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GEOMETRY ON THE BIG TANGENT BUNDLE

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Abstract. In this paper we describe some geometrical structures on the manifold $TM \oplus T^*M$ fibred over M: Liouville vector fields, almost tangent structure, semisprays, nonlinear connections and we show how these can be used in the study of mechanical systems.

1. Introduction

In a geometrical treatment of the mechanical systems a Lagrangian and an Hamiltonian formalism can be alternatively used. The former is based on a Lagrangian defined as a real function on the tangent bundle TM of configuration manifold M, while the latter is based on an Hamiltonian defined as a real function on the cotangent bundle T^*M. For classical mechanical systems the Lagrangian function is usually the kinetic energy provided by a Riemannian metric on the manifold M minus a potential energy. This is a remarkable example of the so-called regular Lagrangian function. Any regular Lagrangian defines by the Legendre map a unique regular Hamiltonian and conversely. Thus in this case the mentioned formalisms are equivalent. The need to extend the standard Lagrangian and Hamiltonian formalism to nonholonomic mechanical systems or singular Lagrangian systems produced a variety of means involving new concepts. Among these we find the use of the big tangent bundle $TM \oplus T^*M$ and in connection with it the so-called Dirac structures.

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The geometry of sections of the vector bundle $\tau \oplus \tau^* : TM \oplus T^*M \to M$ is now well-understood under the name of generalized geometry, see [7],[8] and the reference therein. The total space of the big tangent bundle i.e. the manifold $TM \oplus T^*M$ with the differentiable structure induced by the structure of M and by the vector bundle structure of the big tangent bundle, materializes the idea to consider the velocities and momenta as independent variables. This idea was proposed and developed by R. Skinner and R. Rusk ([5],[6]) and later was used for study of singular Lagrangian systems, [2]. The big tangent bundle is a particular vector bundle and so in the study of its total space the techniques from the book [3] can be applied. In this paper we begin the study of the manifold $\mathcal{T}M = TM \oplus T^*M$ following the said book [3] as well as [4]. For a different point of view we refer to [9]. In the second Section we shall describe the local structure of the manifold $\mathcal{TM}$ having in mind that it is fibred over M. The kernel of the differential of the projection $\tau \oplus \tau^*$ defines a subbundle of the tangent bundle $T(\mathcal{TM})$ called the vertical bundle. As a distribution it is integrable with the leaves $T_xM \oplus T_x^*M$. It naturally decomposes into two sub-distributions of equal dimensions and we define three Liouville vector fields that can be used to characterize the homogeneity of the various geometric objects with respect to velocities, momenta or to the both. In the third Section we define and study the notion of semispray on $TM \oplus T^*M$ regarding this bundle as an anchored vector bundle on the line developed in [1]. In the fourth Section we introduce a nonlinear connection as a distribution which is supplementary to the vertical distribution and partially recover its relationship with semisprays, well-known for the tangent bundle. In the last Section we sketch a possibility to study a singular Lagrangian system using the geometry of the big tangent bundle just developed.

2. Local structure of the manifold $TM \oplus T^*M$

Let M be an n-dimensional C^∞ manifold, $\tau : TM \to M$ its tangent bundle and $\tau^* : T^*M \to M$ its cotangent bundle. We denote by $\pi \equiv \tau \oplus \tau^* : TM \oplus T^*M \to M$ the Whitney sum of the tangent and cotangent bundle of M.

Let $(U, (x^i))$ be a local chart on M. Then $\left(\frac{\partial}{\partial x^i}\Big|_x\right)$, $x \in U$ is a local field of sections in the tangent bundle over U and $(dx^i|_x)$, $x \in U$ is a local field of sections in the cotangent bundle over U. By the definition of the Whitney sum, $\left(\frac{\partial}{\partial x^i}\Big|_x, dx^i|_x\right)$, $x \in U$ is a local field of sections in the bundle $TM \oplus T^*M$ over U. Every section (y, p) of π over U takes the form $(y, p) = y^i \frac{\partial}{\partial x^i} + p_i dx^i$ and the local coordinates on $\pi^{-1}(U)$ will be defined as (x^i, y^i, p_i). The indices $i, j, k, ...$ will run over $\{1, 2, ..., n\}$ and the Einstein convention on summation will be used.

