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# ON CERTAIN GENERALIZATION OF SUPERCONTINUITY/$\delta$-CONTINUITY

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**Abstract.** Two generalizations of supercontinuous functions (Indian J. Pure Appl. Maths. 13(1982), 229-236) and $\delta$-continuous functions (J. Korean Math. Soc. 16(1980), 161-166) are introduced. Several properties of these generalizations and their relationships with other variants of continuity in the literature are investigated. These new variants of supercontinuity / $\delta$-continuity also generalize certain forms of (almost) strong $\theta$-continuity (J. Korean Math. Soc. 18(1981), 21-28; Indian J. Pure Appl. Maths. 15(1) (1984), 1-8).

## 1. Introduction

Supercontinuous functions were introduced by Munshi and Bassan [30] and $\delta$-continuous functions were defined by Noiri [31]. In this paper we introduce two generalizations of supercontinuous/$\delta$-continuous functions called 'quasi supercontinuous' and 'pseudo supercontinuous' functions. The class of quasi (pseudo) supercontinuous functions besides strictly containing the classes of supercontinuous and $\delta$-continuous functions also properly contains each of the classes of (1) (almost) strongly $\theta$ - continuous functions and ([34] [27] [31])

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(2) quasi $\theta$-continuous functions [35] and so contains all the functions which lie above strong $\theta$-continuity in the hierarchy of variants of continuity (see Diagram 2). Organization of the paper is as follows: Section 2 is devoted to preliminaries and basic definitions. In Section 3 we define 'quasi supercontinuous' and 'pseudo supercontinuous' functions and reflect upon their interrelations and interconnections with other variants of continuity that already exist in the mathematical literature and are related to the theme of the present paper. Herein examples are included and observations are made to reflect upon the distinctiveness of the notions so introduced from the existing ones. In Section 4 we study the properties of 'quasi supercontinuous' functions and Section 5 is devoted to the study of 'pseudo supercontinuous' functions.

## 2. Preliminaries and basic definitions

A subset $A$ of a space $X$ is called a ***regular*** $G_\delta$***-set*** [29] if $A$ is an intersection of a sequence of closed sets whose interiors contain $A$, i.e., if $A = \bigcap_{n=1}^{\infty} F_n = \bigcap_{n=1}^{\infty} F_n^o$, where each $F_n$ is a closed subset of $X$ (here $F_n^o$ denote the interior of $F_n$). The complement of a regular $G_\delta$-set is called a ***regular*** $F_\sigma$***-set***. Any union of regular $F_\sigma$-set is called $d_\delta$***-open*** [15].The complement of a $d_\delta$-open set is referred to as a $d_\delta$***-closed*** set. A point $x \in X$ is called a $\theta$***-adherent point*** [47] of A $\subset$ X if every closed neigbourhood of x intersects $A$. Let $cl_\theta\ A$ denote the set of all $\theta$-adherent points of $A$. The set $A$ is called $\theta$***-closed*** [47] if $A = cl_\theta A$. The complement of a $\theta$-closed set is referred to as a $\theta$***-open set***. A subset $A$ of a space $X$ is said to be ***regular open*** if it is the interior of its closure, i.e., $A = \overline{A}^{\circ}$. The complement of a regular open set is referred to as a ***regular closed set***. A union of regular open sets is called $\delta$***-open*** [47]. The complement of a $\delta$-open set is referred to as a $\delta$***-closed set***.

Now we give the definitions of strong and weak variants of continuity related to our discussion in the paper.

2.1. **Definitions.** A function $f : X \to Y$ *from a topological space* $X$ *into a topological space* $Y$ *is said to be*
(a) ***supercontinuous*** [31] *if for each* $x \in X$ *and for each open set* $V$ *containing f(x), there exists a regular open set* $U$ *containing* $x$ *such that* $f(U) \subset V$.
(b) ***strongly*** $\theta$***-continuous*** ([27][31]) if for each $x \in X$ and for each open set V containing f(x), there exists an open set U containing x such that $f(\overline{U}) \subset V$.

(c) $D_\delta$***-continuous*** [16] (respectively ***z-continuous*** [41]) if for each point $x \in X$ and each regular $F_\sigma$-set (respectively cozero set) V containing f(x), there is an open set U containing x such that $f(U) \subset V$.
(d) ***almost continuous*** [42] (respectively ***faintly continuous*** [28]) if for each point $x \in X$ and each regular open set (respectively $\theta$-open set) V containing f(x), there is an open set U containing x such that $f(U) \subset V$.
(e) $\theta$***-continuous***[7] if for each $x \in X$ and each open set V containing f(x), there exists an open set U containing x such that $f(\overline{U}) \subset \overline{V}$.
(f) ***weakly continuous*** *[26] if for each $x \in X$ and each open set V containing f(x), there exists an open set U containing x such that $f(U) \subset \overline{V}$.*
(g)***quasi*** $\theta$***-continuous*** *[35] if for each $x \in X$ and each $\theta$-open set V containing f(x), there exists a $\theta$-open set U containing x such that $f(U) \subset V$.*
(h) ***slightly continuous***[1] *[11] if $f^{-1}(V)$ is open in X for every clopen set $V \subset Y$.*
(i) $d_\delta$***-map*** *[17] if for each regular $F_\sigma$-set U in Y, $f^{-1}(U)$ is a regular $F_\sigma$-set in X.*

In the following we give definitions of those variants of continuity which are independent of continuity and are related to the contents of the paper.

2.2. **Definitions.** *A function $f : X \to Y$ from a topological space X into a topological space Y is said to be*
*(a)* ***almost strongly*** $\theta$***-continuous*** *[34] if for each $x \in X$ and for each regular open set V containing f(x), there exists an open set U containing x such that $f(\overline{U}) \subset V$.*
*(b)* ***quasi perfectly continuous*** *[24] (**pseudo perfectly continuous** [22]) if $f^{-1}(V)$ is clopen in X for every $\theta$-open set (regular $F_\sigma$ -set ) V in Y.*
*(c)* ***quasi z-supercontinuous*** *[23](**quasi cl-supercontinuos** [13]) if for each $x \in X$ and each $\theta$-open set V containing f(x), there exists a cozero (clopen) set U containing x such that $f(U) \subset V$.*
*(d)* ***pseudo z-supercontinuous*** *[23] (**pseudo cl-supercontinuos** [21]) if for each $x \in X$ and each regular $F_\sigma$-set V containing f(x), there exists a cozero (clopen) set U containing x such that $f(U) \subset V$.*

[1]Slightly continuous functions have been referred to as cl-continuous in ([16]).

