"Vasile Alecsandri" University of Bacău
Faculty of Sciences
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COMMON FIXED POINT THEOREMS FOR A PAIR OF WEAKLY COMPATIBLE MAPPINGS IN FUZZY METRIC SPACES USING COMMON LIMIT IN THE RANGE PROPERTY

SAURABH MANRO, S.S. BHATIA, SANJAY KUMAR AND BRIAN FISHER

Abstract. The aim of this work is to establish some new common fixed point theorems for four self mappings satisfying an implicit contractive condition in fuzzy metric spaces by using common limit in range property and give some examples. Our results do not require the condition of closedness of range and so our theorems generalize, unify and extend many results in the literature.

1. Introduction

Fixed point theory in fuzzy metric spaces has been developed starting with the work of Heilpern [9]. In 1981, he introduced the concept of fuzzy contraction mappings and proved some fixed point theorems for fuzzy contraction mappings in metric linear spaces, which are fuzzy extension of the Banach contraction principle. Many authors have contributed to the development of this theory and its applications to fixed point theory, for instance [1,3-5,8,10,11,14-16,19-21,23,25-29].

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In 1976, Jungck [12] introduced the notion of commuting mappings. Afterward, Sessa [24] gave the notion of weakly commuting mappings. Jungck [13] defined the notion of compatible mappings to generalize the concept of weak commutativity and showed that weakly commuting mappings are compatible but the converse is not true. The concept of property $(E.A)$ in metric space has been recently introduced by Aamri and El Moutawakil [2]. In 2009, M. Abbas et al. [1] introduced the notion of common property $(E.A)$.

Sintunavarat and Kumam [26], in 2011, introduced the concept of the common limit in the range property and also established the existence of common fixed point theorems for generalized contractive mappings satisfy this property in fuzzy metric spaces.

Recently, Manro et al. [17] introduced the concept of common limit in the range property for four self maps and established related fixed point theorems. The aim of this work is to use this new property which is the so called "common limit in the range" for four self-mappings and establish some new existence of a common fixed point theorem for implicit contractive mappings.

2. Preliminaries

The concept of triangular norms (t-norms) is originally introduced by Menger [18] in study of statistical metric spaces.

Definition 2.1 (22). *A binary operation* $* : [0,1] \times [0,1] \to [0,1]$ *is continuous t-norm if * satisfies the following conditions*:
*(i) * is commutative and associative*;
*(ii) * is continuous*;
(iii) $a * 1 = a$ *for all* $a \in [0,1]$;
(iv) $a * b \leq c * d$ *whenever* $a \leq c$ *and* $b \leq d$ *for all* $a, b, c, d \in [0,1]$.
Examples of t-norms are: $a * b = min\{a, b\}, a * b = ab$ *and* $a * b = max\{a + b - 1, 0\}$.

Definition 2.2 (6). *A 3-tuple* $(X, M, *)$ *is said to be a fuzzy metric space if X is an arbitrary set, * is a continuous t-norm, and M is fuzzy sets on* $X^2 \times [0, \infty)$ *satisfying the following conditions*: *for all* $x, y \in X$ *and* $s, t > 0$,
(i) $M(x, y, 0) = 0$;
(ii) $M(x, y, t) = 1$ *for all* $t > 0$ *if and only if* $x = y$;
(iii) $M(x, y, t) = M(y, x, t)$;
(iv) $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$;
(v) $M(x, y, .) : [0, \infty) \to [0, 1]$ *is left continuous.*

The function $M(x,y,t)$ denotes the degree of nearness between x and y w.r.t. t respectively.

