"Vasile Alecsandri" University of Bacău
Faculty of Sciences
Scientific Studies and Research
Series Mathematics and Informatics
Vol. 24(2014), No. 1, 129-139

ON {CLAW,ANTENNA,NET}-FREE GRAPHS

MIHAI TALMACIU

Abstract. In this article, we give a characterization of $\{claw, antenna, net\}$-free graphs, a characterization of claw-free graphs, using weakly decomposition. Also, we give a $O(n(n+m))$ recognition algorithm for $\{claw, antenna, net\}$-free graphs, but using weakly decomposition.

During the last three decades, different types of decompositions have been processed in the field of graph theory. Among these we mention: decompositions based on the additivity of some characteristics of the graph, decompositions where the adjacency law between the subsets of the partition is known, decompositions where the subgraph induced by every subset of the partition must have predeterminate properties, as well as combinations of such decompositions.

In various problems in graph theory, for example in the construction of recognition algorithms, frequently appears the so-called weakly decomposition of graphs.

1. Introduction

During the last decades, numerous studies have been undertaken on the classes of net-free graphs, claw-free graphs, the relationship between them.

Keywords and phrases: net-free graphs, claw-free graphs, antenna-free graphs, asteroidal triple-free graphs, weakly decomposition, recognition algorithm.
(2010)Mathematics Subject Classification: 05C85, 68R10

On www.graphclasses.org/classes/AUTO-4.html it is said that net-free graph recognition is in polynomial time.

The interval graphs [19], permutation graphs [14] and co- comparability graphs [16] have a linear structure. Each of these classes is a subfamily of the asteroidal triple graphs (AT-free graphs, for short). An independent set of three vertices is called an asteroidal triple if between any pair in the triple there exists a path that avoids the neighborhood of the third. AT-free graphs were introduced by Lekkerkerker and Boland [19]. Corneil, Olariu and Stewart showed a number of results on the linear structure of AT-free [7, 8, 9].

A maximal subclass of a class of net-free graphs is the class (claw,net)-free graphs (CN-free graphs, for short). Also note that CN-free graphs are exactly the Hamiltonian-hereditary graphs[12] (was cited in [3]). CN-free graphs turn out to be closely related to AT-free graphs form their structure properties [3]. There are, however, few results about the structure of these graphs [3]. In [3] the authors give results on the linear and circular structure of CN-free graphs. AT-free graphs can be generalized in a manner obvious to admit circular structure [3]. CN-free graphs were introduced by Duffus [13]. Although CN-free graphs seems to be quite restrictive, it contains a couple of families of graphs that are interesting in their own right.

Throughout this paper, $G = (V, E)$ is a connected, finite and undirected graph [2], without loops and multiple edges, having $V = V(G)$ as the vertex set and $E = E(G)$ as the set of edges. $\overline{G}$ (or $c-G$) is the complement of G. If $U \subseteq V$, by $G(U)$ we denote the subgraph of G induced by U. By $G-X$ we mean the subgraph $G(V-X)$, whenever $X \subseteq V$, but we simply write $G-v$, when $X = \{v\}$. If $e = xy$ is an edge of a graph G, then x and y are adjacent, while x and e are incident, as are y and e. If $xy \in E$, we also use $x \sim y$, and $x \not\sim y$ whenever x, y are not adjacent in G. A vertex $z \in V$ distinguishes the non-adjacent vertices $x, y \in V$ if $zx \in E$ and $zy \notin E$. If $A, B \subset V$ are disjoint and $ab \in E$ for every $a \in A$ and $b \in B$, we say that A, B are *totally adjacent* and we denote by $A \sim B$, while by $A \not\sim B$ we mean that no edge of G joins some vertex of A to a vertex from B and, in this case, we say that A and B are *non-adjacent*.