*(e) $\delta$-**continuous** [31] if for each $x \in X$ and for each regular open set V containing f(x), there exists a regular open set U containing x such that $f(U) \subset V$.*

2.3. **Definitions.** *A space $X$ is said to be*
*(i) $D_\delta$-**Hausdorff** [16] ($\theta$-**Hausdorff** [5] [44]) if every pair of distinct points in $X$ are contained in disjoint regular $F_\sigma$- sets ($\theta$-open sets).*
*(ii) **weakly Hausdorff** [45]if every singleton is the intersection of regular closed sets containing it.*
*(iii) $D_\delta T_0$-**space** [22] if for each pair of distinct points x, y in $X$, there is a regular $F_\sigma$-set U containing one of the points x and y but not the other.*
*(iii) $D_\delta$-**compact** [17] (respectively **nearly compact** [40], respectively $\theta$-**compact** [12] [10][2]) if every cover of $X$ by regular $F_\sigma$-sets (respectively regular open, respectively $\theta$-open) sets has a finite subcover.*
*(iv) $\delta$-**completely regular space** ([16]) (respectively **almost completely regular** ([39]), respectively $\theta$-**completely regular** [44]) if for each regular $G_\delta$- set (respectively regularly closed set, respectively $\theta$-closed set) F and a point $x \in F$, there exists a continuous function $f : X \to [0, 1]$ such that f (x) = 0 and f (F) = 1.*
*(vi) $D_\delta$-**completely regular** [16] if it has a base of regular $F_\sigma$-sets.*
*(vii) **almost regular** [38] if for each regular closed set $A$ and each point $x \notin A$, there exist disjoint open sets $U$ and $V$ such that $x \in U$, $A \subset V$. (viii) **extremally disconnected** [8] if the closure of every open set is open in X.*
*(ix) **almost zero dimensional** [18] at $x \in X$ if for every regular open set V containing x there exists a clopen set U containing x such that $U \subset V$. The space X is said to be almost zero dimensional if it is almost zero dimensional at each $x \in X$.*

2.4. **Definition.** *A space X is said to be endowed with a $\delta$-**partition** topology [19] if every $\delta$-open set in X is closed.*

2.5. **Definition.** *A subset S of a space X is said to be **regular** $G_\delta$-**embedded** [2] (respectively $\theta$-**embedded** [12], respectively $\delta$-**embedded** [18]) in X if every regular $G_\delta$-set (respectively $\theta$-closed set, respectively regular closed set) in S is the intersection of a regular $G_\delta$-set ( respectively $\theta$-closed set, respectively regular closed set) in X*

[2] $\theta$-sets have been called $\theta$-compact by Jafari [10]. For an example of a $\theta$-set which is not $\theta$-compact see [12,Remark 2.2]

*with S; or equivalently every regular $F_\sigma$-set (respectively $\theta$-open set, respectively regular open) in S is the intersection of a regular $F_\sigma$-set (respectively $\theta$-open set, respectively regular open set) in X with S.*

2.6. **Definition.** *filter $\mathcal{F}$ is said to **$\theta$-converge** [47] (respectively **$d_\delta$-converge** [15], respectively **$\delta$-converge** [47]) to a point x, written as $\mathcal{F} \xrightarrow{\theta} x$ (respectively $\mathcal{F} \xrightarrow{d_\delta} x$, respectively$\mathcal{F} \xrightarrow{\delta} x$) if any closed neighbourhood (respectively regular $F_\sigma$-set, respectively regular open set) of $x$ contains a member of $\mathcal{F}$.*

2.7. **Definition.** *A net $(x_\alpha)$ in a topological space is said to **$\theta$-converge** [47] to x, written as $(x_\alpha) \xrightarrow{\theta} x$, if for each open set $V$ containing $x$ it is eventually in $\overline{V}$.*

2.8. **Definition.** *A net $(x_\alpha)$ in a topological space is said to **$d_\delta$-converge** [15] (**$\delta$-converge** [19]) to x, written as$(x_\alpha) \xrightarrow{d_\delta} x$ ($(x_\alpha) \xrightarrow{\delta} x$),if for each regular $F_\sigma$-set (regular open set) $V$ containing $x$ the net $(x_\alpha)$ is eventually in $V$.*

2.9. **Change of Topology.** Let (X,$\tau$) be a topological space.
i) Let $B_{d_\delta}$ denote the collection of all regular $F_\sigma$-sets in $X$. Since the intersection of two regular $F_\sigma$-sets is a regular $F_\sigma$-set, the collection $B_{d_\delta}$ is a base for a topology $\tau_{d_\delta}$ on $X$ such that $\tau_{d_\delta} \subset \tau$. The topology $\tau_{d_\delta}$ has been used in ([15] [16]).
ii) Let $B_\theta$ denote the collection of $\theta$-open sets of the space (X,$\tau$). Since arbitrary unions and finite intersections of $\theta$ -open sets are $\theta$-open, the collection $B_\theta$ is indeed a topology on $X$. We shall denote this topology by $\tau_\theta$. The topology $\tau_\theta$ has been extensively referred to in the literature (see [28] [47]).
iii) Let $B_\delta$ denote the collection of all regular open subsets of the space (X,$\tau$). Since the intersection of two regular open sets is regular open, the collection $B_\delta$ constitutes a base for a topology $\tau_\delta$ on $X$ and is called the semiregular topology associated with $\tau$. The space (X,$\tau_\delta$) is often called the semiregularization of the space and has been extensively used in the topological literature (see [3, Exercise 20, p.138]).

## 3. Quasi supercontinuous and Pseudo supercontinuous functions

We call a function $f : X \to Y$ from a topological space $X$ into a topological space $Y$ **quasi (pseudo) supercontinuous** at $x \in X$ if for each $\theta$-open set (regular $F_\sigma$-set) $V$ containing $f(x)$ there exists

an open set $U$ containing x such that $f(\overline{U}^0) \subset V$. The function $f$ is quasi (pseudo) supercontinuous if $f$ is quasi (pseudo) supercontinuous at each x $\in X$.
The following diagram well reflects upon the interrelations and interconnections that exist between quasi (pseudo) supercontinuous functions and other variants of continuity that already exist in the literature and are related to the contents of the present paper. The implications in the following diagram are either immediate from definitions or easily verified.

