"Vasile Alecsandri" University of Bacău
Faculty of Sciences
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ON RANDERS SPACES OF CONSTANT CURVATURE

H. G. NAGARAJA

Abstract. In this paper we study the Randers manifolds of constant curvature induced by generalized Sasakian space forms with different almost contact structures.

1. Introduction

A Finsler metric $L(x, y)$ on an n-dimensional manifold M^n is called an (α, β)-metric[9] $L(\alpha, \beta)$ if L is positively homogeneous function of α and β of degree one, where $\alpha^2 = a_{ij}(x)y^i y^j$ is a Riemannian metric and $\beta = b_i(x)y^i$ is a 1-form on M^n. Randers metrics are simplest (α, β)-metrics, which are expressed in the form $F = \alpha + \beta$ with $\|\beta\| < 1$. In Finsler geometry it is difficult to make classification of Finsler metrics of constant curvature, but for the case of special Finsler metrics like Randers metrics, a classification theorem was given by Yasuda and Shimada[16] and Matsumoto and Shimada[14]. But Bao and Robles[8] found some errors in both the proofs and they gave a corrected version of Yasuda and Shimada theorem which characterises Randers spaces of constant curvature.

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Bejancu and Farran [6] gave a classification theorem for a class of proper Randers manifolds of positive constant flag curvature. They have shown that a Sasakian space form induces a Randers space of positive constant curvature. Same authors in [5] proved that under certain natural conditions, Randers manifolds of positive constant flag curvature are diffeomorphic to odd dimensional spheres. Hasegawa et al.[11] established that there are other almost contact structures that induce Randers space of constant negative flag curvature and constant curvature $K = 0$. Motivated by the above results, in this paper we investigate a class of Randers manifolds induced by generalized Sasakian space forms with a-Sasakian, b-Kenmotsu and co-symplcteic structures.

2. Preliminaries

2.1. **Randers Metrics.** A Finsler metric F on a manifold M is a function $F : TM \to (0, \infty)$ such that

(1) F is C^∞ on $TM - \{0\}$
(2) $F_x = F_{|T_xM}$ is a Minkowski norm on T_xM:
$F(x, \lambda y) = \lambda F(x, y)$ for $\lambda > 0$
The fundamental tensor field is positive definite, $(g_{ij}(x, y)) > 0$, where $g_{ij} = \frac{1}{2}\frac{\partial^2 F^2}{\partial y^i \partial y^j}$.

Randers metrics are among the simplest Finsler metrics. Let $F = \alpha + \beta$ be a Randers metric on a manifold M^n, where $\alpha^2 = a_{ij}(x)y^iy^j$ is a Riemannian metric and $\beta = b_i(x)y^i$ is a 1-form on M^n with $\|\beta\| < 1$ [9].The space $R^n = (M^n, \alpha)$ is called the associated Riemannian space with $F^n = (M^n, L(\alpha, \beta))$. The spray coefficients G^i of F are given by

$$G^j = \frac{1}{4}g^{jh}\left(\frac{\partial^2 F^2}{\partial y^h \partial x^k}y^k - \frac{\partial F^2}{\partial x^h}\right), G^i_j = \frac{\partial G^i}{\partial y^j}. \tag{1}$$

The spray coefficients G^i of F and $G^i\alpha$ of α are related by [17]

$$G^i = G^i_\alpha + Py^i + Q^i, \text{ where} \tag{2}$$

$$\begin{aligned} P = \frac{e_{00}}{2F} - s_0, Q^i = \alpha s^i_0, \\ e_{ij} = r_{ij} + b_i s_j + b_j s_i, \\ r_{ij} = \frac{1}{2}(b_{i|j} + b_{j|i}), s_{ij} = \frac{1}{2}(b_{i|j} - b_{j|i}), \\ s^i_j = a^{ih}s_{hj}, s_j = b_i s^i_j = b^i s_{ij}, \\ e_{00} = e_{ij}y^iy^j, s_0 = s_i y^i, s^i_0 = s^i_j y^j, \end{aligned} \tag{3}$$

where | denote the covariant differentiation with respect to the Levi-Civita connection $\gamma^i_{jk}(x)$ of R^n.
The spray coefficients G^i_j are used to define Riemann tensor R^i_j as follows.

$$R^i_j = l^h \left(\frac{\delta}{\delta x^j} \left(\frac{G^k_h}{F} \right) - \frac{\delta}{\delta x^h} \left(\frac{G^k_j}{F} \right) \right), l^h = \frac{y^h}{F}, \tag{4}$$

where

$$\frac{\delta}{\delta x^i} = \frac{\partial}{\partial x^i} - G^j_i \frac{\partial}{\partial y^j}. \tag{5}$$

A Finsler metric F has constant flag curvature λ if

$$\begin{aligned} R^i{}_k &= \lambda F^2 h^i{}_k, \\ \text{where } h^i{}_k &= h_{jk} g^{ij}, h_{ij} = (g_{ij} - l_i l_j), l_i = l^j g_{ij}. \end{aligned} \tag{6}$$

