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GENERAL FIXED POINT THEOREMS FOR MULTIVALUED MAPPINGS OF LATIF - BEG TYPE IN G - METRIC SPACES

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Abstract. In this paper two general fixed point theorems for multivalued mappings in G - metric spaces satisfying a new type of implicit relation which generalizes and improves Theorem 2.1 [1] are proved.

1. Introduction and preliminaries

In [2], [3] Dhage introduced a new class of generalized metric space, named D - metric spaces. Mustafa and Sims [5], [6] proved that most of the claims concerning the fundamental topological structures on D - metric spaces are incorrect and introduced an appropriate notion of generalized metric space, named G - metric space.

In fact, Mustafa, Sims and other authors [7], [8], [9], [10] studied many fixed point results for self mappings in G - metric spaces under certain conditions. In [1], the authors extend the results by [17] for G - metric spaces.

In [11], [12] and in other papers, Popa initiated the study of fixed points for mappings satisfying implicit relations.

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Actually, the method is used in the study of fixed points in metric spaces, symmetric spaces, quasi - metric spaces, ultra - metric spaces, reflexive spaces, convex metric spaces, compact metric spaces, paracompact metric spaces in two and three metric spaces, for single valued mappings, hybrid pairs of mappings and set valued mappings.

Quite recently, the method is used in the study of fixed points for mappings satisfying a contractive/extensive condition of integral type, in fuzzy metric spaces and intuitionistic metric spaces. There exists a vast literature in this topic which cannot be completely cited here. The method unified different types of contractive and extensive conditions. Some of the proofs of fixed point theorems are more simple.

Also, this method allows the study of local and global properties of fixed point structures.

The study of fixed points in G - metric spaces using implicit relations is initiated in [13], [14], [15], [16].

2. Preliminaries

Definition 2.1 ([6])**.** Let X be a nonempty set and $G : X^3 \to \mathbb{R}_+$ be a function satisfying the following properties:

$(G_1) : G(x, y, z) = 0$ if $x = y = z$,

$(G_2) : 0 < G(x, y, y)$ for all $x, y \in X$ with $x \neq y$,

$(G_3) : G(x, x, y) \leq G(x, y, z)$ for all $x, y, z \in X$ with $z \neq y$,

$(G_4) : G(x, y, z) = G(y, z, x) = G(z, x, y)$... (symmetry in all three variables),

$(G_5) : G(x, y, z) \leq G(x, a, a) + G(a, y, z)$ for all $x, y, z, a \in X$ (rectangle inequality).

The function G is called a G - metric on X and the pair (X, G) is called a G - metric space.

Note that if $G(x, y, z) = 0$ then $x = y = z$ [6].

Definition 2.2 ([6])**.** Let (X, G) be a G - metric space. A sequence (x_n) in X is said to be:

a) G - convergent if for $\varepsilon > 0$, there exists $x \in X$ and $k \in \mathbb{N}$ such that for all $m, n \geq k$, $G(x, x_n, x_m) < \varepsilon$.

b) G - Cauchy if for $\varepsilon > 0$, there exists $k \in \mathbb{N}$ such that for all $m, n, p \geq k$, $G(x_n, x_m, x_p) < \varepsilon$, that is $G(x_n, x_m, x_p) \to 0$ as $m, n, p \to \infty$.

c) A G - metric space (X, G) is said to be G - complete if every G - Cauchy sequence in X is G - convergent.

Lemma 2.3 ([6]). *Let (X,G) be a G - metric space. Then, the following properties are equivalent:*

1) the sequence (x_n) is G - convergent to x;
2) $G(x,x_n,x_n)\to 0$ as $n\to\infty$;
3) $G(x_n,x,x)\to 0$ as $n\to\infty$;
4) $G(x_m,x_n,x)\to 0$ as $n,m\to\infty$.

Lemma 2.4 ([6]). *Let (X,G) be a G - metric space. Then, following properties are equivalent:*

1) (x_n) is G - Cauchy;
2) for $\varepsilon>0$, there exists $k\in\mathbb{N}$ such that $G(x_n,x_m,x_m)<\varepsilon$ for $n,m\geq k$.