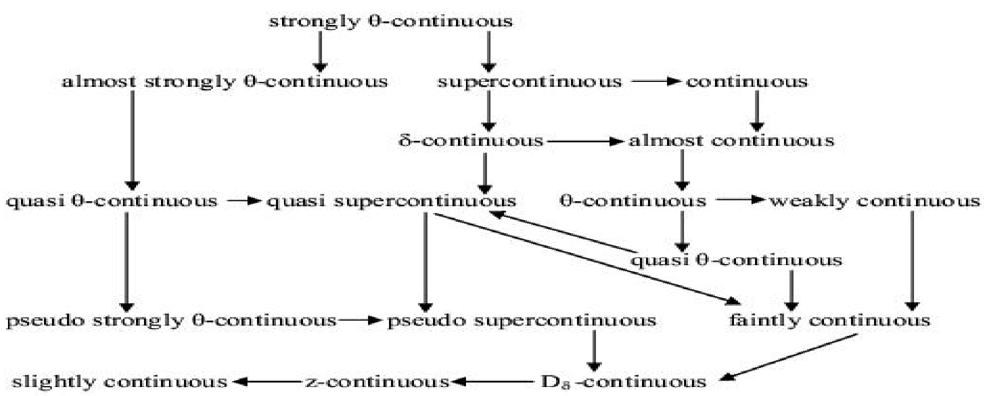


Figure 1.

However, none of the above implications is reversible as is shown by the following examples or examples/observations in ([16] [21] [30] [31]).
3.1 **Example**:Let $X$={a,b,c,d} with topologies $\tau_1$={$\emptyset$, X} and $\tau_2$={$\emptyset$, $X$,{a},{b,c},{a,b,c}}. Let $f : X \to Y$ be the identity function. Then f is quasi supercontinuous but not $\delta$-continuous.
3.2 **Example**: Let $X = \{a, b, c, d\}$ with topology $\tau$={$\emptyset$, $X$, {a}, {b,c}, {a,b,c}} and let $Y$ be the skyline space due to Heldermann [9] which is a $T_1$-regular space. Let $f : X \to Y$ be defined as f(a) = f(b) = f(c) = $p^-$, f(d) = $p^+$. Then $f$ is pseudo supercontinuous, since $Y$ is the only regular $F_\sigma$-set containing both $p^-$ and $p^+$ but it is not quasi supercontinuous as $V(c) = \{(x, y) : c < x\} \cup \{p^+\}$ is a $\theta$-open set containing $p^+$ and its inverse image is not even open.
3.3 **Example**: Let $\mathbb{N}^*$ be the set of positive integers. Define a topology $\tau$ on $\mathbb{N}^*$ by taking every singleton consisting of an odd integer to be open and a set U$\subset \mathbb{N}^*$ is open if for every even integer p $\in U$, the predecessor and successor of p are also in U. Let $Y$ denote the one point compactification of the space ($\mathbb{N}^*$, $\tau$). Let $X$

denote the same set $Y$ endowed with the topology $\tau_1 = \{\emptyset, \mathbb{N}^*, X\}$. Then identity map $f : X \to Y$ is faintly continuous but not pseudo supercontinuous.

3.4 **Example**: Let $Y = \mathbb{N}^* \cup \{\infty\}$ be the one point compactification of the space $\mathbb{N}^*$, where $\mathbb{N}^*$ is endowed with the topology defined in Example 3.3 and let $X = \{a, b, c, d\}$ be equipped with the topology $\tau = \{\emptyset, X, \{a\}, \{b,c\}, \{a,b,c\}\}$. Then $f : X \to Y$ defined as f(a) = f(b) = f(c) = 2, f(d) = $\infty$ is quasi supercontinuous but not quasi $\theta$-continuous.

3.5 **Proposition**: *If $f : X \to Y$ is quasi supercontinuous and $X$ is an almost regular space, then f is quasi $\theta$-continuous.* **Proof**: In an almost regular space every $\delta$-open set is $\theta$-open. 3.6 **Proposition**: *If $f : X \to Y$ is quasi supercontinuous and Y is an almost regular space, then f is $\delta$-continuous.*

3.7 **Proposition**: *Let $f : X \to Y$ be a quasi (pseudo) supercontinuous function. The following statements are true. (a) If X is endowed with a $\delta$-partition topology, then f is quasi (pseudo) perfectly continuous.*

*(b) If X is an almost zero dimensional space, then f is quasi (pseudo) cl-supercontinuous.*

*(c) If X is an extremally disconnected space, then f is quasi (pseudo) cl-supercontinuous.*

3.8 **Proposition**: *For a semiregular space X the following statements are true.*

(a) *If $f : X \to Y$ is faintly continuous, then f is quasi supercontinuous.*

(b) *If $f : X \to Y$ is $D_\delta$-continuous, then f is pseudo supercontinuous.*

The following diagram well exhibits the classes of functions which are properly contained in the classes of quasi (pseudo) supercontinuous functions.

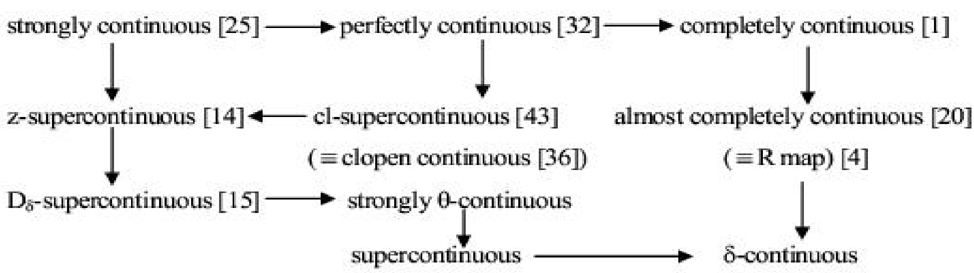


Figure 2.