The *neighbourhood* of the vertex $v \in V$ is the set $N_G(v) = \{u \in V : uv \in E\}$, while $N_G[v] = N_G(v) \cup \{v\}$; we simply write $N(v)$ and $N[v]$, when G appears clearly from the context. The neighbourhood of the vertex v in the complement of G will be denoted by $\overline{N}(v)$.

If $N[v] = V$, then v is called a *dominating vertex* in G. If $D \subset V$ and every vertex from $V - D$ has at least one neighbour in D, then D is called a *dominating set* of G. If $D \subset V$ and $\overline{N}_G(D) \neq \emptyset$, then D is a *non-dominating set* of G.

The neighbourhood of $S \subset V$ is the set $N(S) = \cup_{v \in S} N(v) - S$ and $N[S] = S \cup N(S)$. A *clique* is a subset Q of V with the property that $G(Q)$ is complete. The *clique number* of G, denoted by $\omega(G)$, is the size of the maximum clique.

By P_n, C_n, K_n we mean a chordless path on $n \geq 3$ vertices, a chordless cycle on $n \geq 3$ vertices, and a complete graph on $n \geq 1$ vertices, respectively.

A graph is called *triangulated* if it does not contain chordless cycles having the length greater or equal to four.

A *antenna* graph is isomorphic to $G = (\{a, b, c, d.e.f\}, \{af, fd, fe, db, ec, bc\})$.

Let F denote a family of graphs. A graph G is called F-*free* if none of its subgraphs is in F. The *Zykov sum* of the graphs G_1, G_2 is the graph $G = G_1 + G_2$ having:

$$V(G) = V(G_1) \cup V(G_2),$$
$$E(G) = E(G_1) \cup E(G_2) \cup \{uv : u \in V(G_1), v \in V(G_2)\}.$$

The structure of the paper is the following. In Section 2 we recall the notion of weakly decomposition, we recall a characterization of net-free graphs, a characterization of claw-free graphs, using weakly decomposition. In Section 3 we establish a characterization of $\{claw, antenna, net\}$-free graphs, we give a recognition algorithm for $\{claw, antenna, net\}$-free graphs, using weakly decomposition.

2. Preliminaries

At first, we recall the notions of weakly component and weakly decomposition.

When searching for recognition algorithms, frequently appears a type of partition for the set of vertices in three classes A, B, C, which we call a *weakly decomposition*, such that: A induces a connected subgraph, C is totally adjacent to B, while C and A are totally non-adjacent.

Definition 1. ([?], [?], [?]) *A set $A \subset V(G)$ is called a weakly set of the graph G if $N_G(A) \neq V(G) - A$ and $G(A)$ is connected. If A is a weakly set, maximal with respect to set inclusion, then $G(A)$ is called a weakly component. For simplicity, the weakly component $G(A)$ will be denoted with A.*

Definition 2. ([?], [?], [?]) *Let $G = (V, E)$ be a connected and non-complete graph. If A is a weakly set, then the partition $\{A, N(A), V - A \cup N(A)\}$ is called a weakly decomposition of G with respect to A.*

Below we remind a characterization of the weakly decomposition of a graph.

The name of "*weakly component*" is justified by the following result.

Theorem 1. ([?], [?], [?]) *Every connected and non-complete graph $G = (V, E)$ admits a weakly component A such that $G(V - A) = G(N(A)) + G(\overline{N}(A))$.*

Theorem 2. ([?], [?]) *Let $G = (V, E)$ be a connected and non-complete graph and $A \subset V$. Then A is a weakly component of G if and only if $G(A)$ is connected and $N(A) \sim \overline{N}(A)$.*

The next result, that follows from Theorem 1, ensures the existence of a weakly decomposition in a connected and non-complete graph.

Corollary 1. *If $G = (V, E)$ is a connected and non-complete graph, then V admits a weakly decomposition (A, B, C), such that $G(A)$ is a weakly component and $G(V - A) = G(B) + G(C)$.*

Theorem 2 provides an $O(n + m)$ algorithm for building a weakly decomposition for a non-complete and connected graph.

