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## THE RESEARCH OF UNORDER TABLES AS ALGEBRAIC STRUCTURES

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**Abstract.** The purpose of the paper is research unordered tables as algebraic structures. Algebraic properties were investigated operations tables union, intersection and difference. It was proposed in C++ a generic parameterized class with abstract class records tabular representation which allows modeling work with tables arranged in terms of algebraic structures [1,2].

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### 1. INTRODUCTION

In what follows it will try to examine the tables with different unordered operations in terms of algebraic structures. Will be considered the most important operations on the unordered table, such as an item in the unordered table, concatenation of two unordered difference tables. We will investigate which of these operations a lot of tables and unordered forms a monoid.

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The table is a set of elements, each of which has a particular named key indicator with the property that the elements are added to the table and search by key. In addition to the key element in the table (also called registration form) typically contains data, some bearing information. We say that the table is a key element as unordered, which facilitates the retrieval of the information they contain.

## 2. ALGEBRAIC STRUCTURE OF UNORDERED TABLES

Let  $E = \{e_1, e_2, \dots, e_n\}$  the set of all tabulated records by the same key structure highlighted. Key is determined by the relationship of the order allowing us ordering tabular entries by key.

**Definition 2.1.** The unordered table of set  $E$ , note  $T_1 = T(e_{i_1}, e_{i_2}, \dots, e_{i_{k_i}})$ , we understand unordered elements set  $(e_{l_1}, e_{l_2}, \dots, e_{l_{k_i}})$ , where  $(e_{l_j}) \in E$ .  $j = 1, 2, \dots, k$ , and  $k$  - is the number of elements in the table  $\bigcap T((e_{l_1}, e_{l_2}, \dots, e_{l_{k_i}}))$ , and element  $(e_{l_1})$  - it's called the first element in the table  $T$ ,  $(e_{l_k})$  - it's called the last element of the table  $T$ .

**Definition 2.2** The two simple unordered tables that same elements are equal.

**Remark 2.1** Unordered table may be empty if it has no element. We denote the empty unordered table by " $\emptyset$ ", that is  $T() = \emptyset$ . Unordered table  $T(e)$  it is composed of a single element .

Let  $\Psi = T(1), T(2), \dots, T(m) \dots$  - the set of all possible unordered tables on the crowd.  $E$ . It introduces a set of operations for unordered tables similar to those described in [4,5].

## 3. THE CONCATENATION UNORDERED TABLES

In classic Informatics is used in the operation of inserting a new tabular entries. The operation using two operands of different types (table and tabular record). From the point of view of operation algebra that can be expressed through "add" operation extended to two operand will be presented as a table in a single table record. We will introduce operation that will give us an opportunity to concatenate two unordered tables with the same type of elements and we will note the sign  $\oplus$ .

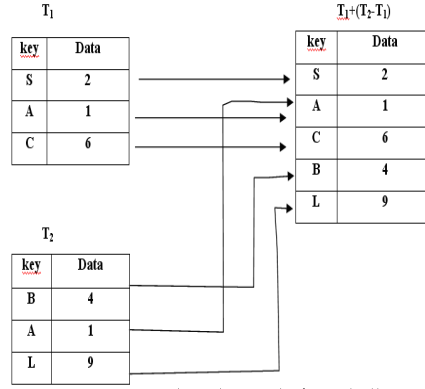


Figure 1. The concatenation of two unordered tables

**Definition 3.1** Operation  $\oplus$  which merges unordered table  $T_1 = T(e_{i_1}, e_{i_2}, \dots, e_{i_{k_i}})$ , with unordered table  $T_2 = T(e_{j_1}, e_{j_2}, \dots, e_{j_{k_j}})$ , is operation  $(T_1(e_{i_1}, e_{i_2}, \dots, e_{i_{k_i}}) \oplus T_2(e_{j_1}, e_{j_2}, \dots, e_{j_{k_j}}))$ , which has resulting unordered table  $T(e_{i_1}, e_{i_2}, \dots, e_{i_{k_i}} \oplus e_{j_1}, e_{j_2}, \dots, e_{j_{k_j}})$ , so that the element  $e_{j_{k_j}}$  is the last element of the unordered  $T_i \oplus T_j$  table for other  $T_i, T_j \in \Psi$ . That the set of all unordered tables on the set  $E$  with extended operation "add" is a groupoid noted  $(\Psi, \oplus)$ .