## 4. Basic properties of quasi supercontinuous functions

4.1 **Theorem**: *For a function $f : (X, \tau) \to (Y, \upsilon)$ the following statements are equivalent.*
(a) *f is quasi supercontinuous*
*(b) for each $\theta$-open set V containing f(x) there exists a regular open set U containing x such that $f(U) \subset V$.*
*(c) $f^{-1}(V)$ is $\delta$-open in X for every $\theta$-open set $V \subset Y$*
*(d) $f^{-1}(V)$ is $\delta$-closed in X for every $\theta$-closed set $V \subset Y$*
*(e) The function $f : (X, \tau_\delta) \to (Y, \upsilon_\theta)$ is continuous*
*(f) The function $f : (X, \tau) \to (Y, \upsilon_\theta)$ is supercontinuous*
*(g) The function $f : (X, \tau_\delta) \to (Y, \upsilon)$ is faintly continuous*
*(h) For every net $(x_\alpha)$ in X with $x_\alpha \xrightarrow{\delta} x, f(x_\alpha) \xrightarrow{\theta} f(x)$*
*(i) For every filter $\mathcal{F}$ with $\mathcal{F} \xrightarrow{\delta} x, f(\mathcal{F}) \xrightarrow{\theta} f(x)$.*
4.2 **Theorem**: *If $f : X \to Y$ is a quasi supercontinuous function and $g : Y \to Z$ is quasi $\theta$-continuous, then gof is quasi supercontinuous.*
**Proof**: Let $W$ be a $\theta$-open set in $Z$. Then in view of quasi $\theta$-continuity of the function $g$, $g^{-1}(W)$ is a $\theta$-open in $Y$ and so $f^{-1}(g^{-1}(W))$ is $\delta$-open in $X$. Since $f^{-1}(g^{-1}(W)) = (gof)^{-1}(W)$, $gof$ is quasi supercontinuous.
4.3 **Corollary**: *If $f : X \to Y$ is a quasi supercontinuous function, $g : Y \to Z$ is either (1) continuous or (2) almost continuous or (3) $\theta$-continuous, then the composition gof is quasi supercontinuous.*
4.4 **Theorem**: *If $f : X \to Y$ is a quasi supercontinuous function and A is a $\delta$-open subset of X, then the restriction $f|_A : A \to Y$ is quasi supercontinuous. In addition, if f (A) is $\theta$-embedded in Y, then $f|_A : A \to f(A)$ is quasi supercontinuous.*
The following result gives a sufficient condition for the preservation of quasi supercontinuity under the shrinking of range.
4.5 **Theorem**: *If $f : X \to Y$ is a quasi supercontinuous function and f (X) is $\theta$-embedded in Y, then $f : X \to f(X)$ is quasi supercontinuous.*
In contrast, quasi supercontinuity is preserved under the expansion of range as shown by the following result.
4.6 **Theorem**: *If $f : X \to Y$ is a quasi supercontinuous function and Y is a subspace of Z, then $g : X \to Z$ defined by g (x) = f (x) for all $x \in X$ is quasi supercontinuous.*
4.7 **Theorem**: *Let $f : X \to Y$ be a surjection which maps $\delta$-open sets in X to $\delta$-open sets in Y and $g : Y \to Z$ is any function. If gof is quasi supercontinuous, then g is quasi supercontinuous.*
**Proof**: Let $W$ be a $\theta$-open set in $Z$. Then $(gof)^{-1}(W) =$

$f^{-1}(g^{-1}(W))$ is a $\delta$-open set in $X$. In view of the hypothesis on $f$, it follows that $f(f^{-1}(g^{-1}(W))) = g^{-1}(W)$ is a $\delta$-open set in $Y$ and so $g$ is quasi supercontinuous.

4.8 **Theorem**: *Let $f : X \to Y$ be a function. The following statements are true. (a) Let $U_\alpha : \alpha \in \Lambda$ be a $\delta$-open cover of $X$ such that each $U_\alpha$ is $\delta$-embedded in X. If for each $\alpha$, $f_\alpha = f|_{U_\alpha}$ is quasi supercontinuous, then f is quasi supercontinuous.*

*(b) Let $\{F_i : i = 1, ..., n\}$ be a cover of $X$ by $\delta$-closed sets such that each $F_i$ is $\delta$-embedded in X. If for each i = 1,...,n, $f_i = f|_{F_i}$ is quasi supercontinuous, then f is quasi supercontinuous.*

**Proof**: (a) Let $V$ be a $\theta$-open set of $Y$. Then $f^{-1}(V) = \cup\{f_\alpha^{-1}(V) : \alpha \in \lambda\}$ and in view of quasi supercontinuity of $f_\alpha$, each $f_\alpha^{-1}(V)$ is $\delta$-open in $U_\alpha$. Suppose that $f_\alpha^{-1}(V) = \cup_\beta V_{\alpha\beta}$, where each $V_{\alpha\beta}$ is a regular open set in $U_\alpha$. Since $U_\alpha$ is $\delta$-embedded in $X$, there exists a regular open set $V^*_{\alpha\beta}$ in $X$ such that $V_{\alpha\beta} = V^*_{\alpha\beta} \cap U_\alpha$. Now, $f_\alpha^{-1}(V) = \cup_\beta V_{\alpha\beta} = \cup_\beta(V^*_{\alpha\beta} \cap U_\alpha) = (\cup V^*_{\alpha\beta}) \cap U\alpha$. Since arbitrary unions and finite intersections of $\delta$-open sets is $\delta$-open, $f_\alpha^{-1}(V)$ is $\delta$-open set in $X$ and hence so is $f^{-1}(V)$.

(b) Let $F$ be any $\theta$-closed subset of $Y$. Then $f^{-1}(F) = \cup_{i=1}^n f_i^{-1}(F)$. Since each $f_i$ (i = 1,...,n), is quasi supercontinuous, each $f_i^{-1}(F)$ is a $\delta$-closed set in $F_i$. Again, since each $F_i$ is $\delta$-embedded in $X$, it is easily shown that each $f_i^{-1}(F)$ is a $\delta$-closed set in $X$. Now, since every finite union of $\delta$-closed sets is $\delta$-closed, $f^{-1}(F)$ is $\delta$-closed in $X$ and so $f$ is quasi supercontinuous.