Lemma 2.5 ([6]). *Let (X,G) be a G - metric space. Then, the function $G(x,y,z)$ is jointly continuous in all three of its variables.*

Let (X,G) be a G - metric space. We denote by $\mathcal{P}(X)$ the family of all nonempty subsets of X, and by $\mathcal{P}_{Cl}(X)$ the family of all nonempty closed subsets of X.

A point $x\in X$ is called a fixed point for a multivalued mapping $T:(X,G)\to\mathcal{P}_{Cl}(X)$ if $x\in Tx$. The collection of all fixed points of T is denoted by $Fix(T)$.

Quite recently, the following theorem is proved in [1].

Theorem 2.6. *Let (X,G) be a complete G - metric space and $T:(X,G)\to\mathcal{P}_{Cl}(X)$ be a multivalued mapping. If for each $x,y\in X$, $u_x\in T(x)$, there exists $u_y\in T(y)$ such that*

$$
\begin{aligned}
G(u_x,u_y,u_y)\leq h\max\{&G(x,y,y),G(x,u_x,u_x),\\
&G(y,u_y,u_y),\frac{G(x,u_y,u_y)+G(y,u_x,u_x)}{2}\},
\end{aligned}
\tag{2.1}
$$

where $h\in[0,1)$. Then T has a fixed point.

Theorem 2.6 generalize and extend the results from Theorem 3.1 [17].

The purpose of this paper is to prove two general fixed point theorems for multivalued mappings in G - metric spaces satisfying a new type of implicit relation, which generalizes and improves Theorem 2.6.

3. Implicit relations

Let $\mathfrak{F}_c$ be the set of all continuous multifunctions $F(t_1,...,t_6):\mathbb{R}_+^6\to\mathbb{R}$ satisfying the following conditions:

Definition 3.1. (F_1) : F is nonincreasing in variables t_3, t_4, t_5.

(F_2) : there exists $h \in [0,1)$ and $g \geq 0$ such that for all $u, v, w \geq 0$ with $F(u, v, v+w, u+w, u+v+w, w) \leq 0$ we have $u \leq hv + gw$.

Example 3.2. $F(t_1, ..., t_6) = t_1 - p\max\{t_2, t_3, t_4, \frac{t_5+t_6}{2}\}$, *where* $p \in [0,1)$.

(F_1) : Obviously.

(F_2) : Let $u, v, w \geq 0$ be and $F(u, v, v+w, u+w, u+v+w, w) = u - p\max\{v, v+w, u+w, \frac{u+v+2w}{2}\} \leq 0$. If $u > v$, then $u \leq p(u+w)$. If $w = 0$, then $u(1-p) \leq 0$, a contradiction. Hence $u \leq v$ which implies $u \leq p(v+w) = hv + gw$, where $0 \leq h = p < 1$ and $g = p \geq 0$.

Example 3.3. $F(t_1, ..., t_6) = t_1 - p\max\{t_2, \frac{t_3+t_4}{2}, \frac{t_5+t_6}{2}\}$, *where* $p \in [0,1)$.

The proof is similar to the proof of Example 3.2.

Example 3.4. $F(t_1, ..., t_6) = t_1 - p\max\{t_2, t_3, t_4, t_5, t_6\}$, *where* $p \in \left[0, \frac{1}{2}\right)$.

(F_1) : Obviously.

(F_2) : Let $u, v, w \geq 0$ be and $F(u, v, v+w, u+w, u+v+w, w) = u - p\max\{v, v+w, u+w, u+v+w, w\} = u - p(u+v+w) \leq 0$ which implies $u \leq hv + gw$, where $0 \leq h = \frac{p}{1-p} < 1$ and $g = \frac{p}{1-p} \geq 0$.

Example 3.5. $F(t_1, ..., t_6) = t_1 - at_2 - bt_3 - ct_4 - dt_5 - et_6$, *where* $a, b, c, d, e \geq 0$ *and* $a + b + c + 2d + e < 1$.