Algorithm for the weakly decomposition of a graph ([?])

Input: A connected graph with at least two nonadjacent vertices, $G = (V, E)$.

Output: A partition $V = (A, N, R)$ such that $G(A)$ is connected, $N = N(A)$, $A \not\sim R = \overline{N}(A)$.

begin
  $A :=$ any set of vertices such that
  $A \cup N(A) \neq V$
  $N := N(A)$
  $R := V - A \cup N(A)$
  while ($\exists n \in N$, $\exists r \in R$ such that $nr \notin E$) *do*
    begin
      $A := A \cup \{n\}$

$N := (N - \{n\}) \cup (N(n) \cap R)$
$R := R - (N(n) \cap R)$
end
end

The notion of weakly decomposition (a partition of the set of vertices in three classes A, B, C such that A induces a connected graph and C is totally adjacent to B and totally non-adjacent to A) and the study of its properties allow us to obtain several important results such as: characterization of cographs, $\{P_4, C_4\}$-free and paw-free graphs.

A new characterization of net-free graphs, using weakly decomposition, is given below.

Theorem 3. [24] *Let $G = (V, E)$ be a connected and non-complete graph. Let (A, N, R) be a weakly decomposition with $G(A)$ a weakly component. $G = (V, E)$ is net-free if and only if:*
i) does not exist P_4 in $G(A)$ and n in N such that n is adjacent with the middle vertices of the P_4 specified;
ii) (does not exist P_4, with extremities in A and the middle vertices in N) or (does not exist t in N such that his neighbors t are not in P_4 specified);
iii) $G(V - R)$, $G(V - A)$ are net-free.

Some interesting properties of claw-free graphs have been established in ([1], [6], [15], [18], [20], [21]).

In [4] the authors consider the algorithmic problem of finding a Hamiltonian path or a Hamiltonian cycle efficiently.

In what follows we recall a characterization of the claw-free graphs. A similar result is found in ([**?**])

Theorem 4. [24] *Let $G = (V, E)$ be a connected and non-complete graph. Let (A, N, R) a weakly decomposition with $G(A)$ a weakly component. $G = (V, E)$ is claw-free if and only if:*
i) R and $N(n) \cap A$ are cliques, $\forall n \in N$
ii) $G(V - R)$, $G(V - A)$ are claw-free.

3. A new characterization of $\{claw, antenna, net\}$-free graphs using the weakly decomposition

A graph is *chordal* if it contains no induced C_k, $k \geq 4$. A graph is *nearly chordal* if for each of its vertices, the subgraph induced by the set of its non-neighbors is a chordal graph. More generally, if P is a graph property then a graph is *nearly* P if for each of its vertices, the subgraph induced by the set of its nonneighbors has the property P.

In [5] is given:
Connected (claw, antenna, net)-free graphs are nearly chordal.

In $www.graphclasses.org/classes/problem_Recognition.html$ it is said that there are polynomial algorithms for recognition $(S_3, claw, net)$-free graph recognition. Also, in $www.graphclasses.org/classes/gc_137.html$ it is said that there are polynomial algorithms for recognition $(claw, net)$-free graph recognition.

A new characterization of $\{claw, antenna, net\}$-free graphs, using weakly decomposition, is given below.

Theorem 5. *Let $G = (V, E)$ be a connected and non-complete graph. Let (A, N, R) be a weakly decomposition with $G(A)$ a weakly component. $G = (V, E)$ is $\{claw, antenna, net\}$-free if and only if:*
i) $G(V - R)$, $G(V - A)$ are $\{claw, antenna, net\}$-free;
ii) R and $N(n) \cap A$ are cliques, $\forall n \in N$;
iii) does not exist P_4 in $G(A)$ and n in N such that n is adjacent with the middle vertices of the P_4 specified;
iv) (does not exist P_4, with extremities in A and the middle vertices in N) or (does not exist t in N such that his neighbors t are not in P_4 specified);
v) (does not exist P_4, with extremities in A and the middle vertices in N) or (does not exist t in N such that his neighbors t are an extremity of P_4 specified);
vi) (does not exist P_4 in $G(A \cup N)$, with an extremity in N and rest the vertices in N) or (does not exist n in N such that n is adjacent with the middle vertices of the P_4 specified);
vii) $A \cup N - N(n) \cap A$ is chordal, $\forall n \in N$;
viii) $A - N(n) \cap A$ is chordal, $\forall n \in N$.