4.9 **Lemma**: *Let $\{X_\alpha : \alpha \in \Lambda\}$ be a family of spaces and let $X = \prod_{\alpha\in\Lambda} X_\alpha$ be the product space. If $x = (x_\alpha) \in X$ and $V$ is a $\theta$-open set containing $x$, then there exists a basic $\theta$-open set $\prod_{\alpha\in\Lambda} V_\alpha$ such that $x \in \prod_{\alpha\in\Lambda} V_\alpha \subset V$, where $V_\alpha$ is a $\theta$-open set in $X_\alpha$ for each $\alpha \in \Lambda$ and $V_\alpha = X_\alpha$ for all except finitely many $\alpha_1, ..., \alpha_n \in \Lambda$.*

4.10 **Theorem**: *Let $\{f_\alpha : X \to X_\alpha : \alpha \in \Lambda\}$ be a family of functions and let $f : X \to \prod_{\alpha\in\Lambda} X_\alpha$ be defined by $f(x) = (f_\alpha(x))$ for each $x \in X$. Then f is quasi supercontinuous if and only if each $f_\alpha$ is quasi supercontinuous.*

**Proof**: Suppose $f$ is quasi supercontinuous. Then for each $\alpha$, $f_\alpha = \pi_\alpha o f$, where $\pi_\alpha$ denotes the projection map $\pi_\alpha : \prod_{\alpha\in\Lambda} X \to X_\alpha$. So in view of Corollary 4.3 each $f_\alpha$ is quasi supercontinuous.

Conversely, suppose that each $f_\alpha : X \to X_\alpha$ is quasi supercontinuous. To prove that $f$ is quasi supercontinuous, it suffices to prove that $f^{-1}(U)$ is $\delta$-open for each $\theta$-open set $U$ in the product space $\prod_{\alpha\in\Lambda} X_\alpha$. Since arbitrary unions and finite intersections of $\delta$-open sets is $\delta$-open, in view of Lemma 4.9 it is sufficient to prove that $f^{-1}(S)$ is $\delta$-open for every subbasic $\theta$-open set S in the product space $\prod_{\alpha\in\Lambda} X_\alpha$. Let $U_\beta \times \prod_{\alpha\in\Lambda} X_\alpha$ be a subbasic $\theta$-open set in $\prod_{\alpha\in\Lambda} X_\alpha$.Then $f^{-1}(U_\beta \times \prod_{\alpha\neq\beta} X_\alpha) = f^{-1}(\pi_\beta^{-1}(U_\beta)) = f_\beta^{-1}(U_\beta)$ is $\delta$-open in $X$ and so $f$ is quasi supercontinuous.

4.11 **Theorem**: *Let $\{f_\alpha : X_\alpha \to Y_\alpha : \alpha \in \Lambda\}$ be a family of functions and let $f : \prod_{\alpha\in\Lambda} X_\alpha \to \prod_{\alpha\in\Lambda} Y_\alpha$ be defined by $f((x_\alpha)) = (f_\alpha(x_\alpha))$ for each $(x_\alpha) \in \prod_{\alpha\in\Lambda} X_\alpha$. Then f is quasi supercontinuous if and only if each $f_\alpha$ is quasi supercontinuous.*

**Proof**: Suppose that $f$ is quasi supercontinuous and let $V_\beta$ be a $\theta$-open set in $Y_\beta$. Then $V_\beta \times (\prod_{\alpha\neq\beta} Y_\alpha)$ is a $\theta$-open set in the product space $\prod_{\alpha\in\Lambda} Y_\alpha$. In view of quasi supercontinuity of $f$, $f^{-1}(V_\beta \times (\prod_{\alpha\neq\beta} Y_\alpha)) = f^{-1}(V_\beta) \times (\prod_{\alpha\neq\beta} X_\alpha)$ is a $\delta$-open set in $\prod X_\alpha$. Consequently, $f^{-1}(V_\beta)$ is a $\delta$-open set in $X_\beta$ and hence $f_\beta$ is quasi supercontinuous for each $\beta \in \Delta$.

Conversely, suppose that each $f_\alpha$ is quasi supercontinuous. To show that $f$ is quasi supercontinuous it suffices to prove that $f^{-1}(V)$ is $\delta$-open for each $\theta$-open set $V$ in the product space $\prod_{\alpha\in\Lambda} Y_\alpha$. In view of Lemma 4.9, $V$ is expressible as a union of basic $\theta$-open sets of the form $\prod_{\alpha\in\Lambda} V_\alpha$, where each $V_\alpha$ is a $\theta$-open set in $X_\alpha$ and $V_\alpha = X_\alpha$ for all but finitely many $\alpha_1, ..., \alpha_n \in \Delta$. Thus $f^{-1}(V) = f^{-1}(\cup \prod_{\alpha\in\Lambda} V_\alpha) = \cup f^{-1}(\prod_{\alpha\in\Lambda} V_\alpha) = \cup(\cap_{i=1}^n f_{\alpha_i}^{-1}(V_{\alpha_i}))$. Since each $f_\alpha$ is quasi supercontinuous and since finite intersections of $\delta$-open sets is $\delta$-open, each $\cap_{i=1}^n f_{\alpha_i}^{-1}(V_{\alpha_i})$ is $\delta$-open and so $f^{-1}(V)$ being a union of $\delta$-open sets is $\delta$-open.

4.12 **Corollary**: *Let $f : X \to Y$ be a function and $g : X \to X \times Y$, defined by $g(x) = (x, f(x))$ for each $x \in X$, be the graph function. Then g is quasi supercontinuous if and only if f is quasi supercontinuous.*

**Proof**: Let $f_1 = 1_X$ be the identity map defined on $X$ and let $f_2 = f$. Then $g(x) = (f_1(x), f_2(x))$.

4.13 **Theorem**: *Let $f, g : X \to Y$ be quasi supercontinuous functions from a space X into a $\theta$-Hausdorff space Y. Then the equalizer $E = \{x : f(x) = g(x)\}$ of f and g is $\delta$-closed in X.*

Proof: Let $x \in (X - E)$. Then $f(x) \neq g(x)$, and so by hypothesis on $Y$, there are disjoint $\theta$-open sets $U$ and $V$ containing $f(x)$ and $g(x)$, respectively. Since $f$ and $g$ are quasi supercontinuous, the sets $f^{-1}(U)$ and $g^{-1}(V)$ are $\delta$-open and contain the point x. Let $G = f^{-1}(U) \cap g^{-1}(V)$. Then $G$ is a $\delta$-open set containing x and $G \cap E = \emptyset$. Thus $E$ is $\delta$-closed in $X$.