(F_1) : Obviously.

(F_2) : Let $u, v, w \geq 0$ be and $F(u, v, v+w, u+w, u+v+w, w) = u - av - b(v+w) - c(u+w) - d(u+v+w) - ew \leq 0$. Then $u \leq hv + gw$, where $0 \leq h = \frac{a+b+d}{1-(c+d)} < 1$ and $g = \frac{b+c+d+e}{1-(c+d)} \geq 0$.

Example 3.6. $F(t_1, ..., t_6) = t_1 - at_2 - b\max\{t_3, t_4\} - c\max\{t_2, t_5, t_6\}$, *where* $a, b, c \geq 0$ *and* $a + b + 2c < 1$.

(F_1) : Obviously.

(F_2) : Let $u, v, w \geq 0$ be and $F(u, v, v+w, u+w, u+v+w, w) = u - av - b\max\{v+w, u+w\} - c\max\{v, u+v+w, w\} \leq 0$. If $u > v$, then $u - au - b(u+w) - c(2u+w) \leq 0$. If $w = 0$, then $u[1-(a+b+2c)] \leq 0$, a contradiction. Hence $u \leq v$ which implies $u \leq hv + gw$, where $0 \leq h = \frac{a+b+c}{1-c} < 1$ and $g = \frac{b+c}{1-c} \geq 0$.

Example 3.7. $F(t_1,...,t_6) = t_1 - at_2 - b(t_3+t_4) - c\min\{t_5,t_6\}$, *where* $a,b,c \geq 0$ *and* $a+2b<1$.

(F_1) : Obviously.

(F_2) : Let $u,v,w \geq 0$ be and $F(u,v,v+w,u+w,u+v+w,w) = u - av - b(u+v+2w) - cw \leq 0$, which implies $u \leq hv + gw$, where $0 \leq h = \frac{a+b}{1-b} < 1$ and $g = \frac{2b+c}{1-b} \geq 0$.

Example 3.8. $F(t_1,...,t_6) = t_1 - at_2 - b\max\{2t_4+t_6, 2t_4+t_5, t_3+t_5+t_6\}$, *where* $a,b \geq 0$ *and* $a+5b<1$.

(F_1) : Obviously.

(F_2) : Let $u,v,w \geq 0$ be and $F(u,v,v+w,u+w,u+v+w,w) = u - av - b\max\{2u+3w, 3u+v+3w, u+2v+3w\} \leq 0$, which implies $u - av - b(3u+2v+3w) \leq 0$. Therefore, $u \leq hv + gw$, where $0 \leq h = \frac{a+2b}{1-3b} < 1$ and $g = \frac{3b}{1-3b} \geq 0$.

Example 3.9. $F(t_1,...,t_6) = t_1^2 + \frac{t_1}{1+t_5+t_6} - (at_2^2 + bt_3^2 + ct_4^2)$, *where* $a,b,c \geq 0$ *and* $a+b+c < \frac{1}{4}$.

(F_1) : Obviously.

(F_2) : Let $u,v,w \geq 0$ be and $F(u,v,v+w,u+w,u+v+w,w) = u^2 + \frac{u}{u+v+2w} - [au^2 + b(v+w)^2 + c(u+w)^2] \leq 0$, which implies $u^2 - [au^2 + b(v+w)^2 + c(u+w)^2] \leq 0$. Then $u^2 - (a+b+c)(u+v+w)^2 \leq 0$. Hence $u \leq \sqrt{a+b+c}(u+v+w)$. Therefore, $u \leq hv + gw$, where $0 \leq h = \frac{\sqrt{a+b+c}}{1-\sqrt{a+b+c}} < 1$ and $g = \frac{\sqrt{a+b+c}}{1-\sqrt{a+b+c}} \geq 0$.

Example 3.10. $F(t_1,...,t_6) = t_1^2 - p\max\{t_2^2, t_3t_4, t_5t_6\}$, *where* $0 \leq p < \frac{1}{4}$.

(F_1) : Obviously.