Proof. Let G be {claw, net, antenna}-free. Since the property of being {claw, net, antenna}-free is hereditary as follows $G(V - A)$ and $G(V - R)$ {claw,antenna,net}-free graphs, so i) holds.

If there would be $r_1, r_2 \in R$ such that $r_1 r_2 \notin E$, because $R \sim N$, $\forall n \in N$ and $a \in N(n) \cap A$ ($N(n) \cap A \neq \emptyset, \forall n \in N$ according to his N), $G(\{a, n, r_1, r_2\})$ is isomorphic to *claw*. If there would be $\exists n \in N$ such that $\exists a_1, a_2 \in A \cap N_G(n)$ with $a_1 a_2 \notin E$, because $R \sim N$, $\forall r \in R$, $G(\{r, n, a_1, a_2\})$ is isomorphic to *claw*. So ii) holds.

If $\exists n \in N$, $\exists P_4 \subseteq G(A)$ such that n is adjacent to the middle vertices in P_4, so $\exists P_4 : a, b, c, d$ with $a, b, c, d \in A$, $ab, bc, cd \in E$, $ac, ad, bd \notin E$ and $nb, nc \in E$, then, because $N \sim R$, it follows that $\forall r \in R$: $G(\{a, b, c, d, n, r\})$ is net, a contradiction. So iii) holds.

If $\exists P_4 : ab, bc, cd$ with $a, d \in A$, $b, c \in N$ and $\exists t \in N$ with $ta, tb, tc, td \notin E$ then $G(\{a, b, c, d, r, t\})$ is net $\forall r \in R$, a contradiction. So iv) holds.

If ($\exists P_4 : a, b, c, d$, with extremities a, b in A and the middle vertices in N) and ($\exists t$ in N such that his neighbors t are an extremity of P_4 specified) then, $\forall r \in R$, $G(\{a, b, c, d, t, r\})$ is antenna, a contradiction. So v) holds.

If ($\exists P_4 : a, b, c, d$, with an extremity d in N and the rest vertices in A) and ($\exists n$ in N such that $nb, nc \in E$) then, $\forall r \in R$, $G(\{a, b, c, d, n, r\})$ is antenna, a contradiction. So vi) holds.

If ($\exists C_4 : a, b, c, d$ in $G(A \cup N)$, with two adjacent vertices in A and rest the vertices in N) and ($\exists t$ in N such that his neighbors t are not in C_4 specified) then, $\forall r \in R$, $G(\{a, b, c, d, t, r\})$ is antenna, a contradiction. So vii) holds.

If $\exists C_4 : a, b, c, d$ in $G(A)$ and n in N such that n is adjacent with two adjacent vertices of the C_4 specified then, $\forall r \in R$, $G(\{a, b, c, d, n, r\})$ is antenna, a contradiction. So viii) holds.

Conversely, we suppose that i), ii), iii), iv), v), vi), vii), viii) holds and to show that G is $\{claw, antenna, net\}$-free graph.

We assume that there are $\{x, a, b, c\}$ a claw with center in x. From i) results that $G(A \cup N)$ and $G(N \cup R)$ are claw-free graphs. So $x \notin A \cup R$, that is $x \in N$. From ii), two of the vertices a, b, c are necessarily in N, that is $\{x, a, b, c\}$ are in $A \cup N$ or in $N \cup R$, thereby contradicting with i).

