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ON (b, μ_Y)-CONTINUOUS FUNCTIONS

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Abstract. The notion of γ-b-open sets in generalized topological spaces was introduced and studied by Sivagami [5] in 2008. By using the notion of μ-b-open sets, we introduce and study (b, μ_Y)-continuous functions, μ-b-kernel and (μ_X, μ_Y)-b-open functions in generalized topological spaces. Also some characterizations of (b, μ_Y)-continuous functions are obtained.

1. Introduction and preliminaries

In 1998, A. Csaszar [2] under suitable conditions on $\gamma : expX \to expX$ gave explicitly formulas for $i_\gamma(A)$ and $c_\gamma(A)$ for a subset A of X. Also in 2002, A. Csaszar [1] introduced the notions of generalized topology and generalized continuity. Let X be a non-empty set and μ be the collection of subsets of X.

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Then μ is called a *generalized topology* (briefly GT) on X if $\emptyset \in \mu$ and arbitrary union of elements of μ belongs to μ. By a space (X, μ), we mean a *generalized topological space* (briefly GTS). A subset A of a GTS (X, μ) is said to be μ-open if $A \in \mu$ and μ-closed if $X \setminus A \in \mu$. For $A \subseteq X$, the union of all μ-open subsets of A is denoted by $i_\mu(A)$ and the intersection of all μ-closed supersets of A is denoted by $c_\mu(A)$. Clearly $i_\mu(A)$ is μ-open and $c_\mu(A)$ is μ-closed and $i_\mu(A) \subset A \subset c_\mu(A)$ for every $A \subset X$. Moreover, $A \subset i_{(A)}$ if and only if A is μ-open, while $c_\mu(A) \subset A$ if and only if A is μ-closed. Also $x \in c_\mu(A)$ if and only if $x \in G \in \mu$ implies $G \cap A \neq \emptyset$. Moreover the operators i_μ and c_μ are dual, that is $c_\mu(A) = X \setminus i_\mu(A)$ and $i_\mu(A) = X \setminus c_\mu(X \setminus A)$ for every $A \subseteq X$. A subset A of a GTS (X, μ) is μ-b-*open* if $A \subseteq i_\mu(c_\mu(A)) \cup c_\mu(i_\mu(A))$. The complement of μ-b-open set is called μ-b-closed. The class of μ-b-open subsets of a GTS (X, μ) denoted by $b(\mu)$ is a generalized topology. Moreover, $\mu \subset b(\mu)$ and consequently, every μ-closed set is also μ-b-closed. The μ-b-interior [5] of a subset A of (X, μ) denoted by $i_b(A)$ is the union of all μ-b-open sets contained in A. The μ-b-closure [5] of a subset A of (X, μ) denoted by $c_b(A)$ is the intersection of all μ-b-closed sets containing A. Observe that $i_\mu(A) \subset i_b(A)$ and $c_b(A) \subset c_\mu(A)$, for every $A \subset X$. A subset N_x of the GTS (X, μ) is said to be a μ-b-neighborhood of a point $x \in X$ if there exists a μ-b-open set U such that $x \in U \subseteq N_x$. A function $f : (X, \mu_X) \to (Y, \mu_Y)$ is (μ_X, μ_Y)-continuous if $G' \in \mu_Y$ implies that $f^{-1}(G') \in \mu_X$ [1]. In this paper we investigate a new particular type of generalized continuity, called (b, μ)-continuity, as well as the corresponding notions of open and closed functions.