Remark 2.1 (6). *In the fuzzy metric space $(X,M,*)$, the function $M(x,y,.):[0,\infty)\to[0,1]$ is non-decreasing for all $x,y\in X$.*

Definition 2.3 (6). *Let $(X,M,*)$ be a fuzzy metric space. Then a sequence $\{x_n\}$ in X is said to be*
(i) convergent to a point $x\in X$ if, for all $t>0$,

$$lim_{n\to\infty}M(x_n,x,t)=1.$$

(ii) a Cauchy sequence if, for all $t>0$ and $p>0$,

$$lim_{n\to\infty}M(x_{n+p},x_n,t)=1.$$

Definition 2.4 (6). *A fuzzy metric space $(X,M,*)$ is said to be complete if and only if every Cauchy sequence in X is convergent.*

Remark 2.2 (6). *Let (X,d) be a metric space. Denote $a*b=ab$ for all $a,b\in[0,1]$. For each $t>0$ and $x,y\in X$, define $(X,M,*)$ by $M(x,y,t)=\frac{t}{t+|x-y|}$. Then, $(X,M,*)$ is a fuzzy metric space called Standard fuzzy metric space.*

Lemma 2.1 (7). *(i) If for two points $x,y\in X$ and some positive number $k<1$, we have $M(x,y,kt)\geq M(x,y,t)$ for all $t>0$, then $x=y$.*
(ii) If $lim_{n\to\infty}M(x_n,x,t)=lim_{n\to\infty}M(y_n,y,t)=1$, then $lim_{n\to\infty}M(x_n,y_n,t)=M(x,y,t)$.
(iii) $M(x,y,t)>0$ for all $x,y\in X$ and $t>0$.

Definition 2.5 (13). *A pair of self mappings (f,g) of a fuzzy metric space $(X,M,*)$ is said to be compatible if $lim_{n\to\infty}M(fgx_n,gfx_n,t)=1$ whenever $\{x_n\}$ is a sequence in X such that $lim_{n\to\infty}fx_n=lim_{n\to\infty}gx_n=z$ for some $z\in X$.*

Definition 2.6 (13). *Two self-mappings f and g of a fuzzy metric space $(X,M,*)$ are called non-compatible if there exists at least one sequence such that $lim_{n\to\infty}fx_n=lim_{n\to\infty}gx_n=z$ for some $z\in X$ but either $lim_{n\to\infty}M(fgx_n,gfx_n,t)\neq 1$ or the limit does not exist.*

Definition 2.7 (13). *Let f and g be two self-mappings on a fuzzy metric space $(X,M,*)$. A point $x\in X$ is said to be point of coincidence of maps f and g if $fx=gx$.*

Definition 2.8 (8). *Two self-mappings f and g on a non empty set X are said to be weakly compatible if $fgx=gfx$ for all x at which $fx=gx$.*

Definition 2.9 (2). *A pair of self mappings (f, g) on a fuzzy metric space $(X, M, *)$ is said to satisfy the property $(E.A)$ if there exists a sequence $\{x_n\}$ in X such that $lim_{n\to\infty} f x_n = lim_{n\to\infty} g x_n = z$ for some $z \in X$.*

The class of E.A. mappings contains the class of non compatible mappings.

Definition 2.10 (1). *The pairs (A, S) and (B, T) on a fuzzy metric space $(X, M, *)$ are said to satisfy the common property $(E.A)$ if there exist two sequences $\{x_n\}$ and $\{y_n\}$ in X such that $lim_{n\to\infty} Ax_n = lim_{n\to\infty} Sx_n = lim_{n\to\infty} Ty_n = lim_{n\to\infty} By_n = p$ for some $p \in X$.*

Definition 2.11 (26). *A pair of self mappings (f, g) on a fuzzy metric space $(X, M, *)$ is said to satisfy the common limit in the range of g property (CLR_g) if there exists a sequence $\{x_n\}$ in X such that $lim_{n\to\infty} f x_n = lim_{n\to\infty} g x_n = gz$ for some $z \in X$.*

Remark 2.3 (26). *The notion of pair of mappings (f, g) satisfying the common limit in the range of g property is closely related to the notion of pair of mappings with property $(E.A)$. If (f, g) satisfies the property $(E.A)$ and $g(X)$ is closed, then (f, g) satisfies the common limit in the range of g.*

Inspired by Sintunavarat et al. [26], Manro et al. [17] introduced the following:

Definition 2.12. *The pairs (A, S) and (B, T) on a fuzzy metric space $(X, M, *)$ are said to share the common limit in the range of S property if there exist two sequences $\{x_n\}$ and $\{y_n\}$ in X such that $lim_{n\to\infty} Ax_n = lim_{n\to\infty} Sx_n = lim_{n\to\infty} Ty_n = lim_{n\to\infty} By_n = Sz$ for some $z \in X$.*