Suppose, however, that there is $H = G(\{a, b, c, 1, 2, 3\})$ an subgraph net, with the vertices a, b, c of the degree 1, the vertices 1, 2, 3 of degree 3, and $a1, b2, c3 \in E$.

Case 1. Let $|V(H) \cap R| = 1$. We assume 1.1. $V(H) \cap R = \{a\}$. 1.2. $V(H) \cap R = \{1\}$. 1.1. From $R \sim N$ it follows $V(H) \cap N = \{1\}$. So $V(H) \cap A = \{c, 3, 2, b\}$. But $G(\{c, 3, 2, b\})$ is P_4 and 1 is adjacent with the middle vertices in P_4, thereby contradicting with iii). 1.2. From $R \sim N$ it follows $V(H) \cap N = \{a, 2, 3\}$. So $V(H) \cap A = \{b, c\}$. P=$G(\{b, c, 2, 3\})$ is an P_4, with extremities $b, c \in A$ and the middle vertices $2,3 \in N$. For $t = a$ we have $N(t) \cap V(P) = \emptyset$, thereby contradicting with iv). So, Case 1 holds not.

Case 2. Let $|V(H) \cap R| = 2$. There are subcases 2.1. $V(H) \cap R = \{a,1\}$; 2.2. $V(H) \cap R = \{1,2\}$; 2.3. $V(H) \cap R = \{a,2\}$; 2.4. $V(H) \cap R = \{a,b\}$.

2.1.cannot hold because the vertices a and 1 have no common neighbors. 2.2. cannot hold because the vertices 1 and 2 not have only common neighbors. 2.3. cannot hold because the vertices a and 2 not have only common neighbors. 2.4. cannot hold because the vertices a and b have no common neighbors.

Case 3. $|V(H) \cap R| = 3$ cannot hold because the vertices in $\{a,1,2\}$ and in $\{a,1,b\}$ not have only common neighbors.

Case 4. Let $|V(H) \cap R| = 4$. Any subset $X \subset V(H)$ with $|X| = 4$ has the property that its vertices are not only common neighbors, that is $\exists v \in V - X$ such that v is adjacent some of the vertices of X, and with the rest of them, v it is not adjacent.

$|V(H) \cap R| \in \{5,6\}$ it is not possible, because $V(H) \cap A \neq \emptyset$ and $V(H) \cap N \neq \emptyset$.

Suppose that there is $F = G(\{a,b,x,y,i,j\},\{ab,bx,by,xy,xi,yj,ij\})$ an subgraph antenna.

Case 1. Let $|V(F) \cap R| = 1$. We assume 1.1. $V(F) \cap R = \{a\}$. 1.2. $V(F) \cap R = \{b\}$. 1.3. $V(F) \cap R = \{x\}$. 1.4. $V(F) \cap R = \{i\}$. From 1.1 $b \in N$, $x,y,j,i \in A$, $G(\{x,y,j,i\})$ is C_4, a contradiction (viii) does not hold). From 1.2. $a,x,y \in N$, $i,j \in A$, a contradiction (vii) does not hold). From 1.3. $b,y,i \in N$, $a,j \in A$, a contradiction (v) does not hold). From 1.4. $x,j \in N$, $a,b,y \in A$, a contradiction (vi) does not hold).

Case 2. Let $|V(F) \cap R| = 2$ (we suppose $V(F) \cap R = \{u,v\}$). Case 2 does not hold because either $N(u) \cap N(V) = \emptyset$ or $\exists s \in N(u) - N(v)$ or $\exists w \in N(v) - N(u)$.

Case 3. Let $|V(F) \cap R| = 3$. Because $N \sim R$, only $\{a,x,y\} \sim \{b\}$, so $a,x.y \in R$, $b \in N$. So $i,j \in A$, a contradiction ($A \not\sim R$ does not hold).

Case 4. Let $|V(F) \cap R| = 4$. Because $R \sim N$ and $\nexists v \in V(F)$ with $d_F(v) = 4$.