2. (b, μ_Y)-CONTINUOUS FUNCTIONS

Definition 2.1. *A function $f : (X, \mu_X) \to (Y, \mu_Y)$ is (b, μ_Y)-continuous if $f^{-1}(V)$ is μ_X-b-open in X for each μ_Y-open set V of Y.*

Theorem 2.2. *Every (μ_X, μ_Y)-continuous function is (b, μ_Y)-continuous but not conversely.*

Example 2.3. *Let $X = \mathbb{R}$ be the set of real numbers, $\mu = \{\emptyset, \mathbb{R} - \mathbb{Q}\}$ and $\lambda = \{\emptyset, \mathbb{Q}\}$ where $\mathbb{Q}$ denotes the set of all rational numbers and $\mathbb{R} - \mathbb{Q}$ denotes the set of all irrational numbers. Define an identity* *function* $f:(\mathbb{R},\mu)\to(\mathbb{R},\lambda)$. *Then* f *is* (b,λ)*-continuous but not* (μ,λ)*-continuous.*

Theorem 2.4. *Let* $f:(X,\mu_X)\to(Y,\mu_Y)$ *be a function. Then the following are equivalent :*

(1) f *is* (b,μ_Y)*-continuous.*
(2) *For each* $x\in X$ *and each* μ_Y*-open set* V *of* Y *with* $f(x)\in V$, *there exists a* μ_X-b*-open set* U *containing* x *such that* $f(U)\subseteq V$.
(3) *For each* $x\in X$ *and each* μ_Y*-open set* V *of* Y *with* $f(x)\in V$, $f^{-1}(V)$ *is a* μ_X-b*-neighborhood of* x.
(4) *The inverse image of each* μ_Y*-closed set in* Y *is* μ_X-b*-closed.*
(5) $c_b(f^{-1}(B))\subseteq f^{-1}(c_{\mu_Y}(B))$ *for every set* B *in* Y.
(6) $f(c_b(A))\subseteq c_{\mu_Y}(f(A))$ *for every set* A *in* X.
(7) $f^{-1}(i_{\mu_Y}(B))\subseteq i_b(f^{-1}(B))$ *for every set* B *in* Y.

Proof. 1. $\Rightarrow$ 2. Let $x\in X$ and let V be a μ_Y-open set such that $f(x)\in V$. Since f is (b,μ_Y)-continuous, $f^{-1}(V)$ is μ_X-b-open. By putting $U=f^{-1}(V)$ which is containing x, we have $f(U)\subseteq V$.

2. $\Rightarrow$ 3. Let V be a μ_Y-open in Y and let $f(x)\in V$. Then by 2, there exists a μ_X-b-open set U containing x such that $f(U)\subseteq V$. So each $x\in U\subseteq f^{-1}(V)$. Hence $f^{-1}(V)$ is a μ_X-b-neighborhood of x.

3. $\Rightarrow$ 1. Let V be a μ_Y-open set in Y and let $f(x)\in V$. Then by 3, $f^{-1}(V)$ is a μ_X-b-neighborhood of x. Thus for each $x\in f^{-1}(V)$, there exists a μ_X-b-open set U_x containing x such that $x\in U_x\subseteq f^{-1}(V)$. Hence $f^{-1}(V)\subseteq\cup_{x\in f^{-1}(V)}U_x$ and so $f^{-1}(V)\in b(\mu_X)$.

1. $\Leftrightarrow$ 4. This is obvious.

1. $\Rightarrow$ 5. Let B be a subset of Y. Since $c_{\mu_Y}(B)$ is μ_Y-closed and f is (b,μ_Y)-continuous, $f^{-1}(c_{\mu_Y}(B)$ is μ_X-b-closed. Therefore $c_b(f^{-1}(B))\subseteq c_b(f^{-1}(c_{\mu_Y}(B)))=f^{-1}(c_{\mu_Y}(B))$.

5. $\Rightarrow$ 6. Let A be a subset of X. By 5, we have $f^{-1}(c_{\mu_Y}(f(A)))\supseteq c_b(f^{-1}(f(A)))\supseteq c_b(A)$. Therefore $f(c_b(A))\subseteq c_{\mu_Y}(f(A))$.