4.14 **Corollary**: *Let X be a $\theta$-Hausdorff space and let $f : X \to X$ be a quasi supercontinuous function. Then the set of fixed points of f is $\delta$-closed in X.*

4.15 **Theorem**: *Let $f : X \to Y$ be a quasi supercontinuous function. If X is almost completely regular, then f is quasi z-supercontinuous.*

**Proof**: Let $x \in X$ and let $V$ be a $\theta$-open set containing f(x). Since $f$ is quasi supercontinuous, there exists a regular open set $U$ containing x such that $f(U) \subset V$. In view of almost complete regularity of $X$ there exists a continuous function $h : X \to [0, 1]$ such that h(x) = 0 and h(X-U) = 1. Then $W = h^{-1}[0, 1)$ is a cozero set containing x and contained in $U$ and Thus $f$ is quasi z-supercontinuous.

4.16 **Definition**: *Let $f : X \to Y$ be a function from a topological space X into a topological space Y. The graph $G(f)$ of a f is said to be* $\delta\theta$***-closed*** *with respect to $X \times Y$ if for each $(x, y) \notin G(f)$ there exists a regular open set U containing x and a $\theta$ -open set V containing y such that $(U \times V) \cap G(f) = \emptyset$.*

4.17 **Theorem**: *Let $f : X \to Y$ be a quasi supercontinuous function from a space X into a $\theta$-Hausdorff space Y. Then G(f), the graph of f is $\delta\theta$-closed with respect to $X \times Y$.*

**Proof**: Let $(x, y) \notin G(f)$. Then $y \neq f(x)$. Since $Y$ is $\theta$-Hausdorff, there exist disjoint $\theta$-open sets $V$ and $W$ containing f(x) and y, respectively. In view of quasi supercontinuity of $f$, there exists a regular open set $U$ containing x such that $f(U) \subset V \subset (Y - \overline{W})$. Consequently, $(U \times W) \cap G(f) = \emptyset$ and so $G(f)$ is $\delta\theta$-closed with respect to $X \times Y$.

4.18 **Theorem**: *Let $f : X \to Y$ be a quasi supercontinuous injection which maps open sets in X to $\theta$-open sets in Y. Then X is a semi regular space. Further, if X is almost regular, then X is a regular space.*

**Proof**: To prove that $X$ is a semiregular space it suffices to prove that every open set in $X$ is $\delta$-open. To this end, let $U$ be an open set in $X$ containing x. Then $f(U)$ is a $\theta$-open set containing $f(x)$. By quasi supercontinuity of $f$ there exists a regular open set $U_x$ containing x such that $f(U_x) \subset f(U)$. Since $f$ is an injection, $U_x \subset f^{-1}(f(U_x)) \subset f^{-1}(f(U)) \subset U$. Thus $U$ being a union of regular open sets is $\delta$-open and hence $X$ is a semiregular space. The last assertion is immediate, since a semiregular space is regular if and only if it is almost regular [38].

4.19 **Theorem**: *Let $f : X \to Y$ be a quasi supercontinuous open closed surjection. If X is an almost completely regular space, then Y is*

*$\theta$-completely regular. Moreover, if $Y$ is regular, then $Y$ is completely regular.*

**Proof**: Let $K \subset Y$ be a $\theta$-closed set and let $z \notin K$. Since $f$ is quasi supercontinuous, $f^{-1}(K)$ is $\delta$-closed. Let $x_0 \in f^{-1}(z)$. Then $x_0 \notin f^{-1}(K)$ and so there exists a regular closed set $B$ containing $f^{-1}(K)$ and $x_0 \notin B$. Since $X$ is almost completely regular, there exists a continuous function $\phi : X \to [0,1]$ such that $\phi(x_0) = 0$ and $\phi(B) = 1$. Define $\hat{\varphi} : Y \to [0,1]$ by taking $\hat{\varphi}(y) = sup\{\phi(x) : x \in f^{-1}(y)\}$ for each $y \in Y$. Then $\hat{\varphi}(z) = 0$, $\hat{\varphi}(K) = 1$ and by [6, Exercise 16] $\hat{\varphi}$ is continuous. Hence $Y$ is a $\theta$-completely regular space. The last assertion is immediate in view of the fact that every regular, $\theta$-completely regular space is completely regular.

4.20 **Theorem**: *Let $f : X \to Y$ be a quasi supercontinuous surjection from a nearly compact space $X$ onto a space $Y$. Then $Y$ is $\theta$ -compact.*

**Proof**: Let $\Omega = \{U_\alpha : \alpha \in \Lambda\}$ be a cover of $Y$ by $\theta$-open sets. Since $f$ is quasi supercontinuous, the collection $\beta = \{f^{-1}(U_\alpha) : \alpha \in \Lambda\}$ is a $\delta$-open cover of $X$. Since $X$ is nearly compact, let $\{f^{-1}(U_{\alpha_1}), ..., f^{-1}(U_{\alpha_n})\}$ be a finite subcollection of $\beta$ which covers $X$. Then $\{U_{\alpha_1}, ..., U_{\alpha_n}\}$ is a finite subcollection of $\Omega$ which covers $Y$. Hence $Y$ is $\theta$-compact.

4.21 **Theorem**: *Let $f : X \to Y$ be a quasi supercontinuous injection. The following statements are true.*
*(a) If $Y$ is Hausdorff, then $X$ is weakly Hausdorff.*
*(b) If $Y$ is $\theta$-Hausdorff, then $X$ is Hausdorff.*

**Proof**: (a). Let $x \in X$. Since $Y$ is Hausdorff and since every compact set in a Hausdorff space is $\theta$-closed, the singleton $\{f(x)\}$ is a $\theta$-closed subset of $Y$ and so $\{x\} = f^{-1}(f(x))$ is $\delta$-closed in $X$. Thus $X$ is weakly Hausdorff.

(b). Let $x_1, x_2 \in X$, $x_1 \neq x_2$. Then Since $Y$ is $\theta$-Hausdorff, there exist disjoint $\theta$-open sets $V_1$ and $V_2$ containing $f(x_1)$ and $f(x_2)$ respectively. Since $f$ is quasi supercontinuous $f^{-1}(V_1)$ and $f^{-1}(V_2)$ are disjoint $\delta$-open sets containing $x_1$ and $x_2$, respectively and so $X$ is Hausdorff.