$|V(F) \cap R| \in \{5,6\}$ does not hold because $V(F) \cap A \neq \emptyset$ and $V(F) \cap N \neq \emptyset$.

Theorem 5 provides the following recognition algorithm for net-free graphs.

Algorithm Recognition
Input: A connected, non-complete graph $G = (V, E)$.
Output: An answer to the question: "Is G $\{claw, antenna, net\}$-free"?
begin
1. $L_G \leftarrow \{G\}$
2. *while* $L_G \neq \emptyset$ *do*
3. extract an element H in L
4. determine the weakly decomposition (A, N, R) with $[A]_H$ weakly component
5. if { ii) does not take place} *then*
   G is not $\{claw, antenna, net\}$-free *else*
6. if { iii) does not take place} *then*
   G is not $\{claw, antenna, net\}$-free *else*
7. if { iv) does not take place} *then*
   G is not $\{claw, antenna, net\}$-free *else*
8. if { v) does not take place} *then*
   G is not $\{claw, antenna, net\}$-free *else*
9. if { vi) does not take place} *then*
   G is not $\{claw, antenna, net\}$-free *else*
10. if { vii) does not take place} *then*
   G is not $\{claw, antenna, net\}$-free *else*
11. if { viii) does not take place} *then*
   G is not -free *else*
12. enter in L subgraphs $[V - R]$, $[V - A]$
13. Return: G is $\{claw, antenna, net\}$-free
*1*4. end
EndRecognition

Determine the degree each the vertex from $G(A \cup N)$, $G(A \cup R)$, $G(N \cup R)$, $G(A)$, $G(N)$, $G(R)$. R is clique if and only if $d_{G(A\cup R)}(r) = |N| - d_{G(R)}(r)$, $\forall r \in R$. For each $n \in N : N_G(n) \cap A$ is a clique if and only if $d_{G(A\cup N)}(n) - d_{G(N)}(n) = |N_G(n) \cap A|$ (for each vertex a in A; $a \in N_G(n)$?). For the recognition *chordal* graphs is necessary $O(n+m)$ ([25]) time. Theorem 2 provides an $O(n+m)$ algorithm for building a weakly decomposition for a non-complete and connected graph. Because for the recognition P_4-free graphs is necessary $O(n + m)$ ([17]) time, it follows that, in total, the algorithm is run in $O(n \cdot (n+m))$ time.

4. Conclusions and future work

In this paper we give a recognition algorithm for $\{claw, net, antenna\}$-free graphs. Our future work concerns giving some applications of $\{claw, antenna, net\}$-free graphs.

Acknowledgment. The research of Mihai Talmaciu was supported by the project entitled Decomposition of graphs and solving discrete optimization problems, "AR-FRBCF", 2014-2015, a Bilateral Cooperation Research Project, involving Romanian Academy ("Vasile Alecsandri" University of Bacau is partner), National Academy of Sciences of Belarus, Belarusian Republican Foundation for Fundamental Research

References

[1] A.S. Asratian, **Every 3-connected, locally connected, claw-free graph is Hamilton-connected**,J. Graph Theory, 23, pp. 191-201, (1996).
[2] C. Berge, **Graphs**, North-Holland, Amsterdam, (1985).
[3] A. Brandstedta, F. F. Dragan, **On linear and circular structure of (claw, net)-free graphs,** Discrete Applied Mathematics, Volume 129, Issues 23, 285-303, (2003).
[4] A. Brandstadt, F. F. Dragan, and E. Kohler, **Linear time algorithms for Hamiltonian problems on (claw,net)-free graphs**, SIAM J. COMPUT., Vol. 30, No. 5, pp. 1662-1677, (2000).
[5] A. Brandstadta, C. T. Hoang, **On clique separators, nearly chordal graphs, and the Maximum Weight Stable Set Problem**, Theoretical Computer Science, Vol. 389, 295-306, (2007).
[6] J. Brousek, **Minimal 2-connected non-hamiltonian claw-free graphs**, Discrete Math., 191, pp. 57-64, (1998).
[7] D.G. Corneil, S. Olariu, L. Stewart, **A linear time algorithm to compute a dominating path in an AT-free graph**, Inform. Proc. Lett. 54, 253-257, (1995).
[8] D.G. Corneil, S. Olariu, L. Stewart, **Asteroidal triple-free graphs**, SIAM J. Discrete Math., 10, 399-430, (1997).
[9] D.G. Corneil, S. Olariu, L. Stewart, **Linear time algorithms for dominating pairs in asteroidal triple-free graphs**, SIAM J. Comput. 28, 1284-1297, (1999).
[10] C. Croitoru, E. Olaru, M. Talmaciu, **Confidentially connected graphs**, The annals of the University "Dunarea de Jos" of Galati, Proceedings of the