We prove that the pace unordered tables have neutral element is groupoid.

**Remark 3.2** Usually neutral element is denoted by 0 and is called the zero element.

**Theorem 3.1** Groupoid  $(\Psi, \oplus)$  has a neutral element.

**Proof** As a neutral element will be empty table  $(\emptyset)$ . We take any arbitrary table  $T_i \in \Psi$ . Let  $T_1 = T(e_{i_1}, e_{i_2}, \dots, e_{i_{k_i}})$ , then  $T(e_{i_1}, e_{i_2}, \dots, e_{i_{k_i}}) \oplus \emptyset = \emptyset \oplus T(e_{i_1}, e_{i_2}, \dots, e_{i_{k_i}}) = T(e_{i_1}, e_{i_2}, \dots, e_{i_{k_i}}) = T_i$ , Theorem is proved.

**Theorem 3.2** Groupoid  $(\Psi, \oplus)$  is a semigroup.

**Proof** Let  $T_i, T_j, T_r$  three arbitrary tables in  $\Psi$ . We prove that  $(T_i \oplus T_j) \oplus T_r = T_i \oplus (T_j \oplus T_r)$ . Let  $T_i = T(e_{i_1}, e_{i_2}, \dots, e_{i_{k_i}})$ ,  $T_j = T(e_{j_1}, e_{j_2}, \dots, e_{j_{k_j}})$ ,  $T_r = T(e_{r_1}, e_{r_2}, \dots, e_{r_{k_r}})$ . Then  $(T_i \oplus T_j) \oplus T_r = (T(e_{i_1}, e_{i_2}, \dots, e_{i_{k_i}}) \oplus T(e_{j_1}, e_{j_2}, \dots, e_{j_{k_j}})) \oplus T(e_{r_1}, e_{r_2}, \dots, e_{r_{k_r}}) = T(e_{i_1}, e_{i_2}, \dots, e_{i_{k_i}}) \oplus (T(e_{j_1}, e_{j_2}, \dots, e_{j_{k_j}}) \oplus T(e_{r_1}, e_{r_2}, \dots, e_{r_{k_r}})) = T_i \oplus (T_j \oplus T_r)$ , for other  $T_i, T_j, T_r \in \Psi$ . Theorem is proved.

**Theorem 3.3** Groupoid  $(\Psi \oplus)$  is a monoid.

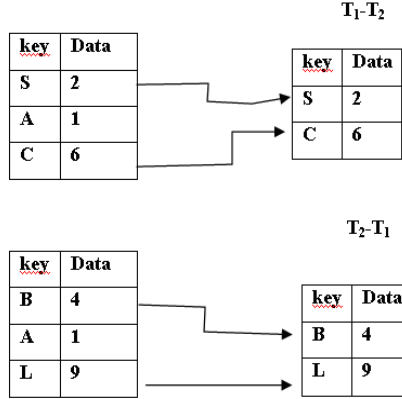
**Proof** From the theorem 1 result that groupoid  $(\Psi \oplus)$  posses a identity element, but from theorem 2 result that groupoid  $(\Psi \oplus)$  is a semigroup. Then, groupoid  $(\Psi \oplus)$  is a monoid.