4.22 **Definitions**: *A space $X$ is said to be **nearly paracompact** [37] (respectively $\theta$-**paracompact**, respectively $D_\delta$-**paracompact)** if every cover by regular open sets (respectively $\theta$-open sets, respectively regular $F_\sigma$-sets) has a locally finite open refinement.*

Following implications are immediate from definitions.

paracompact $\longrightarrow$ nearly paracompact $\longrightarrow$ $\theta$-paracompact $\longrightarrow$ $D_\delta$-paracompact

4.23 **Theorem**: *Let $f : X \to Y$ be a closed, quasi supercontinuous almost open surjection such that $f^{-1}(y)$ is compact for each $y \in Y$. If $X$ is a nearly paracompact space, then $Y$ is a $\theta$-paracompact space. Moreover, if $Y$ is regular, then $Y$ is paracompact.*
**Proof**: Let $B = \{U_\alpha : \alpha \in \Lambda\}$ be a $\theta$-open cover of $Y$. Since $f$ is quasi supercontinuous, $\Sigma = \{f^{-1}(U_\alpha) : \alpha \in \Lambda\}$ is a $\delta$-open cover of $X$. Let $E = \{V_\beta : \beta \in \Gamma\}$ be the natural regular open refinement of $X$. Since $X$ is nearly paracompact, there exists a locally finite open refinement $\omega = \{W_\delta : \delta \in \Omega\}$ of $E$ which covers $X$. Since each $V_\beta$ is regular open, it is easily verified that each $W_\delta$ may be chosen to be regular open. Since $f$ is almost open, $A = \{f(W_\delta) : \delta \in \Omega\}$is an open refinement of $B$ which covers $Y$. Again, since $f$ is a closed function such that $f^{-1}(y)$ is compact for each $y \in Y$,A $= \{f(W_\delta) : \delta \in \Omega\}$ is a locally finite open cover of $Y$. To complete the proof it suffices to show that the collection $A$ is locally finite. To this end, let $y \in Y$. For each $x \in f^{-1}(y)$, there exists an open set $U_x$ containing x which intersects at most finitely many members of $\omega$. Then $H = \{U_x : x \in f^{-1}(y)\}$ is an open cover of the compact set $f^{-1}(y)$ and so there exists a finite subcollection $\{U_{x_1}, ..., U_{x_n}\}$ which covers $f^{-1}(y)$. Let U $= \cup_{i=1}^{n} U_{x_i}$. Then $U$ is an open set containing $f^{-1}(y)$ and intersects at most finitely many members of $\omega$. Since $f$ is a closed function, $Y - f(X - U)$ is an open set containing y which intersects at most finitely many members of $A$ and so $A$ is locally finite. Thus $Y$ is $\theta$-paracompact. The last assertion is immediate in view of the fact that a regular $\theta$-paracompact space is paracompact.

## 5. Properties and characterizations of pseudo supercontinuous functions

5.1 **Theorem**: *Let be a function from a topological space $X$ into a topological space $Y$. The following statements are equivalent.*
*(a) The function f is pseudo supercontinuous.*
*(b) for each regular $F_\sigma$-set $V$ containing f(x) there exists a regular open set $U$ containing $x$ such that $f(U) \subset V$.*
*(c) $f^{-1}(V)$ is $\delta$-open in $X$ for every regular $F_\sigma$-set $V \subset Y$.*
*(d) $f^{-1}(V)$ is $\delta$-open in $X$ for every $d_\delta$-open set $V \subset Y$.*
*(e) $f^{-1}(B)$ is $\delta$-closed in $X$ for every regular $G_\delta$-set $B \subset Y$.*
*(f) $f^{-1}(B)$ is $\delta$-closed in $X$ for every $d_\delta$-closed set $B \subset Y$.*

*(g) The function* $f : (X, \tau_\delta) \to (Y, \vartheta_{d_\delta})$ *is continuous.*
*(h) The function* $f : (X, \tau) \to (Y, \vartheta_{d_\delta})$ *is supercontinuous.*
*(i) The function* $f : (X, \tau_\delta) \to (Y, \vartheta)$ *is* $D_\delta$*-continuous.*
*(j) For every net* $(x_\alpha)$ *in* $X$ *with* $x_\alpha \xrightarrow{\delta} x, f(x_\alpha) \xrightarrow{d_\delta} f(x)$.
*(k) For every filter* $\mathcal{F}$ *with* $\mathcal{F} \xrightarrow{\delta} x, f(\mathcal{F}) \xrightarrow{d_\delta} f(x)$.

5.2 **Theorem**: *If* $f : X \to Y$ *is a quasi supercontinuous function and* $g : Y \to Z$ *is* $d_\delta$*-map, then* $gof$ *is a pseudo supercontinuous.*

5.3 **Corollary**: *If* $f : X \to Y$ *is pseudo supercontinuous and* $g : Y \to Z$ *is continuous, then the composition* $gof$ *is pseudo supercontinuous.*

5.4 **Theorem**: *If* $f : X \to Y$ *is pseudo supercontinuous and* $A$ *is a* $\delta$*-open subset of X, then the restriction* $f|_A : A \to Y$ *is pseudo supercontinuous, further, if f(A) is regular* $G_\delta$*-embedded in Y, then* $f|_A : A \to f(A)$ *is pseudo supercontinuous.*

5.5 **Theorem**: *If* $f : X \to Y$ *is pseudo supercontinuous and Y is a subspace of Z, then* $g : X \to Z$ *defined by* $g(x) = f(x)$ *for all* $x \in X$ *is pseudo supercontinuous.*

5.6 **Theorem**: *If* $f : X \to Y$ *is pseudo supercontinuous and f (X) is regular* $G_\delta$*-embedded in Y, then* $f : X \to f(X)$ *is pseudo supercontinuous.*

5.7 **Theorem**: *Let* $f : X \to Y$ *be a surjection which maps* $\delta$*-open sets in* $X$ *to* $\delta$*-open sets in* $Y$ *and* $g : Y \to Z$ *is any function. If* $gof$ *is pseudo supercontinuous, then g is pseudo supercontinuous.*