international conference "The risk in contemporany economy", Supplement to Tome XVIII (XXII), 17-20, (2000).

[11] C. Croitoru, M. Talmaciu, **On Confidentially connected graphs**, Bul. Stiint. Univ. Baia Mare, Ser B, Matematica - Informatica, Vol.XVI, Nr. 1, 13-16, (2000).

[12] P. Damaschke, **Hamiltonian-hereditary graphs**, unpublished.

[13] D. Duffus, M.S. Jacobson, and R.J. Gould, **Forbidden subgraphs and the Hamiltonian theme, in The Theory and Applications of Graphs**, (Kalamazoo, Mich, 1980), Wiley, New York, pp. 297-316, (1981).

[14] S. Even, A. Pnueli, A. Lempel, **Permutation graphs and transitive graphs**, J. ACM, 19, pp. 400-410, (1972).

[15] E. Flandrin, J.L. Fouquet, and H. Li, **On Hamiltonian claw-free graphs**, Discrete Math., 111, pp. 221-229, (1993).

[16] M.C. Golumbic, C.L. Monma, W.T. Trotter Jr., **Tolerance graphs**, Discrete Appl. Math., 9, pp. 157-170, (1984).

[17] M. Habib, C. Paul, **A simple linear time algorithm for cograph recognition**, Discrete Applied Mathematics 145, 183 - 197, (2005).

[18] E. Kohler, **Linear Time Algorithms for Hamiltonian Problems in Claw-Free AT-Free Graphs**, manuscript, (1999).

[19] C.G. Lekkerkerker, J.C. Boland, **Representation of a finite graph by a set of intervals on the real line**, Fund. Math., 51, pp. 45-64, (1962).

[20] M. Li, **Hamiltonian cycles in 3-connected claw-free graphs**, J. Graph Theory, 17, pp. 303-313, (1993).

[21] F.B. Shepherd, **Hamiltonicity in claw-free graphs**, J. Combin. Theory Ser. B, 53, pp. 173-194, (1991).

[22] M. Talmaciu, **Decomposition Problems in the Graph Theory with Applications in Combinatorial Optimization** - Ph. D. Thesis, University "Al. I. Cuza" Iasi, Romania, (2002).

[23] M. Talmaciu, E. Nechita, **Recognition Algorithm for diamond-free graphs,** Informatica, Vol. 18, No. 3, 457-462, (2007).

[24] M. Talmaciu and V. Lepin, **Algorithms for the recognition of net-free graphs and for computing maximum cardinality matchings in claw-free graphs**, accepted for publication in Studies in Informatics and Control.

[25] R.E. Tarjan, M. Yannakakis, **Simple linear-time algorithms to test chordality of graphs, test acyclicity of hypergraphs, and selectively reduce acyclic hypergraphs.**, SIAM J. Comput. 13, 566-579 (1984).

"Vasile Alecsandri" University of Bacău
Faculty of Sciences
Department of Mathematics, Informatics and Education Sciences
157 Calea Mărăşeşti, Bacău, 600115, ROMANIA
email: mtalmaciu@ub.ro