**Theorem 3.4** Monoid  $(\Psi \oplus)$  is a commutative.

**Proof** Let  $T_i, T_j$  two arbitrary tables in  $(\Psi)$ . Is proved that  $T_i \oplus T_j = T_j \oplus T_i$ . Let  $T_i = T(e_{i_1}, e_{i_2}, \dots, e_{i_{k_i}}), T_j = T(e_{j_1}, e_{j_2}, \dots, e_{j_{k_j}})$ . Then  $T_i \oplus T_j = T(e_{i_1}, e_{i_2}, \dots, e_{i_{k_i}}) \oplus T(e_{j_1}, e_{j_2}, \dots, e_{j_{k_j}}) = T(e_{j_1}, e_{j_2}, \dots, e_{j_{k_j}}) \oplus T(e_{i_1}, e_{i_2}, \dots, e_{i_{k_i}})$  for other  $T_i, T_j \in \Psi$ . Theorem is proved.

#### 4. THE DIFFERENCE UNORDERED TABLES

We introduce the difference operation will enable us to extract records from unordered table and is denoted by " $\ominus$ " sign. We will show how the drawing occurs difference of two unordered tables. **Remark 4.1** The unordered table  $T(e_{i_1}, e_{i_2}, \dots, e_{i_{k_i}}) \ominus 0 = T(e_{i_1}, e_{i_2}, \dots, e_{i_{k_i}})$  but if  $0 \ominus T(e_{i_1}, e_{i_2}, \dots, e_{i_{k_i}}) = T()$  is obtain a empty table.



**Figure 2.** The difference a two unordered table.

**Definition 2.4** Operation " $\ominus$ " that extracts in the unordered table  $T_i = T(e_{i_1}, e_{i_2}, \dots, e_{i_{k_i}})$ , unordered table  $T_j = T(e_{j_1}, e_{j_2}, \dots, e_{j_{k_j}})$  is operation  $T(e_{i_1}, e_{i_2}, \dots, e_{i_{k_i}}) \ominus T(e_{j_1}, e_{j_2}, \dots, e_{j_{k_j}})$  which results in table  $T(e_{i_1}, e_{i_2}, \dots, e_{i_{k_i}}) \ominus T(e_{j_1}, e_{j_2}, \dots, e_{j_{k_j}})$  where the elements are equal and obtain empty table, or get items elements are that in  $T_i$  and not are in  $T_j$  for other  $T_i, T_j \in \Psi$ . That the set of all unordered tables on the set  $E$  with the operation " $\ominus$ " is a groupoid noted  $(\Psi, \ominus)$ .

#### 5. THE INTERSECTION UNORDERED TABLES

The intersection of two tables  $T = T_1 \cap T_2$  does it mean the set that contains the common elements of  $T_1$  and  $T_2$  with equal keys.

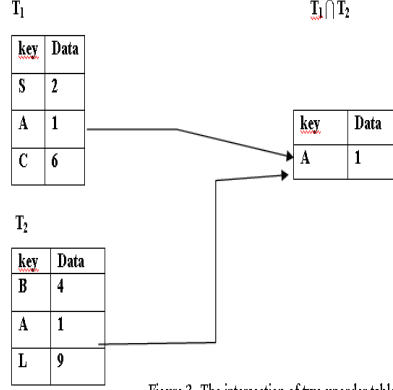


Figure 3. The intersection of two unordered table.

**Definition 5.1** The operation " $\cap$ " intersecting unordered table  $T_1 = T(e_{r_1}, e_{r_2}, \dots, e_{r_{k_i}})$  with unordered table  $T_2 = T(e_{l_1}, e_{l_2}, \dots, e_{l_{k_i}})$ , is operation  $(T_1(e_{r_1}, e_{r_2}, \dots, e_{r_{k_i}}) \cap T_2(e_{l_1}, e_{l_2}, \dots, e_{l_{k_i}}))$  which resulting unordered table  $T(e_{j_1}, e_{j_2}, \dots, e_{j_{k_i}})$  so that every element belongs unordered table  $T_1 = T(e_{r_1}, e_{r_2}, \dots, e_{r_{k_i}})$  with unordered table  $T_2 = T(e_{l_1}, e_{l_2}, \dots, e_{l_{k_i}})$ , and unordered table  $T_2 = T(e_{l_1}, e_{l_2}, \dots, e_{l_{k_i}})$ , for other  $T_r, T_l \in \Psi$ . That way the set of all unordered tables on the set  $E$  with the operation " $\cap$ " is a groupoid noted  $(\Psi, \cap)$ .