6. $\Rightarrow$ 7. Let B be a subset of Y. By 6, $f(c_b(X-f^{-1}(B)))\subseteq c_{\mu_Y}(f(X-f^{-1}(B)))$ and $f(X-i_b(f^{-1}(B)))\subseteq c_{\mu_Y}(Y-B)=$

$Y - i_{\mu_Y}(B)$. Therefore $X - i_b(f^{-1}(B)) \subseteq f^{-1}(Y - i_{\mu_Y}(B))$ and $f^{-1}(i_{\mu_Y}(B)) \subseteq i_b(f^{-1}(B))$.

7. $\Rightarrow$ 1. Let B be μ_Y-open in Y and $f^{-1}(i_{\mu_Y}(B)) \subseteq i_b(f^{-1}(B))$. Then $f^{-1}(B) \subseteq i_b(f^{-1}(B))$. But $i_b(f^{-1}(B)) \subseteq f^{-1}(B)$. Hence $f^{-1}(B) = i_b(f^{-1}(B))$. Therefore $f^{-1}(B)$ is μ_X-b-open.

Remark 2.5. *The above theorem is similar to a characterizations of m-continuous functions [4], when we change the GT, μ_X by a minimal structure and the GT, μ_Y by a topology on Y. Recall that a collection m of subsets of X is a minimal structure on X if the $\emptyset$ and X belong to m.*

Theorem 2.6. *Let $f : (X, \mu_X) \to (Y, \mu_Y)$ be a function. Then the following are equivalent :*

(1) *f is (b, μ_Y)-continuous.*
(2) *$i_{\mu_X}(c_{\mu_X}(f^{-1}(B))) \cap c_{\mu_X}(i_{\mu_X}(f^{-1}(B))) \subseteq f^{-1}(c_{\mu_Y}(B))$ for each B in Y.*
(3) *$f[i_{\mu_X}(c_{\mu_X}(A)) \cap c_{\mu_X}(i_{\mu_X}(A))] \subseteq c_{\mu_Y}(f(A)))$ for each A in X.*

Proof. 1. $\Rightarrow$ 2. Let B be a subset of Y. Since f is (b, μ_Y)-continuous, $f^{-1}(c_{\mu_Y}(B))$ is μ_X-b-closed by Theorem 2.4. Then $i_{\mu_X}(c_{\mu_X}(f^{-1}(B))) \cap c_{\mu_X}(i_{\mu_X}(f^{-1}(B))) \subseteq i_{\mu_X}(c_{\mu_X}(f^{-1}(c_{\mu_Y}(B)))) \cap c_{\mu_X}(i_{\mu_X}(f^{-1}(c_{\mu_Y}(B)))) \subseteq f^{-1}(c_{\mu_Y}(B))$.

2. $\Rightarrow$ 3. Let A be a subset of X. Set $f^{-1}(B) = A$. Then by 2, $i_{\mu_X}(c_{\mu_X}(A)) \cap c_{\mu_X}(i_{\mu_X}(A)) \subseteq f^{-1}(c_{\mu_Y}(f(A)))$. Hence $f[i_{\mu_X}(c_{\mu_X}(A)) \cap c_{\mu_X}(i_{\mu_X}(A))] \subseteq c_{\mu_Y}(f(A)))$.

3. $\Rightarrow$ 1. Let V be μ_Y-open in Y. Then $W = Y \setminus V$ is μ_Y-closed Y. By 3, $f[i_{\mu_X}(c_{\mu_X}(f^{-1}(W))) \cap c_{\mu_X}(i_{\mu_X}(f^{-1}(W)))] \subseteq c_{\mu_Y}(f(f^{-1}(W))) \subseteq c_{\mu_Y}(W) = W$. Thus $f^{-1}(W)$ is μ_X-b-closed. Hence f is (b, μ_Y)-continuous.