Let $(\tilde{U}, (\tilde{x}^i))$ be another local chart on M with $U \cap \tilde{U} \neq \phi$. On $U \cap \tilde{U}$ we have

$$\tilde{x}^i = \tilde{x}^i(x^1, ...x^n), \quad \text{rank}\left(\frac{\partial \tilde{x}^i}{\partial x^j}\right) = n. \tag{2.1}$$

It follows that a change of coordinates $(x^i, y^i, p_i) \to (\tilde{x}^i, \tilde{y}^i, \tilde{p}_i)$ on the manifold $\mathcal{T}M = TM \oplus T^*M$ has the form

$$\begin{aligned} &\tilde{x}^i = \tilde{x}^i(x^1, ..., x^n), \quad \text{rank}\left(\frac{\partial \tilde{x}^i}{\partial x^j}\right) = n \\ &\tilde{y}^i = \frac{\partial \tilde{x}^i}{\partial x^j}(x) y^j, \\ &\tilde{p}_i = \frac{\partial x^j}{\partial \tilde{x}^i} p_j, \end{aligned} \tag{2.2}$$

where $\left(\frac{\partial x^j}{\partial \tilde{x}^i}\right)$ is the inverse of the Jacobian matrix $\left(\frac{\partial \tilde{x}^i}{\partial x^k}\right)$.

Let $(\partial_i := \dfrac{\partial}{\partial x^i}, \dot{\partial}_i = \dfrac{\partial}{\partial y^i}, \partial^i = \dfrac{\partial}{\partial p_i})$ be the natural basis in $T_{(y,p)}\mathcal{T}M$. The change of coordinates (2.2) implies

$$\begin{aligned} &\partial_i = \frac{\partial \tilde{x}^j}{\partial x^i}\tilde{\partial}_j + \frac{\partial \tilde{y}^j}{\partial x^i}\tilde{\dot{\partial}}_j + \frac{\partial \tilde{p}_j}{\partial x^i}\tilde{\partial}^j, \\ &\dot{\partial}_i = \frac{\partial \tilde{x}^j}{\partial x^i}(x)\tilde{\dot{\partial}}_j, \\ &\partial^i = \frac{\partial x^i}{\partial \tilde{x}^j}(x)\partial^j. \end{aligned} \tag{2.3}$$

The natural cobasis (dx^i, dy^i, dp_i) from $T^*_{(y,p)}\mathcal{T}M$ transforms as follows:

$$d\tilde{x}^i = \frac{\partial \tilde{x}^i}{\partial x^j}(x)dx^j,$$

$$d\tilde{y}^i = \frac{\partial^2 \tilde{x}^i}{\partial x^j \partial x^k}(x)y^j dx^k + \frac{\partial \tilde{x}^i}{\partial x^j}(x)dy^j \tag{2.4}$$

$$d\tilde{p}_i = \frac{\partial}{\partial x^k}\left(\frac{\partial x^j}{\partial \tilde{x}^i}\right)p_j dx^k + \frac{\partial x^j}{\partial \tilde{x}^i}dp_j.$$

The kernel of the tangent map $\pi_* : T(\mathcal{T}M) \to TM$ is called the vertical bundle over $\mathcal{T}M$. By (2.3), a vector field $X^i\partial_i + Y^i\dot{\partial}_i + P_i\partial^i$ on $\mathcal{T}M$ is vertical if and only if $X^i = 0$. Thus the vertical bundle $\mathcal{V}$ is locally spanned by $(\dot{\partial}_i, \partial^i)$ and its fibre is $2n-$dimensional.

We note that $\mathcal{V}$ has the following decomposition

$$\mathcal{V} = \mathcal{V}_1 \oplus \mathcal{V}_2, \tag{2.5}$$

where $\mathcal{V}_1$ is locally spanned by $(\dot{\partial}_i)$ and $\mathcal{V}_2$ is locally spanned by (∂^i).