**Remark 5.1** If the key in both tables is different (does not intersect) then they get empty set  $T_1 \cap T_2 = \emptyset$ .

**Theorem 5.1** Groupoid  $(\Psi, \cap)$  posses identity element.

**Proof** As a neutral element will get an empty table  $\emptyset$ . We take any arbitrary table  $T_i \in \Psi$ . Let  $T_r = T(e_{r_1}, e_{r_2}, \dots, e_{r_{k_i}})$ , then.  $T_r \cap \emptyset = T(e_{r_1}, e_{r_2}, \dots, e_{r_{k_i}}) \cap \emptyset = \emptyset \cap T(e_{r_1}, e_{r_2}, \dots, e_{r_{k_i}}) = T_r$ . Theorem is proved.

**Theorem 5.2** Groupoid  $(\Psi, \cap)$  is semigroup.

**Proof** Let  $T_i, T_j, T_r$  three arbitrary tables of the  $\Psi$ . Is proved that  $(T_i \cap T_j) \cap T_r = T_i \cap (T_j \cap T_r)$ . Let  $T_i = T(e_{i_1}, e_{i_2}, \dots, e_{i_{k_i}})$ ,  $T_j = T(e_{j_1}, e_{j_2}, \dots, e_{j_{k_i}})$ ,  $T_r = T(e_{r_1}, e_{r_2}, \dots, e_{r_{k_i}})$ . Then  $(T_i \cap T_j) \cap T_r = (T(e_{i_1}, e_{i_2}, \dots, e_{i_{k_i}}) \cap T(e_{j_1}, e_{j_2}, \dots, e_{j_{k_i}})) \cap T(e_{r_1}, e_{r_2}, \dots, e_{r_{k_i}}) = T(e_{i_1}, e_{i_2}, \dots, e_{i_{k_i}}) \cap (T(e_{j_1}, e_{j_2}, \dots, e_{j_{k_i}}) \cap T(e_{r_1}, e_{r_2}, \dots, e_{r_{k_i}})) = T_i \cap (T_j \cap T_r)$  for other  $T_i, T_j, T_r \in \Psi$ . Theorem is proved.

**Teorema 5.3** Groupoid  $(\Psi, \cap)$  is monoid.

**Proof** From the theorem 1 it follows that groupoid  $(\Psi, \cap)$  posses a neutral element, and theorem 2 that groupoid  $(\Psi, \cap)$  is semigroup. So, groupoid  $(\Psi, \cap)$  is monoid.

**Teorema 5.4** Monoid  $(\Psi, \cap)$  is commutative.

**Proof** Let  $T_i, T_j$  two arbitrary tables of the  $\Psi$ . We prove that  $T_i \cap T_j = T_j \cap T_i$ . Let  $T_i = T(e_{i_1}, e_{i_2}, \dots, e_{i_{k_i}})$ ,  $T_j = T(e_{j_1}, e_{j_2}, \dots, e_{j_{k_j}})$ . Then  $T_i \cap T_j = T(e_{i_1}, e_{i_2}, \dots, e_{i_{k_i}}) \cap T(e_{j_1}, e_{j_2}, \dots, e_{j_{k_j}}) = T(e_{j_1}, e_{j_2}, \dots, e_{j_{k_j}}) \cap T(e_{i_1}, e_{i_2}, \dots, e_{i_{k_i}}) = T_j \cap T_i$  for other  $T_i, T_j \in \Psi$ . Theorem is proved.

## 6. CONCLUSIONS

The purpose of the article was to show that unordered tables shows can be an algebraic structure. It has been shown that T unordered table with commutative monoid operation is concatenation. We prove that the set of common and uncommon records from two tables that are unique keys can be combined.

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