Theorem 2.7. *A function $f : (X, \mu_X) \to (Y, \mu_Y)$ is (b, μ_Y)-continuous if one of the following holds :*

(1) *$f^{-1}(i_b(B)) \subseteq i_{\mu_X}(f^{-1}(B))$ for every set B in Y.*
(2) *$c_{\mu_X}(f^{-1}(B)) \subseteq f^{-1}(c_b(B))$ for every set B in Y.*
(3) *$f(c_{\mu_X}(A)) \subseteq c_b(f(A))$ for every set A in X.*

Proof. 1. Let B be any μ_Y-open set in Y. If $f^{-1}(i_b(B)) \subseteq i_{\mu_X}(f^{-1}(B))$ is satisfied then $f^{-1}(B) \subseteq i_{\mu_X}(f^{-1}(B))$. Therefore

$f^{-1}(B)$ is μ_X-open which implies $f^{-1}(B)$ is μ_X-b-open. Hence f is (b, μ_Y)-continuous.

2. Let $B \subset Y$ be any μ_Y-closed set. Then B is μ_Y-b-closed, hence $c_b(B) = B$. According to (2), $c_{\mu_X}(f^{-1}(B)) \subset f^{-1}(B)$, which implies that $f^{-1}(B)$ is μ_X-b-closed. By Theorem 2.4 ((4) $\Leftrightarrow$ (1)), f is (b, μ_Y)-continuous.

3. Let B be any subset in Y. Then $f^{-1}(B)$ is a set in X and $f(c_{\mu_X}(f^{-1}(B))) \subseteq c_b(f(f^{-1}(B)))$ as (3) is satisfied. This implies $f(c_{\mu_X}(f^{-1}(B))) \subseteq c_b(B)$. Then by (2), f is (b, μ_Y)-continuous.

Proposition 2.8. *Let $f : (X, \mu_X) \to (Y, \mu_Y)$ and $g : (Y, \mu_Y) \to (Z, \mu_Z)$ be two functions. Then $g \circ f$ is (b, μ_Z)-continuous if f is (b, μ_Y)-continuous and g is (μ_Y, μ_Z)-continuous.*

Proof. Let W be μ_Z-open in Z. Since g is (μ_Y, μ_Z)-continuous, $g^{-1}(W)$ is μ_Y-open in Y and since f is (b, μ_Y)-continuous, $f^{-1}(g^{-1}(W))$ is μ_X-b-open in X. Thus $g \circ f$ is (b, μ_Z)-continuous.

Remark 2.9. *Let $f : (X, \mu_X) \to (Y, \mu_Y)$ and $g : (Y, \mu_Y) \to (Z, \mu_Z)$ be two functions. Then $g \circ f$ is not (b, μ_Z)-continuous if f is (b, μ_Y)-continuous and g is (b, μ_Z)-continuous.*

Example 2.10. *Let $X = \{a, b, c\}$, $Y = \{p, q, r\}$ and $Z = \{x, y, z\}$. Consider $\mu_X = \{\emptyset, \{a\}, \{a, b\}, \{a, c\}, X\}$, $\mu_Y = \{\emptyset, \{p, q\}, \{q, r\}, Y\}$ and $\mu_Z = \{\emptyset, \{x\}, \{y\}, \{x, y\}\}$. Define $f : (X, \mu_X) \to (Y, \mu_Y)$ by $f(a) = f(b) = q$, $f(c) = p$ and $g : (Y, \mu_Y) \to (Z, \mu_Z)$ by $g(p) = g(r) = y$, $g(q) = x$. Then f is (b, μ_Y)-continuous and g is (b, μ_Z)-continuous but $g \circ f$ is not (b, μ_Z)-continuous.*

Definition 2.11. *Let (X, μ) be a GTS and $A \subseteq X$. Then the μ-b-kernel of A denoted by μ-b-$Ker(A)$ is defined to be the set, μ-b-$Ker(A) = \bigcap \{U : U \in b(\mu), A \subseteq U\}$.*

Remark 2.12. *If in the above definition, we take μ as a topology, we obtain the definition of kernel [3].*

Theorem 2.13. *Let (X, μ) be a GTS and $x \in X$. Then $y \in \mu$-b-$Ker(\{x\})$ if and only if $x \in c_b(\{y\})$.*

Proof. Assume that $y \notin \mu$-b-$Ker(\{x\})$. Then there exists a μ-b-open set U containing x such that $y \notin U$. Therefore, we have $x \notin c_b(\{y\})$. The converse is similarly shown.