(F_2) : Let $u,v,w \geq 0$ be and $F(u,v,v+w,u+w,u+v+w,w) = u^2 - p\max\{v^2, (v+w)(u+w), (u+v+w)w\} \leq 0$, which implies $u^2 - p(u+v+w)^2 \leq 0$. Hence, $u \leq hv + gw$, where $0 \leq h = \frac{\sqrt{p}}{1-\sqrt{p}} < 1$ and $g = \frac{\sqrt{p}}{1-\sqrt{p}} \geq 0$.

Example 3.11. $F(t_1,...,t_6) = t_1 - p\max\{t_2, t_3, t_4, \frac{2t_4+t_6}{3}, \frac{2t_4+t_3}{3}, \frac{t_5+t_6}{3}\}$, *where* $0 \leq p < 1$.

(F_1) : Obviously.

(F_2) : Let $u,v,w \geq 0$ be and $F(u,v,v+w,u+w,u+v+w,w) = u - p\max\{v, v+w, u+w, \frac{2u+3w}{3}, \frac{2u+v+3w}{3}, \frac{u+v+2w}{3}\} = u - p\max\{u+

$w, v+w, \frac{2u+v+3w}{3}\} \leq 0$. If $u > v$, then $u - p(u+w) \leq 0$. If $w = 0$ we have $u(1-p) \leq 0$, a contradiction. Hence $u \leq v$ which implies $u \leq p(v+w)$. Then $u \leq hv + gw$, where $0 \leq h = p < 1$ and $g = p \geq 0$.

4. Main results

Theorem 4.1. *Let (X, G) be a complete G - metric space and $T : (X, G) \to \mathcal{P}_{Cl}(X)$ such that for each $x \in X$, $u_x \in T(x)$ and for each $y \in X$ there exists $u_y \in T(y)$ satisfying the inequality*

$$\begin{gathered} \phi(G(u_x, u_y, u_y), G(x, y, y), G(x, u_x, u_x), \\ G(y, u_y, u_y), G(x, u_y, u_y), G(y, u_x, u_x)) \leq 0 \end{gathered} \tag{4.1}$$

where $\phi \in \mathfrak{F}_c$. Then $Fix(T) \neq \phi$.

Proof. Let $x_0 \in X$ be arbitrary and $x_1 \in T(x_0)$. It follows that there exists $x_2 \in T(x_1)$ such that

$$\begin{gathered} \phi(G(x_1, x_2, x_2), G(x_0, x_1, x_1), G(x_0, x_1, x_1), \\ G(x_1, x_2, x_2), G(x_0, x_2, x_2), 0) \leq 0 \end{gathered}$$

By (G_5) and (F_1) we obtain

$$\begin{gathered} \phi(G(x_1, x_2, x_2), G(x_0, x_1, x_1), G(x_0, x_1, x_1), \\ G(x_1, x_2, x_2), G(x_0, x_1, x_1) + G(x_1, x_2, x_2), 0) \leq 0. \end{gathered}$$

Since $\phi \in \mathfrak{F}_c$ we have

$$G(x_1, x_2, x_2) \leq hG(x_0, x_1, x_1).$$

Similarly, by (4.1) we have that there exists $x_3 \in T(x_2)$ such that

$$\begin{gathered} \phi(G(x_2, x_3, x_3), G(x_1, x_2, x_2), G(x_1, x_2, x_2), \\ G(x_2, x_3, x_3), G(x_1, x_3, x_3), 0) \leq 0. \end{gathered}$$

By (G_5) and (F_1) we obtain

$$\begin{gathered} \phi(G(x_2, x_3, x_3), G(x_1, x_2, x_2), G(x_1, x_2, x_2), \\ G(x_2, x_3, x_3), G(x_1, x_2, x_2) + G(x_2, x_3, x_3), 0) \leq 0. \end{gathered}$$