By (2.2) and (2.3), the vector fields

$$C_1 = y^i\dot{\partial}_i, \quad C_2 = p_i\partial^i \text{ and } C_1 + C_2 \tag{2.6}$$

are globally defined on $\mathcal{T}M$. They are called the Liouville vector fields and can be used to characterize the homogeneity with respect to the variables $(y^i), (p_i)$ or to the both sets of these variables for various geometrical objects on $\mathcal{T}M$.

We note also that on $\mathcal{T}M$ we have a globally defined function $f(x, y, p) = p_i y^i$ and a 1-form $\alpha = p_i dx^i$. The 2-form $d\alpha$ is closed but degenerate. Thus it defines on $T\mathcal{T}M$ a presymplectic form.

3. Semisprays on $TM \oplus T^*M$

We recall that a semispray on the manifold TM is a vector field on TM which at the same time is a section in the vector bundle $\tau_* : TTM \to TM$, that is we have $\tau_{TM}(S(u)) = u$ and $\tau_{*,u}(S(u)) = u$, $\forall u \in TM$, where τ_{TM} is the vector bundle projection $TTM \to TM$.

Locally, if $S = X^i\partial_i + S^i\dot{\partial}_i$ is a vector field on TM, it is a semispray if and only if $X^i = y^i$, $i = 1, 2, ..., n$. Thus the integral curves $t \to (x^i(t), y^i(t))$ of S are solution of the system of differential equation

$$\frac{dx^i}{dt} = y^i, \quad \frac{dy^i}{dt} = S^i. \tag{3.1}$$

With the notation $S^i = -2G^i$, this system is equivalent to

$$\frac{d^2x^i}{dt^2} + 2G^i(x, \dot{x}) = 0, \tag{3.2}$$

which is the usual form of a second order differential equation (SODE). The notion of semispray can be extended only to anchored vector bundle, that is, to the vector bundles $E \to M$ that are endowed with a morphism (anchor) $\rho : E \to TM$. For details we refer to [1].

The vector bundle $\pi : \mathcal{T}M \to M$ is anchored with the anchor $\rho : TM \oplus T^*M \to TM$, $\rho(X, \omega) = X$. The above definition of a semispray is extended as follows:

A vector field S on $\mathcal{T}M$ will be called a semispray if

$$\pi_{*,u}(S(u)) = (\rho \circ \tau_{\mathcal{T}M})(S(u)),$$

$\forall u \in \mathcal{T}M$, where $\tau_{\mathcal{T}M} : T(\mathcal{T}M) \to \mathcal{T}M$ is the natural projection.

Locally, if $S = X^i(x, y, p)\partial_i + S^i(x, y, p)\dot{\partial}_i + G_i(x, y, p)\partial^i$, it is a semispray if and only if $X^i(x, y, p) = y^i$.

The integral curves $t \to (x(t), y(t), p(t))$, $t \in \mathbb{R}$ of a semispray S are solutions of the following system of differential equations:

$$\frac{dx^i}{dt} = y^i, \ \frac{dy^i}{dt} = S^i(x, y, p), \ \frac{dp_i}{dt} = G_i(x, y, p). \tag{3.3}$$

This is equivalent to

$$\frac{d^2x^i}{dt^2} = S^i\left(x, \frac{dx}{dt}, p\right), \ \frac{dp_i}{dt} = G_i\left(x, \frac{dx}{dt}, p\right). \tag{3.4}$$

The first equation (3.4) is a general form of the Euler - Lagrange equation and the second is a part of Jacobi - Hamilton equation.

The local components (S^j, G_j) of a semispray S change to $(\tilde{S}^i, \tilde{G}_i)$ according to the following formulae:

$$\begin{aligned} \tilde{S}^i &= \frac{\partial \tilde{x}^i}{\partial x^j} S^j + \frac{\partial \tilde{y}^i}{\partial x^k} y^k \\ \tilde{G}_i &= \frac{\partial x^j}{\partial \tilde{x}^i} G_j + \frac{\partial \tilde{p}_i}{\partial x^k} y^k. \end{aligned} \tag{3.5}$$

Conversely, a vector field $S = y^i\partial_i + S^i\dot{\partial}_i + G_i\partial^i$ whose components change as in (3.5) is a semispray.