Theorem 2.14. *Let* (X,μ) *be a GTS and* A *a subset of* X. *Then* $\mu\text{-}b\text{-}Ker(A) = \{x : x \in X, c_b(\{x\}) \cap A \neq \emptyset\}$.

Proof. Let $x \in \mu\text{-}b\text{-}Ker(A)$ and $c_b(\{x\}) \cap A = \emptyset$. Therefore, $x \notin X \backslash c_b(\{x\})$ which is a $\mu\text{-}b$-open set containing A. But this is impossible, since $x \in \mu\text{-}b\text{-}Ker(A)$. Consequently, $c_b(\{x\}) \cap A \neq \emptyset$.

Conversely, let $x \in X$ such that $c_b(\{x\}) \cap A \neq \emptyset$. Suppose that $x \notin \mu\text{-}b\text{-}Ker(A)$. Then there exists $\mu\text{-}b$-open set U containing A and $x \notin U$. Let $y \in c_b(\{x\}) \cap A$. Then $y \in c_b(\{x\})$ and $y \in A$. Thus $x \in \mu\text{-}b\text{-}Ker(\{y\})$ and $y \in U \in \mu$ implies $x \in U$. By this contradiction, $x \in \mu\text{-}b\text{-}Ker(A)$.

Theorem 2.15. *The following are equivalent for any points* x *and* y *in a GTS* (X,μ) *:*

(1) $\mu\text{-}b\text{-}Ker(\{x\}) \neq \mu\text{-}b\text{-}Ker(\{y\})$.
(2) $c_b(\{x\}) \neq c_b(\{y\})$.

Proof. $1 \Rightarrow 2$. Let $\mu\text{-}b\text{-}Ker(\{x\}) \neq \mu\text{-}b\text{-}Ker(\{y\})$. Then there exists a point z in X such that $z \in \mu\text{-}b\text{-}Ker(\{x\})$ and $z \notin \mu\text{-}b\text{-}Ker(\{y\})$ (say). Then by Theorem 2.13, $x \in c_b(\{z\})$ and $y \notin c_b(\{z\})$. Since $x \in c_b(\{z\})$, $c_b(\{x\}) \subseteq c_b(\{z\})$ and thus $c_b(\{x\}) \cap \{y\} = \emptyset$. Hence $c_b(\{x\}) \neq c_b(\{y\})$.

$2 \Rightarrow 1$. Suppose that $c_b(\{x\}) \neq c_b(\{y\})$. Then there exists a point z in X such that $z \in c_b(\{x\})$ and $z \notin c_b(\{y\})$ (say). So there exists a $\mu\text{-}b$-open set U containing z such that $x \in U$ and $y \notin U$. Thus $y \notin \mu\text{-}b\text{-}Ker(\{x\})$ and hence $\mu\text{-}b\text{-}Ker(\{x\}) \neq \mu\text{-}b\text{-}Ker(\{y\})$.

Theorem 2.16. *Let* (X,μ) *be a GTS and* $A \subseteq X$. *Then*

(1) $x \in \mu\text{-}b\text{-}Ker(A)$ *if and only if* $A \cap F \neq \emptyset$ *for any* $\mu\text{-}b$*-closed subset* F *of* X *with* $x \in F$.
(2) $A \subseteq \mu\text{-}b\text{-}Ker(A)$ *for every* $A \subset X$.
(3) $A = \mu\text{-}b\text{-}Ker(A)$ *if* A *is* $\mu\text{-}b$*-open in* X.
(4) *If* $A \subseteq B$ *then* $\mu\text{-}b\text{-}Ker(A) \subseteq \mu\text{-}b\text{-}Ker(B)$.