Since $\phi \in \mathfrak{F}_c$ we have

$$G(x_2, x_3, x_3) \leq hG(x_1, x_2, x_2) \leq h^2 G(x_0, x_1, x_1).$$

By induction we obtain a sequence (x_n) with $x_0 \in X$, $x_n \in T(x_{n-1})$, $n = 1, 2, ...$ and $G(x_n, x_{n+1}, x_{n+1}) \leq h^n G(x_0, x_1, x_1)$. For $m > n \geq 1$, repeating use of property (G_5) gives:

$$\begin{aligned}G(x_n, x_m, x_m) &\leq G(x_n, x_{n+1}, x_{n+1}) + G(x_{n+1}, x_{n+2}, x_{n+2}) + ...+ \\ &+ G(x_{m-1}, x_m, x_m) \\ &\leq [h^n + h^{n+1} + ... + h^{m-1}]G(x_0, x_1, x_1) \\ &\leq \frac{h^n}{1-h} G(x_0, x_1, x_1)\end{aligned}$$

and so $G(x_n, x_m, x_m) \to 0$ as $n, m \to \infty$. Hence (x_n) is a G - Cauchy sequence in X. Since (X, G) is G - complete, then (x_n) is G - convergent.

Let $x = \lim_{n\to\infty} x_n$. We prove that x is a fixed point of T. Since $x_n \in T(x_{n-1})$, there exists $u_n \in T(x)$ such that

$$\phi(G(x_n, u_n, u_n), G(x_{n-1}, x, x), G(x_{n-1}, x, x), \\ G(x, u_n, u_n), G(x_{n-1}, u_n, u_n), G(x, x_n, x_n)) \leq 0$$

for each $n = 1, 2, ...$

By (F_1) and (G_5) we obtain

$$\phi(G(x_n, u_n, u_n), G(x_{n-1}, x, x), G(x_{n-1}, x, x) + G(x, x_n, x_n), \\ G(x, x_n, x_n) + G(x_n, u_n, u_n), G(x_{n-1}, x, x) + G(x, x_n, x_n)+ \\ +G(x_n, u_n, u_n), G(x, x_n, x_n)) \leq 0.$$

Since $\phi \in \mathfrak{F}_c$ we obtain

$$G(x_n, u_n, u_n) \leq hG(x_{n-1}, x, x) + gG(x, x_n, x_n).$$

On the other hand, by (G_5) we have

$$\begin{aligned}G(x, u_n, u_n) &\leq G(x, x_n, x_n) + G(x_n, u_n, u_n) \\ &\leq G(x, x_n, x_n) + hG(x_{n-1}, x, x) + gG(x, x_n, x_n).\end{aligned}$$

Letting n tend to infinity we obtain $x = \lim_{n\to\infty} u_n$. Since $u_n \in T(x)$ for all $n \in \mathbb{N}$ and $T(X)$ is closed it follows that $x \in T(x)$. Hence $Fix(T) \neq \phi$. □

Corollary 4.2. *Theorem 2.6.*

Proof. The proof it follows by Theorem 4.1 and Example 3.2. □

Theorem 4.3. *Let (X, G) be a complete G - metric space and $T : (X, G) \to \mathcal{P}_{Cl}(X)$ such that for each $x \in X$, $u_x \in T(x)$ and for each $y \in X$ there exists $u_y \in T(y)$ satisfying the inequality*

$$\phi(G(u_x, u_x, u_y), G(x, x, y), G(x, x, u_x), \\ G(y, y, u_y), G(x, x, u_y), G(y, y, u_x)) \leq 0 \tag{4.2}$$

where $\phi \in \mathfrak{F}_c$. *Then* $Fix(T) \neq \phi$.

