on $Y$. A function $f : (X, \tau) \to (Y, m_Y)$ is said to be *mlc-closed* if $f(F)$ is an $mlc$-set for each closed set $F$ of $(X, \tau)$.

**Remark 5.3.** Let $f : (X, \tau) \to (Y, \sigma)$ be a function and $m_Y = \sigma$ (resp. SO($Y$), PO($Y$), $\alpha(Y)$, $\beta(Y)$, BO($Y$)). If $f$ is $mlc$-closed, then $f$ is $LC$-closed (resp. $B$-closed, $A$-7-closed, $\eta$-closed, $BT$-closed, $BC$-closed).

**Theorem 5.1.** *Let $(Y, \sigma)$ be a topological space and $m_Y$ an m-structure on $Y$ having property $\mathcal{B}$. A function $f : (X, \tau) \to (Y, m_Y)$ is m-closed if and only if $f$ is gm-closed and mlc-closed.*

**Proof**. This follows immediately from Theorem 4.1.

**Corollary 5.1.** *Let $f : (X, \tau) \to (Y, \sigma)$ be a function. Then, the following properties hold:*

*(1) $f$ is closed if and only if it is g-closed and LC-closed,*
*(2) $f$ is semi-closed if and only if it is gs-closed and B-closed,*
*(3) $f$ is preclosed if and only if it is gp-closed and A-7-closed,*
*(4) $f$ is $\alpha$-closed if and only if it is $\alpha g$-closed and $\eta$-closed,*
*(5) $f$ is b-closed if and only if it is gb-closed and BC-closed,*
*(6) $f$ is $\beta$-closed if and only if it is $g\beta$-closed and BT-closed.*

**Proof**. This is an immediate consequence of Corollary 4.1 and Theorem 5.1.

**Lemma 5.1.** *Let $(X, \tau)$ be a topological space and $m_X$ an m-structure on $X$ having property $\mathcal{B}$. If $A$ is gm-closed and open, then $A$ is $m_X$-closed.*

**Proof**. Let $A$ be $gm$-closed and open. Then, $\mathrm{mCl}(A) \subset A$ and by Lemma 3.1 $\mathrm{mCl}(A) = A$. Since $m_X$ has property $\mathcal{B}$, by Lemma 3.2 $A$ is $m_X$-closed.

**Definition 5.4.** A function $f : (X, \tau) \to (Y, \sigma)$ is said to be *contra-closed* [6] if $f(F)$ is open for every closed set $F$ of $(X, \tau)$.

**Theorem 5.2.** *Let $(Y, \sigma)$ be a topological space and $m_Y$ an m-structure on $Y$ having property $\mathcal{B}$. If a function $f : (X, \tau) \to (Y, \sigma)$ is gm-closed and contra-closed, then $f$ is m-closed.*

**Proof**. The proof follows immediately from Lemma 5.1.

**Corollary 5.2.** *Let $f : (X, \tau) \to (Y, \sigma)$ be a contra-closed function. If $f$ is g-closed (resp. gs-closed, gp-closed, $\alpha$g-closed gb-closed g$\beta$-closed), then $f$ is closed (resp. semi-closed, preclosed, $\alpha$-closed, b-closed, $\beta$-closed).*

**Proof**. Let $m_Y = \sigma$ (resp. $\mathrm{SO}(Y)$, $\mathrm{PO}(Y)$, $\alpha(Y)$, $\mathrm{BO}(Y)$, $\beta(Y)$), then the proof follows from Theorem 5.2.

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**Takashi Noiri**

2949-1 Shiokita-cho, Hinagu, Yatsushiro-shi, Kumamoto-ken, 869-5142 JAPAN, e-mail: t.noiri@nifty.com

**Valeriu Popa**

Department of Mathematics and Informatics, Faculty of Sciences, "Vasile Alecsandri" University of Bacău, Calea Mărăşeşti 157, Bacău 600115, ROMANIA, e-mail: vpopa@ub.ro