Proof. 1. Let $x \in \mu\text{-}b\text{-}Ker(A)$. Then by Theorem 2.14, $A \cap c_b(\{x\}) \neq \emptyset$. Conversely, assume that $A \cap F \neq \emptyset$ for every $\mu\text{-}b$-closed subset F of X. By taking $F = c_b(\{x\})$, we have $A \cap c_b(\{x\}) \neq \emptyset$ which implies $x \in \mu\text{-}b\text{-}Ker(A)$.

2. It is obvious.

3. Let A be μ-b-open in X. Then always $A \subseteq \mu$-b-$Ker(A)$. On the other hand, assume that $x \in \mu$-b-$Ker(A)$. Then $x \in \bigcap \{U : U \in b(\mu), A \subseteq U\}$, since A is μ-b-open implies that $x \in A$. Thus μ-b-$Ker(A) \subseteq A$. Hence $A = \mu$-b-$Ker(A)$.

4. It is obvious.

Theorem 2.17. *Let $f : (X, \mu_X) \to (Y, \mu_Y)$ be a function. If f is (b, μ_Y)-continuous then for every subset A of X, $f(i_b(A)) \subset \mu_Y -\ b$-$Ker(f(A))$.*

Proof. Let A be any subset of X and $y \in Y$. Suppose that $y \notin \mu_Y -\ b$-$Ker(f(A))$. Then by Theorem 2.16, there exists a μ_Y-b-closed subset F of Y such that $y \in F$ and $f(A) \cap F = \emptyset$. Thus we have $A \cap f^{-1}(F) = \emptyset$ and $i_b(A) \cap f^{-1}(F) = \emptyset$. Therefore, we obtain $f(i_b(A)) \cap F = \emptyset$, hence $y \notin f(i_b(A))$. This implies that $f(i_b(A)) \subseteq \mu_Y -\ b$-$Ker(f(A))$.

3. (μ_X, μ_Y)-b-open functions

Definition 3.1. *A function $f : (X, \mu_X) \to (Y, \mu_Y)$ is called (μ_X, μ_Y)-b-open in X if for each $U \in \mu_X$, $f(U)$ is μ_Y-b-open in Y. The complement of (μ_X, μ_Y)-b-open function is (μ_X, μ_Y)-b-closed.*

Theorem 3.2. *Let $f : (X, \mu_X) \to (Y, \mu_Y)$ be a (μ_X, μ_Y)-b-open (resp. (μ_X, μ_Y)-b-closed) function. If W is any subset of Y and F is a μ_X-closed (resp. μ_X-open) set of X containing $f^{-1}(W)$ then there exists a μ_Y-b-closed (resp. μ_Y-b-open) subset H of Y containing W such that $f^{-1}(H) \subseteq F$.*

Proof. Suppose that f is (μ_X, μ_Y)-b-open function. Let W be any subset of Y and F a μ_X-closed subset of X containing $f^{-1}(W)$. Then $X - F$ is μ_X-open and since f is (μ_X, μ_Y)-b-open, $f(X - F)$ is μ_Y-b-open. Hence $H = Y - f(X - F)$ is μ_Y-b-closed. It follows from $f^{-1}(W) \subseteq F$ such that $W \subseteq H$. Moreover, we obtain $f^{-1}(H) \subseteq F$. The proof is similar for (μ_X, μ_Y)-b-closed function.

Theorem 3.3. *A function $f : (X, \mu_X) \to (Y, \mu_Y)$ is (μ_X, μ_Y)-b-open if and only if $f(i_{\mu_X}(A)) \subseteq i_b(f(A))$ for each set A in X.*

Proof. Suppose that f is (μ_X, μ_Y)-b-open function. Since $i_{\mu_X}(A) \subseteq A$, $f(i_{\mu_X}(A)) \subseteq f(A)$. By hypothesis, $f(i_{\mu_X}(A)) \subseteq i_b(f(A))$.