Proof. Let $x_0 \in X$ be arbitrary and $x_1 \in T(x_0)$. It follows that there exists $x_2 \in T(x_1)$ such that

$$\phi(G(x_1,x_1,x_2),G(x_0,x_0,x_1),G(x_0,x_0,x_1),\\ G(x_1,x_1,x_2),G(x_0,x_0,x_2),0) \leq 0$$

By (G_5) and (F_1) we obtain

$$\phi(G(x_1,x_1,x_2),G(x_0,x_0,x_1),G(x_0,x_0,x_1),\\ G(x_1,x_1,x_2),G(x_0,x_0,x_1)+G(x_1,x_1,x_2),0) \leq 0.$$

Since $\phi \in \mathfrak{F}_c$ we have

$$G(x_1,x_1,x_2) \leq hG(x_0,x_0,x_1).$$

Similarly, by (4.2) we have that there exists $x_3 \in T(x_2)$ such that

$$\phi(G(x_2,x_2,x_3),G(x_1,x_1,x_2),G(x_1,x_1,x_2),\\ G(x_2,x_2,x_3),G(x_1,x_1,x_3),0) \leq 0.$$

By (G_5) and (F_1) we have

$$\phi(G(x_2,x_2,x_3),G(x_1,x_1,x_2),G(x_1,x_1,x_2),\\ G(x_2,x_2,x_3),G(x_1,x_1,x_2)+G(x_2,x_2,x_3),0) \leq 0.$$

Since $\phi \in \mathfrak{F}_c$ we have

$$G(x_2,x_2,x_3) \leq hG(x_1,x_1,x_2) \leq h^2 G(x_0,x_0,x_1).$$

By induction we obtain a sequence $(x_n)_{n\in\mathbb{N}}$ with $x_0 \in X$, $x_n \in T(x_{n-1})$, $n = 1,2,...$ and $G(x_n,x_n,x_{n+1}) \leq h^n G(x_0,x_0,x_1)$.

As in Theorem 4.1, (x_n) is a G - convergent.

Let $x = \lim_{n\to\infty} x_n$. We prove that x is a fixed point of T.

From $x_n \in T(x_{n-1})$, we have by the hypothesis that there exists $u_n \in T(x)$ such that

$$\phi(G(x_n,x_n,u_n),G(x_{n-1},x_{n-1},x),G(x_{n-1},x_{n-1},x),\\ G(x,x,u_n),G(x_{n-1},x_{n-1},u_n),G(x,x,x_n)) \leq 0,$$

for each $n = 1,2,...$, which implies by (F_1) and (G_5) that

$$\phi(G(x_n,x_n,u_n),G(x_{n-1},x_{n-1},x),G(x_{n-1},x_{n-1},x)+G(x,x,x_n),\\ G(x,x,x_n)+G(x_n,x_n,u_n),G(x_{n-1},x_{n-1},x)+G(x,x,x_n)+\\ +G(x_n,x_n,u_n),G(x,x,x_n)) \leq 0.$$

Since $\phi \in \mathfrak{F}_c$ then

$$G(x_n,x_n,u_n) \leq hG(x_{n-1},x_{n-1},x) + gG(x,x,x_n).$$

On the other hand, by (G_5) we have

$$\begin{aligned} G(x,x,u_n) &\leq G(x,x,x_n)+G(x_n,x_n,u_n) \\ &\leq G(x,x,x_n)+hG(x_{n-1},x_{n-1},x)+gG(x,x,x_n). \end{aligned}$$

Letting n tend to infinity we obtain $x=\lim_{n\to\infty} u_n$. Since $u_n \in T(x)$ for all $n \in \mathbb{N}$ and $T(X)$ is closed it follows that $x \in T(x)$. Hence $Fix(T) \neq \phi$. □

Corollary 4.4. *Let (X,G) be a complete G - metric space and $T : (X,G) \to \mathcal{P}_{Cl}(X)$ be a multivalued mapping. If for each $x,y \in X$, $u_x \in T(x)$ there exists $u_y \in T(y)$ such that*

$$\begin{aligned} G(u_x,u_x,u_y) \leq h \max\{&G(x,x,y), G(x,x,u_x), \\ &G(y,y,u_y), \frac{G(x,x,u_y)+G(y,y,u_x)}{2}\} \leq 0 \end{aligned} \tag{4.3}$$

where $h \in [0,1)$. Then T has a fixed point.

Proof. The proof it follows from Theorem 4.3 and Example 3.2. □

Remark 4.5. *By Theorems 4.1, 4.3 and Examples 3.3 - 3.11 we obtain new particular results.*

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