Remark 2.4 (17). *If the pairs of mappings (A, S) and (B, T) satisfy the common property $(E.A)$ and if at least one of these mappings has closed range, that the two pairs share the common limit in the respective range property.*

Example 2.1 (17). *Let $(X, M, *)$ be a fuzzy metric space with $X = [-1, 1]$, for all $a*b = a.b$ and $M(x, y, t) = e^{-\frac{|x-y|}{t}}$ if $t > 0$, $M(x, y, 0) = 0$ for all $x, y \in X$. Define self mappings A, B, S and T on X by $Ax = (\frac{x}{3})$, $Sx = x$, $Rx = -x$, $Bx = (\frac{-x}{3})$ for all $x \in X$. Then with sequences $\{x_n = 1/n\}$ and $\{y_n = -1/n\}$ in X, one can easily verify that $lim_{n\to\infty} Ax_n = lim_{n\to\infty} Sx_n = lim_{n\to\infty} Ty_n = lim_{n\to\infty} By_n = S(0)$. This shows that the pairs (A, S) and (B, T) share the common limit in the range of S property.*

Definition 2.13 (8). *Two finite families of self mappings* $\{A_i\}_{i=1}^m$ *and* $\{B_j\}_{j=1}^n$ *on a set* X *are said to be pairwise commuting if*
(*i*) $A_iA_j = A_jA_i$, $i, j \in \{1, 2, 3, ..., m\}$,
(*ii*) $B_iB_j = B_jB_i$, $i, j \in \{1, 2, 3, ..., n\}$,
(*iii*) $A_iB_j = B_jA_i$, $i \in \{1, 2, 3, ..., m\}$,$j \in \{1, 2, 3, ..., n\}$.

Definition 2.14. *Let* $*$ *be a continuous* t *- norm and* Ψ_6 *be the set of all continuous functions* $F : (0, 1]^6 \to \mathbb{R}$ *satisfying the following conditions:*
(F_1) *F is non- increasing in the fifth and sixth variables*;
(F_2) *if for some constant* $k \in (0, 1)$, *we have*
$(F_2(a))$ $F(u(kt), v(t), v(t), u(t), 1, u(\frac{t}{2}) * v(\frac{t}{2})) \geq 1$, *or*
$(F_2(b))$ $F(u(kt), v(t), u(t), v(t), u(\frac{t}{2}) * v(\frac{t}{2}), 1) \geq 1$
for any $t > 0$ *and any non decreasing functions* $u, v : (0, \infty) \to (0, 1]$, *then there exists* $h \in (0, 1)$ *with* $u(ht) \geq u(t) * v(t)$;
$(F3)$ *if, for some constant* $k \in (0, 1)$, *we have* $F(u(kt), u(t), 1, 1, u(t), u(t)) \geq 1$ *for any fixed* $t > 0$ *and any non decreasing function* $u : (0, \infty) \to (0, 1]$, *then* $u(kt) \geq u(t)$.

Example 2.2. *Let* $F : (0, 1]^6 \to \mathbb{R}$ *be defined by* $F(u_1, u_2, u_3, u_4, u_5, u_6) = \frac{u_1}{min\{u_2,u_3,u_4,u_5,u_6\}}$. *Clearly,* $F \in \Psi_6$.

3. Main Results

Theorem 3.1. *Let* A, B, S *and* T *be self mappings of a fuzzy metric space* $(X, M, *)$ *satisfying following conditions, for some* $F \in \Psi_6$*:*
(*i*) *the pairs of mappings* (A, S) *and* (B, T) *satisfy the common property* $(E.A)$;
(*ii*) $A(X) \subset T(X)$ (*or* $B(X) \subset S(X)$);
(*iii*) *one of mappings* S *or* T *has closed range*;
(*iv*) *for some* $k \in (0, 1)$, *for any* $x, y \in X$, *and* $t > 0$ *the following inequality holds*
(3.1)
$$F(M(Ax, By, kt), M(Sx, Ty, t), M(Sx, Ax, t),$$
$$M(Ty, By, t), M(Ty, Ax, t), M(Sx, By, t)) \geq 1.$$
Then each of the pairs (A, S) *and* (B, T) *has a point of coincidence. Moreover,* A, B, S *and* T *have a unique common fixed point provided that both the pairs* (A, S) *and* (B, T) *are weakly compatible.*