5.8 **Theorem**: *Let* $f : X \to Y$ *be a function. Then the following statements are true.*
*(a) Let* $\{U_\alpha : \alpha \in \Lambda\}$ *be a* $\delta$*-open cover of* $X$ *such that each* $U_\alpha$ *is* $\delta$*-embedded in X. If for each* $\alpha$, $f_\alpha = f|_{U\alpha}$ *is pseudo supercontinuous, then f is pseudo supercontinuous.*
*(b) Let* $\{F_i : i = 1, ..., n\}$ *be a cover of* $X$ *by* $\delta$*-closed sets such that each* $F_i$ *is* $\delta$*-embedded in X. If for each* $i = 1, ..., n$, $f_i = f|_{F_i}$ *is pseudo supercontinuous, then f is pseudo supercontinuous.*

5.9 **Lemma**: *Let* $\{X_\alpha : \alpha \in \Lambda\}$ *be a family of spaces and let* $X = \prod_{\alpha\in\Lambda} X_\alpha$ *be the product space. If* $x = (x_\alpha) \in X$ *and* $V$ *is a* $d_\delta$*-open set containing x, then there exists a basic regular* $F_\sigma$*- set* $\prod_{\alpha\in\Lambda} V_\alpha$ *such that* $x \in \prod_{\alpha\in\Lambda} V_\alpha \subset V$, *where* $V_\alpha$ *is a regular* $F_\sigma$*- set in* $X_\alpha$ *for each* $\alpha \in \Lambda$ *and* $V_\alpha = X_\alpha$ *for all except finitely many* $\alpha_1, ..., \alpha_n \in \Lambda$.

5.10 **Theorem**: *Let* $\{f_\alpha : X \to X_\alpha : \alpha \in \Lambda\}$ *be a family of functions and let* $f : X \to \prod_{\alpha\in\Lambda} X_\alpha$ *be defined by* $f(x) = (f_\alpha(x))$ *for each* $x \in X$. *Then f is pseudo supercontinuous if and only if each* $f_\alpha$ *is pseudo supercontinuous.*

Proof is similar to that of Theorem 4.10 and makes use of Lemma 5.9 instead of Lemma 4.9.

5.11 **Theorem**: *Let* $\{f_\alpha : X_\alpha \to Y_\alpha : \alpha \in \Lambda\}$ *be a family of functions and let* $f : \prod_{\alpha\in\Lambda} X_\alpha \to \prod_{\alpha\in\Lambda} Y_\alpha$ *be defined by* $f((x_\alpha)) = (f_\alpha(x_\alpha))$ *for each* $(x_\alpha) \in \prod_{\alpha\in\Lambda} X_\alpha$. *Then f is pseudo supercontinuous if and only if each* $f_\alpha$ *is pseudo supercontinuous.*

Proof is similar to that of Theorem 4.11 and makes use of Lemma 5.9 instead of Lemma 4.9.

5.12 **Corollary**: *Let* $f : X \to Y$ *be a function and* $g : X \to X \times Y$, *defined by* $g(x) = (x, f(x))$ *for each* $x \in X$, *be the graph function. Then g is pseudo supercontinuous if and only if f is pseudo supercontinuous.*

5.13 **Theorem**: *Let* $f, g : X \to Y$ *be pseudo supercontinuous functions from a space X into a* $D_\delta$*-Hausdorff space Y. Then the equalizer* $E = \{x : f(x) = g(x)\}$ *of f and g is* $\delta$*-closed in X.*

5.14 **Corollary**: *Let* $f : X \to X$ *be a pseudo supercontinuous function defined on a* $D_\delta$*-Hausdorff space X. Then the set of fixed points of f is* $\delta$*-closed.*

5.15 **Theorem**: *Let* $f : X \to Y$ *be a pseudo supercontinuous function. If X is almost completely regular, then f is quasi z-supercontinuous.*

5.16 **Definition**: *Let* $f : X \to Y$ *be a function from a topological space X into a topological space Y. The graph* $G(f)$ *of f is said to be* $\delta\Delta$***-closed*** *with respect to* $X \times Y$ *if for each* $(x, y) \notin G(f)$ *there exists a regular open set U containing x and a regular* $F_\sigma$*-set V containing y such that* $(U \times V) \cap G(f) = \emptyset$.

5.17 **Theorem**: *Let* $f : X \to Y$ *be a pseudo supercontinuous function from a space X into a* $D_\delta$*-Hausdorff space Y. Then G(f), the graph of f is* $\delta\Delta$*-closed with respect to* $X \times Y$.

5.18 **Theorem**: *Let* $f : X \to Y$ *be a pseudo supercontinuous injection which maps open sets in X to* $d_\delta$*-open sets in Y. Then X is a semi regular space. Further, if X is almost regular, then X is a regular space.*

5.19 **Theorem**: *Let* $f : X \to Y$ *be a pseudo supercontinuous open, closed surjection. If X is an almost completely regular space, then Y is* $\delta$*-completely regular.*

5.20 **Theorem**: *Let* $f : X \to Y$ *be a pseudo supercontinuous surjection from a space X onto a space Y. If X is a nearly compact space, then Y is* $D_\delta$*-compact.*

5.21 **Theorem**: *Let* $f : X \to Y$ *be a pseudo supercontinuous injection. Then the following statements are true.*

*(a) If Y is* $D_\delta T_0$*-space, then X is weakly Hausdorff.*

*(b) If Y is $D_\delta$-Hausdorff, then X is Hausdorff.*

**Proof**: (a). Let $x_1, x_2 \in X$, $x_1 \neq x_2$. Then $f(x_1) \neq f(x_2)$. Since Y is $D_\delta T_0$-space, there exists a regular $F_\sigma$-set $V$ containing one of the points $f(x_1)$ and $f(x_2)$ but not both. To be precise assume that $f(x_1) \in V$. Since $f$ is pseudo supercontinuous, $f^{-1}(V)$ is a $\delta$-open set containing $x_1$ but not $x_2$. So there is a regular open set containing $x_1$ but not $x_2$. So $\{x_1\}$ is the intersection of regular open sets containing $x_1$. Hence $X$ is a weakly Hausdorff space.

We omit proof of part (b).

5.22 **Theorem**: *Let $f : X \to Y$ be a closed, almost open, pseudo supercontinuous surjection such that $f^{-1}(y)$ is compact for each $y \in Y$. If X is a nearly paracompact space, then Y is a $D_\delta$-paracompact space. Moreover, if Y is $D_\delta$ completely regular, then Y is paracompact.*

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