We note that $S = y^i\partial_i + S^i(x, y, p)\dot{\partial}_i$ can not be a semispray on $\mathcal{T}M$ because of the second equation in (3.5). If the functions $S^i(x, y, p)$ do not depend on p, S may be viewed as a semispray on TM.

Let us consider a linear operator J on vector fields given in the natural basis by

$$J(\partial_i) = \dot{\partial}_i,\ J(\dot{\partial}_i) = 0,\ J(\partial^i) = 0. \tag{3.6}$$

By (2.3) the operator J is well-defined. It has the properties $J^2 = 0$, $\ker J = \mathcal{V}$, $\operatorname{im} J = \mathcal{V}_1$ and it is easy to prove that

Theorem 3.1. *A vector field S on $\mathcal{T}M$ is a semispray if and only if $JS = C_1$.*

4. Nonlinear connections on $TM \oplus T^*M$

Let $\mathcal{V}$ be the vertical bundle over $\mathcal{T}M$. It may be regarded as a distribution of dimension $2n$ on the manifold $\mathcal{T}M$. This distribution is locally spanned by $(\dot{\partial}_i, \partial^i)$. Hence it is integrable.

Definition 4.1. *A nonlinear connection is a distribution $\mathcal{H}$ on $\mathcal{T}M$ called horizontal which is supplementary to the vertical distribution, that is, we have*

$$T\mathcal{T}M = \mathcal{H} \oplus \mathcal{V} = \mathcal{H} \oplus \mathcal{V}_1 \oplus \mathcal{V}_2. \tag{4.1}$$

The distribution $\mathcal{H}$ is of dimension n. We choose a local basis (δ_i) of it such that

$$\pi_*(\delta_i) = \partial_i, \quad \delta_i = \frac{\partial \tilde{x}^j}{\partial x^i}(x)\overline{\delta}_j, \tag{4.2}$$

where $(\overline{\delta}_j)$ is a similar basis in a different local chart. It follows that

$$\delta_i = \partial_i - N_i^j(x,y,p)\dot{\partial}_j + N_{ij}(x,y,p)\partial^j, \tag{4.3}$$

where the sign "$-$" is taken for convenience and in a different local chart the functions $\tilde{N}_i^j(\tilde{x}(x), \tilde{y}(x,y), \tilde{p}(x,p))$, $\tilde{N}_{ij}(\tilde{x}(x), \tilde{y}(x,y), \tilde{p}(x,p))$ are given by

$$\begin{aligned} \tilde{N}_h^i \frac{\partial \tilde{x}^h}{\partial x^j} &= N_j^h \frac{\partial \tilde{x}^i}{\partial x^h} - \frac{\partial \tilde{y}^i}{\partial x^j} \\ \tilde{N}_{ij} &= \frac{\partial x^h}{\partial \tilde{x}^i}\frac{\partial x^k}{\partial \tilde{x}^j} N_{hk} + p_h \frac{\partial^2 x^h}{\partial \tilde{x}^i \partial \tilde{x}^j}. \end{aligned} \tag{4.4}$$

The equations (4.4) are implied by the second condition (4.2). Conversely, if on each domain of local chart on $\mathcal{T}M$ we have defined the set of functions (N_j^i, N_{ij}) that satisfies (4.4) on overlaps, the basis (δ_i) given by (4.3) satisfies (4.2) and spans a horizontal distribution. Thus, a nonlinear connection is completely determined by the functions (N_j^i, N_{ij}) verifying (4.4). Given a nonlinear connection we may choose $(\delta_i, \dot{\partial}_i, \partial^i)$ as local basis on $\mathcal{T}M$. This basis is adapted to the decomposition (4.1). The dual basis of it is $(dx^i, \delta y^i, \delta p_i)$, where

$$\delta y^i = dy^i + N^i_j dx^j, \quad \delta p_i = dp_i - N_{ji} dx^j. \tag{4.5}$$

Thus any vector field on $\mathcal{T}M$ can be written in the form

$$X = X^i \delta_i + Y^i \dot{\partial}_i + Z_i \partial^i. \tag{4.6}$$

The components $(X^i(x,y,p))$, $(Y^i(x,y,p))$, transform by a change of coordinates as the components of a vector field on M while $Z_i(x,y,p)$ as the components of an one form. One says that they define $d-$objects on $\mathcal{T}M$ (d is for distinguished).