Proof. Since the pairs of mappings (A, S) and (B, T) satisfy the common property $(E.A)$ then there exist two sequences $\{x_n\}$ and $\{y_n\}$ in X such that $lim_{n\to\infty}Ax_n = lim_{n\to\infty}Sx_n = lim_{n\to\infty}Ty_n = lim_{n\to\infty}By_n =$

u for some $u \in X$. Suppose $S(X)$ is a closed subspace of X. This gives, $u = Sz$ for some $z \in X$. Thus, the pairs (A, S) and (B, T) share the common limit in the range of S property. Firstly, we assert that $Az = Sz$. By (3.1), we have

$$F(M(Az, By_n, kt), M(Sz, Ty_n, t), M(Sz, Az, t),$$
$$M(Ty_n, By_n, t), M(Ty_n, Az, t), M(Sz, By_n, t)) \geq 1.$$

For $n \to \infty$ the above inequality implies

$$F(M(Az, Sz, kt), M(Sz, Sz, t), M(Sz, Az, t),$$
$$M(Sz, Sz, t), M(Sz, Az, t), M(Sz, Sz, t)) \geq 1,$$

$$F(M(Az, Sz, kt), 1, M(Sz, Az, t), 1, M(Sz, Az, t), 1) \geq 1.$$

On the other hand, since

$$M(Sz, Az, t) \geq M(Sz, Az, \frac{t}{2}) = M(Sz, Az, \frac{t}{2}) * 1,$$

and as F is non-increasing in the fifth variable, we have, for any $t > 0$,

$$F(M(Az, Sz, kt), 1, M(Sz, Az, t), 1, M(Sz, Az, t), 1)$$
$$\geq F(M(Az, Sz, kt), 1, M(Sz, Az, t), 1, M(Sz, Az, \frac{t}{2}) * 1, 1) \geq 1,$$

which, by using (F_2) gives, $Az = Sz$.
Since $A(X) \subset T(X)$, there exists $v \in X$ such that $Az = Tv$.
Secondly, we assert that $Bv = Tv$. By (3.1), we get

$$F(M(Az, Bv, kt), M(Sz, Tv, t), M(Sz, Az, t),$$
$$M(Tv, Bv, t), M(Tv, Az, t), M(Sz, Bv, t)) \geq 1,$$

$$F(M(Tv, Bv, kt), M(Tv, Tv, t), M(Tv, Tv, t),$$
$$M(Tv, Bv, t), M(Tv, Tv, t), M(Tv, Bv, t)) \geq 1,$$

$$F(M(Tv, Bv, kt), 1, 1, M(Tv, Bv, t), 1, M(Tv, Bv, t)) \geq 1.$$

On the other hand, since

$$M(Tv, Bv, t) \geq M(Tv, Bv, \frac{t}{2}) = M(Tv, Bv, \frac{t}{2}) * 1,$$

and as F is non- increasing in the sixth variable, we have, for any $t > 0$,

$$F(M(Tv, Bv, kt), 1, 1, M(Tv, Bv, t), 1, M(Tv, Bv, \frac{t}{2}) * 1)$$
$$\geq F(M(Tv, Bv, t), 1, 1, M(Tv, Bv, t), 1, M(Tv, Bv, t)) \geq 1,$$

which, by using (F_2) gives $Tv = Bv = Az = Sz$.
Since the pairs (A, S) and (B, T) are weakly compatible and $Az = Sz$ and $Tv = Bv$, we therefore have $ASz = SAz = AAz = SSz, BTv = TBv = TTv = BBv$.
Finally, we assert that $AAz = Az$. Again by (3.1), we have

$$F(M(AAz, Bv, kt), M(SAz, Tv, t), M(SAz, AAz, t),\\ M(Tv, Bv, t), M(Tv, AAz, t), M(SAz, Bv, t)) \geq 1,$$