Conversely, suppose that A be μ_X-open and $f(i_{\mu_X}(A)) \subseteq i_b(f(A))$. Then $f(A) \subseteq i_b(f(A))$. On the other hand, $i_b(f(A)) \subseteq f(A)$. Therefore $f(A) = i_b(f(A))$ which implies $f(A)$ is μ_Y-b-open and hence f is (μ_X, μ_Y)-b-open.

Theorem 3.4. *A function $f : (X, \mu_X) \to (Y, \mu_Y)$ is (μ_X, μ_Y)-b-closed if and only if $c_b(f(A)) \subseteq f(c_{\mu_X}(A))$ for each set A in X.*

Proof. Suppose that f is (μ_X, μ_Y)-b-closed function. Since $c_{\mu_X}(A)$ is μ_X-closed, $f(c_{\mu_X}(A))$ is μ_Y-b-closed. Also since $f(A) \subseteq f(c_{\mu_X}(A))$, $c_b(f(A)) \subseteq f(c_{\mu_X}(A))$.

Conversely, let A be μ_X-closed in X. Then $f(A) \subseteq c_b(f(A)) \subseteq f(c_{\mu_X}(A)) = f(A)$. Thus $f(A) = c_b(f(A))$. Therefore $f(A)$ is μ_Y-b-closed in Y and hence f is (μ_X, μ_Y)-b-closed.

Theorem 3.5. *Let $f : (X, \mu_X) \to (Y, \mu_Y)$ and $g : (Y, \mu_Y) \to (Z, \mu_Z)$ be two functions.*

(1) *If $g \circ f$ is (μ_Z, μ_X)-b-open and f is (μ_X, μ_Y)-continuous surjective then g is (μ_Y, μ_Z)-b-open.*
(2) *If $g \circ f$ is (μ_Z, μ_X)-open and g is (b, μ_Y)-continuous injective then f is (μ_X, μ_Y)-b-open.*

Proof. 1. Let A be μ_Y-open in Y. Then $f^{-1}(A)$ is μ_X-open in X. By hypothesis, $(g \circ f)(f^{-1}(A)) = g(f(f^{-1}(A))) = g(A)$ is μ_Z-b-open in Z. Hence g is (μ_Y, μ_Z)-b-open.

2. Let B be μ_X-open in X. Then by hypothesis, $g(f(B))$ is μ_Z-open in Z and $g^{-1}(g(f(B))) = f(B)$ is μ_Y-b-open in Y. Hence f is (μ_X, μ_Y)-b-open.

Theorem 3.6. *For any bijective function $f : (X, \mu_X) \to (Y, \mu_Y)$, the following are equivalent :*

(1) *$f^{-1} : (Y, \mu_Y) \to (X, \mu_X)$ is (b, μ_X)-continuous.*
(2) *f is (μ_X, μ_Y)-b-open.*
(3) *f is (μ_X, μ_Y)-b-closed.*

Proof. 1. $\Rightarrow$ 2. Let A be a μ_X-open set in X. Since f^{-1} is (b, μ_X)-continuous, $(f^{-1})^{-1}(A) = f(A)$ is a μ_Y-b-open set in Y. Thus f is (μ_X, μ_Y)-b-open.

2. $\Rightarrow$ 3. Let B be a μ_X-closed set in X. Then $X - B$ is μ_X-open set in X and by 2, $f(X - B)$ is μ_Y-b-open set in Y. Since f is bijective, $f(X - B) = Y - f(B)$. Hence $f(B)$ is μ_Y-b-closed in Y. Thus f is (μ_X, μ_Y)-b-closed.

3. $\Rightarrow$ 1. Let f be a (μ_X, μ_Y)-b-closed and B be μ_X-closed set in X. Since f is bijective, $(f^{-1})^{-1}(B) = f(B)$ is a μ_Y-b-open set in Y. By Theorem 2.4, f^{-1} is (b, μ_X)-continuous.

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