A semispray decomposes in the form

$$S = y^i \delta_i + k^i \dot{\partial}_i + h_i \partial^i, \tag{4.7}$$

where $k^i = S^i + N^i_j y^j$, $h_i = G_i - N_{ji} y^j$.

From (4.7) it comes out

Lemma 4.1. *The difference of two semisprays is a vertical vector field.*

The usual relationship between the semisprays and the nonlinear connections, [3], is partially recovered in this setting as follows:

Theorem 4.1. *(i) Let be $S = y^i \partial_i + S^i \dot{\partial}_i + G_i \partial^i$ be a semispray on $\mathcal{T}M$. Then the functions*

$$N^i_j = \partial^i G_j, \quad N_{ij} = \dot{\partial}_i G_j, \tag{4.8}$$

are the local coefficients of a nonlinear connection.

(ii) Let (N^i_j, N_{ij}) be the local coefficients of a nonlinear connection. Then $G_i = N_{ij} y^j$ is the second coefficient of a semispray whose first coefficient (S^i) remains undetermined.

Proof. One applies ∂^i to the law of transformation of G_j and one verifies that $\partial^i G_j$ satisfies the first equation from (4.4). If applies $\dot{\partial}_i$ to the same law of transformation, it comes out that $\dot{\partial}_i N_j$ verifies the second equation from (4.4). The assertion (ii) follows by checking that $(N_{ij} y^j)$ verifies the second equation from (3.5).

5. Application to mechanical systems

Let be a mechanical system on M described by a Lagrangian $L : TM \to \mathbb{R}$. The manifold M is called the configuration space. The variation with fixed endpoints of the action integral $\int_a^b L(x(t), \dot{x}(t))dt$

for curves nearby a fixed curve $t \to x(t), y(t)$, $t \in [a,b] \subset \mathbb{R}$ provides the Euler - Lagrange equations

$$\frac{\partial L}{\partial x^i}(x,\dot{x}) - \frac{d}{dt}\frac{\partial L(x,\dot{x})}{\partial \dot{x}^i} = 0,\ \dot{x}^i = \left(\frac{dx^i}{dt}\right). \tag{5.1}$$

These equations describe the dynamics of the given mechanical system.

Recall that on $T\mathcal{T}M$ we have defined the so-called *inertial form*

$$\omega = -d(p_i dx^i) = dx^i \wedge dp_i.$$

If L is a regular Lagrangian i.e. the matrix with the entries $g_{ij}(x,y) = \dot{\partial}_i\dot{\partial}_j L(x,y)$ is of rank n, one may define a local diffeomorphism $\phi : TM \to T^*M$ by $\phi(x,y) = (x, \dot{\partial}_j L(x,y))$ called the Legendre map. Moreover, in this case one may define a local function $H : T^*M \to \mathbb{R}$ by $H(x,p) = p_i y^i - L(x,y)$, where $y = (y^i)$ is the unique solution of the equation

$$p_i = \dot{\partial}_i L(x,y).$$

In the following, we shall assume that L is degenerate. Thus, we may no longer consider the Legendre map and H. However we may replace H by a function H^g on $TM \oplus T^*M$ defined by

$$(x,y,p) \to H^g(x,y,p) = p_i y^i - L(x,y). \tag{5.2}$$

This function is globally defined on $TM \oplus T^*M$. We call it a generalized Hamiltonian.

Now we have on $TM \oplus T^*M$ a closed 2-form ω and a function H^g. As usual, we may consider the equation

$$i_X\omega = dH^g,$$

in the unknown X, a vector field on $\mathcal{T}M$. Since ω is degenerate this equation has no unique solution and we shall see that even if it has solutions, these are not defined on the whole $\mathcal{T}M$. More precisely, we have

Theorem 5.1. *Let be the first order equation of motion*

$$i_X\omega = dH^g \tag{5.3}$$

in the unknown X, a vector field on $\mathcal{T}M$.