$$F(M(AAz, Bv, kt), M(AAz, Bv, t), M(AAz, AAz, t),\\ M(Bv, Bv, t), M(Bv, AAz, t), M(AAz, Bv, t)) \geq 1,$$

$$F(M(AAz, Az, kt), M(AAz, Az, t), 1, 1, M(Az, AAz, t), M(AAz, Az, t)) \geq 1,$$

which, by using (F_3), we have $AAz = Az = SAz$ and so Az is a common fixed point of A and S. Similarly, one can easily prove that $BBv = Bv = TBv$, that is, Bv is common fixed point of B and T. As $Az = Bv$, we therefore have that Az is common fixed point of A, S, B and T. Similarly, A, S, B and T have a common fixed point if $T(X)$ is a closed subspaces of X and $B(X) \subset S(X)$. The uniqueness of the common fixed point is an easy consequence of inequality (3.1) and (F_3). ∎

By choosing A, B, S and T suitably, one can derive corollaries involving two or three mappings.

Corollary 3.1. *Let A and S be self mappings of a fuzzy metric space $(X, M, *)$ satisfying:*
(i) the pair (A, S) satisfies the property $(E.A)$;
(ii) $A(X) \subset S(X)$;
(iii) $S(X)$ is a closed subspace of X;
(iv) for any $x, y \in X$, $k \in (0, 1)$, $F \in \Psi_6$ and $t > 0$ such that

$$F(M(Ax, Ay, kt), M(Sx, Sy, t), M(Sx, Ax, t),\\ M(Sy, Ay, t), M(Sy, Ax, t), M(Sx, Ay, t)) \geq 1.$$

Then, A and S have a point of coincidence. Moreover, A and S have a unique common fixed point provided that A and S are weakly compatible.

Proof. Taking $B = A$ and $T = S$ in Theorem 3.1, the result follows. ∎

Corollary 3.2. *Let A, B, S and T be self mappings of a fuzzy metric space $(X, M, *)$ satisfying inequality (3.1). Suppose that*
(i) the pairs (A, S) and (B, T) satisfy the common limit in the range

of S *(respectively, in the range of* T*) property;*
(ii) $A(X) \subset T(X)$ *(respectively,* $B(X) \subset S(X)$*).*
Then the pairs (A, S) *and* (B, T) *have a point of coincidence each. Moreover,* A, B, S *and* T *have a unique common fixed point provided that both the pairs* (A, S) *and* (B, T) *are weakly compatible.*

Proof. Proof easily follows on same lines of Theorem 3.1. ∎

Theorem 3.2. *Let* A, B, S *and* T *be self mappings of a fuzzy metric space* $(X, M, *)$ *satisfying inequality* (3.1)*. Suppose that*
(i) the pair (A, S) *(or* (B, T)*) satisfies property* $(E.A)$ *and* $S(X)$ *is a closed subspace of* X*;*
(ii) $A(X) \subset T(X)$ *(or* $B(X) \subset S(X)$*).*
Then the pairs (A, S) *and* (B, T) *each have a point of coincidence. Moreover,* A, B, S *and* T *have a unique common fixed point provided that both the pairs* (A, S) *and* (B, T) *are weakly compatible.*

Proof. Suppose the pair (A, S) satisfies property $(E.A)$. Then there exists a sequence $\{x_n\}$ in X such that $lim_{n\to\infty} Ax_n = lim_{n\to\infty} Sx_n = p$ for some $p \in X$. It follows from $S(X)$ being a closed subspace of X that $p = Sz$ for some $z \in X$ and then the pair (A, S) satisfies the common limit in the range of S property. By Theorem 3.1, we get A, B, S and T have a unique common fixed point. ∎

Since the pair of non compatible mappings imply to the pair satisfying property $(E.A)$, we get the following corollary.

Corollary 3.3. *Let* A, B, S *and* T *be self mappings of a fuzzy metric space* $(X, M, *)$ *satisfying inequality* (3.1)*. Suppose that*
(i) the pair (A, S) *(or* (B, T)*) is non compatible mappings and* $S(X)$ *is a closed subspace of* X*;*
(ii) $A(X) \subset T(X)$ *(or* $B(X) \subset S(X)$*).*
Then the pairs (A, S) *and* (B, T) *each have a point of coincidence. Moreover,* A, B, S *and* T *have a unique common fixed point provided that both the pairs* (A, S) *and* (B, T) *are weakly compatible.*

Proof. Since the pair of mappings (A, S) are non-compatible mappings, we get A and S satisfy the property $(E.A)$. Therefore, by Theorem 3.4, we get A, B, S and T have a unique common fixed point in X. ∎

As an application of Theorem 3.1, we prove a common fixed point theorem for four finite families of mappings on fuzzy metric spaces. While proving our result, we utilize Definition 2.14 which is a natural extension of commutativity condition to two finite families.

Theorem 3.3. *Let* $\{A_1, A_2, ..., A_m\}$, $\{B_1, B_2, ..., B_n\}$, $\{S_1, S_2, ..., S_p\}$ *and* $\{T_1, T_2, ..., T_q\}$ *be four finite families of self mappings of a fuzzy metric space* $(X, M, *)$ *such that* $A = A_1.A_2.....A_m$, $B = B_1.B_2.....B_n$, $S = S_1.S_2.....S_p$ *and* $T = T_1.T_2.....T_q$ *satisfying the conditions* (3.1) *and*
(i) *the pair* (A, S)(*or* (B, T)) *satisfies the common limit in the range of* S *property;*
(ii) $A(X) \subset T(X)$(*or* $B(X) \subset S(X)$).
Then
(a) *each of the pairs* (A, S) *and* (B, T) *has a point of coincidence.*
(b) A_i, S_k, B_r *and* T_t *have a unique common fixed point provided that the pairs of families* $(\{A_i\}, \{S_k\})$ *and* $(\{B_r\}, \{T_t\})$ *commute pairwise for all* $i = 1, 2, ..., m$, $k = 1, 2, ..., p$, $r = 1, 2, ..., n$, $t = 1, 2, ..., q$.

Proof. (a) Theorem 3.1 is applied.
(b) Similar results are proved in the literature and it is standard to prove that $AS = SA$ and $BT = TB$. Hence that the pair (A, S) is obviously compatible and (B, T) is weakly compatible. Now using Theorem 3.1, we conclude that A, S, B and T have a unique common fixed point in X, say z. ∎

Lastly, we give example to illustrate the validity of Corollary 3.3.

Example 3.4. *Let* $(X, M, *)$ *be a fuzzy metric space where* $X = [0, 2)$ *and a t-norm* $*$ *be defined by* $a * b = min\{a, b\}$ *for all* $a, b \in [0, 1]$ *and* M *be a fuzzy set on* $X^2 \times (0, \infty)$ *defined by* $M(x, y, t) = e^{-(\frac{|x-y|}{t})}$ *for all* $x, y \in X$ *and* $t > 0$.
Let $F : (0, 1]^6 \to \mathbb{R}$ *be defined by* $F(u_1, u_2, u_3, u_4, u_5, u_6) = \frac{u_1}{min\{u_2, u_3, u_4, u_5, u_6\}}$.
Clearly, $F \in \Psi_6$.*Define* A, B, S *and* T *by*
$Ax = Bx = 1$,
$Sx = 1$ *if* $x \in \mathbb{Q}$, $Sx = \frac{2}{3}$ *otherwise,*
and
$Tx = 1$ *if* $x \in \mathbb{Q}$, $Tx = \frac{1}{3}$ *otherwise.*
Clearly, the pair (A, S) *satisfies the common limit in the range of* S *property and* $A(X) \subset T(X)$. A, B, S *and* T *have a unique common fixed point* $x = 1$.

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[1,2]School of Mathematics and Computer Applications
Thapar University, Patiala, Punjab, INDIA
[1]sauravmanro@hotmail.com, [2]ssbhatia@thapar.edu
[3]Deenbandhu Chhotu Ram University of Science and Technology,
Murthal (Sonepat), INDIA
sanjuciet@rediff.com
[4]Department of Mathematics, University of Leicester, Leicester LE1 7RH, UK
fbr@leicester.ac.uk
[1]Corresponding Author