(i) The equation (5.3) has no solution on the whole $TM \oplus T^*M$ but only on a submanifold $C_1 \subset TM \oplus T^*M$ with $C_1 = \{(x,y,p) \mid p_i = \dfrac{\partial L}{\partial y^i}\}$.

(ii) The function $\left(\dfrac{\partial L}{\partial x^i}(x,y)\right)$ *are the local coefficients for a semispray on the submanifold* C_1*.*

(iii) The solutions X *of (5.3) are semisprays on the submanifold* C_1*.*

Proof. We search for solutions X of (5.3) in the form $X = X^i\partial_i + S^i\dot{\partial}_i + G_i\partial^i$. We find

$$i_X\omega = -P_i dx^i + X^i dp_i,\ dH^g = -\frac{\partial L}{\partial x^i}dx^i + \left(p_i - \frac{\partial L}{\partial y^i}\right)dy^i + y^i dp_i.$$

Then (5.3) yields

$$X^i = y^i,\ p_i - \frac{\partial L}{\partial y^i} = 0,\ G_i = \frac{\partial L}{\partial x^i}. \tag{5.4}$$

Thus, the solutions X exists only on the submanifold C_1 and they are given by $X = y^i\partial_i + S^i\dot{\partial}_i + \dfrac{\partial L}{\partial x^i}\partial^i$, a form similar to that of a semispray. Thus, (i) is proved.

To prove (ii), first we have

$$\frac{\partial L}{\partial x^i} = \frac{\partial L}{\partial \tilde{x}^j}\frac{\partial \tilde{x}^j}{\partial x^i} + \frac{\partial L}{\partial \tilde{y}^j}\frac{\partial \tilde{y}^j}{\partial x^i}$$

and on C_1 this equality is equivalent to $\tilde{G}_k = G_i\dfrac{\partial x^i}{\partial x^k} - \tilde{p}_j\dfrac{\partial \tilde{y}^j}{\partial x^i}\dfrac{\partial x^i}{\partial \tilde{x}^k}$. Using the chain rule one finds

$$\tilde{G}_k = \frac{\partial x^i}{\partial x^k}G_i + \frac{\partial \tilde{p}_k}{\partial x^j}y^j.$$

Thus the functions $G_k = \dfrac{\partial L}{\partial x^k}$ on C_1 verify the second formula (3.5) and so (ii) is proved.

As $(S^i(x,y,p))$ verify the first formula (3.5), by (ii) it comes out that on C_1, the vector field X is a semispray with the function $(S^i(x,y,p))$ undetermined. Thus the proof is complete.

The integral curves of the solution X of (5.3) are solutions of the following system of differential equations:

$$\frac{d^2x^i}{dt^2} = S^i\left(x, \frac{dx}{dt}, p\right),\ \frac{dp_i}{dt} = \frac{\partial L}{\partial x^i}(x). \tag{5.5}$$

These solutions are the trajectories of the given mechanical system. In the case when L is degenerate, the functions (S^i) are not determined and we need new constraints to fix them. Such constraints are suggested in [5]. If L is a non-degenerate Lagrangian, the submanifold

C_1 has the form $U \times U^*$, where $U \subset TM$ and $U^* = \phi(U) \subset T^*M$ are such that the Legendre map is diffeomorphism. If ϕ is a global diffeomorphism, a case when it is said that L is hyperregular, then C_1 coincides to $\mathcal{T}M$. In the both two cases, a natural solution X one obtains if one takes $S^i(x, y, p) = S^i(x, y)$ as the local coefficients of the semispray determined by L, that is

$$S^i(x, y, p) = g^{ij}\left(\frac{\partial L}{\partial x^j} - \frac{\partial^2 L}{\partial x^k \partial y^j} y^k\right). \tag{5.6}$$

In this case, the system (5.5) splits into independent equations. The first equation determines the trajectory of the system and the second its momenta.

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