The Ricci curvature and Ricci scalar are defined by

$$Ric = R^i{}_i, \quad R = \frac{1}{n+1} Ric. \tag{7}$$

A Finsler metric F has constant Ricci curvature if

$$Ric = (n-1)\lambda F^2. \tag{8}$$

2.2. **Almost contact metric structures and Generalized Sasakian space forms.** An n- dimensional Riemannian manifold(M, α) is said to be an almost contact metric manifold if there exist on M, a (1,1) tensor field ϕ, a unit vector field ξ, a 1-form η satisfying

$$\varphi^2 = -I + \eta \otimes \xi, \eta(\xi) = 1, \varphi\xi = 0, \ \eta \circ \varphi = 0. \tag{9}$$

$$a(\phi X, \phi Y) = a(X, Y) - \eta(X)\eta(Y), \tag{10}$$

$$a(X, \phi Y) + a(\phi X, Y) = 0, \forall X, Y \in TM. \tag{11}$$

We denote an almost contact metric manifold by $M(\phi, \xi, \eta, a)$. An almost contact metric manifold is called a contact metric manifold if $d\eta = \Phi$, where Φ is the fundamental 2-form on M given by $\Phi(X, Y) = a(X, \phi Y)$. An almost contact metric manifold $M(\phi, \xi, \eta, a)$ is called

- Sasakian manifold if

$$(\nabla_X \phi)Y = a(X, Y)\xi - \eta(Y)X, \tag{12}$$

- Kenmotsu manifold if

$$(\nabla_X \phi)Y = -a(X, Y)\xi + \eta(Y)\phi X, \tag{13}$$

- a (a,b)-trans-Sasakian manifold if

(14) $$(\nabla_X \phi)Y = a(a(X,Y)\xi - \eta(Y)X) + b(a(\phi X,Y)\xi - \eta(Y)\phi X),$$

where a and b are some functions.

In a trans-Sasakian manifold M, the following hold:

(15)
$$(\nabla_X \xi) = -\phi X + \beta(X - \eta(X)\xi), \;\; (\nabla_X \eta)Y = a(\nabla_X \xi, Y), \forall X \in TM.$$

If $b = 0$, M is said to be an a- Sasakian manifold and Sasakian manifolds are the examples of a- Sasakian manifolds with $a = 1$.
If $a = 0$,M is said to be an b- Kenmotsu manifold and Kenmotsu manifolds are examples of b- Kenmotsu manifolds with $b = 1$.
If X is orthonormal to ξ then the plane section $\{X, \phi X\}$ is called ϕ-section and the curvature associated with this section is called ϕ-sectional curvature which is given by

(16) $$H(X) = K(X, \phi X) = R(X, \phi X, X, \phi X).$$

A Sasakian manifold of constant ϕ-sectional curvature c is called a Sasakian space form $M(c)$.
A Kenmotsu manifold of constant ϕ-sectional curvature c is called a Kenmotsu space form.
An almost contact metric manifold (M, ϕ, ξ, η, a) is called a generalized Sasakian space forms if the curvature tensor R is given by

(17)
$$\begin{aligned} R(X,Y)Z &= f_1\{g(Y,Z)X - g(X,Z)Y\} \\ &+ f_2\{g(X,\phi Z)\phi Y - g(Y,\phi Z)\phi X + 2g(X,\phi Y)\phi Z\} \\ &+ f_3\{\eta(X)\eta(Z)Y - \eta(Y)\eta(Z)X + g(X,Z)\eta(Y)\xi - g(Y,Z)\eta(X)\xi\}, \end{aligned}$$

where f_1, f_2, f_3 differential functions on M.
Sasakian space forms are examples of generalized Sasakian space forms with constant functions $f_1 = \frac{c+3}{4}, f_2 = f_3 = \frac{c-1}{4}$.

3. Randers spaces of constant curvature induced by generalized Sasakian space forms

Yasuda-Shimada[16] gave the following calssical theorem for Randers spaces of constant curvature.

Theorem 1. *Let $F^n = (M, F = \alpha + \beta), \alpha = \sqrt{a_{ij}(x)y^i y^j}, \beta = b_i(x)y^i$ be a Randers space. Then*

[A] F^n has constant negative flag curvature λ and $curl_{b_i} = b^j(b_{j|i} - b_{i|j}) = 0$ if and only if the Riemannian space (M, α) is of negative

constant sectional curvature $-\lambda^2$. $b_{i|j} = \lambda(a_{ij} - b_i b_j)$,
[B] F^n *is flat (i.e.* $\lambda = 0$ *) and* $curl_{bi} = 0$ *if and only if it is locally Minkowski,*
[C] F^n *has constant positive flag curvature* λ *and* $curl_{bi} = 0$ *if and only if the Riemannian curvature tensor* R *of the Riemanian space* (M, α) *satisfies*

$$\begin{aligned} R_{hikj} &= \lambda(b_h b_j a_{ik} - b_k b_h a_{ij}) + \lambda(b_i b_k a_{hj} - b_i b_j a_{kh}) \\ &+ \lambda(1 - \|b\|^2)(a_{hj} a_{ik} - a_{kh} a_{ij}) \\ &+ 2b_{i|h} b_{j|k} - b_{i|k} b_{h|j} - b_{i|j} b_{k|h} \end{aligned}$$

and $\|b\|$ *is a constant and* $b_{j|k} + b_{k|j} = 0$.

In an a-Sasakian manifold,

$$\eta_{i|j} = -a\phi_i^k a_{jk}, \tag{18}$$

$$\eta_{i|j} + \eta_{j|i} = 0, \tag{19}$$

$$\eta_{i|j} - \eta_{j|i} = 2a\phi_i^j a_{jk}, \tag{20}$$

$$\eta_{i|j}\xi^i = \eta_{i|j}\xi^j = 0, \tag{21}$$

$$\xi^i \left(\eta_{i|j} - \eta_{j|i}\right) = 0. \tag{22}$$

The expression for R in a generalized Sasakian space form is

$$\begin{aligned} R_{hijk} = f_1 \left(a_{hj} a_{ik} - a_{hk} a_{ij}\right) - \frac{f_2}{a^2} \left(\eta_{i|j}\eta_{h|k} - \eta_{h|j}\eta_{i|k} + 2\eta_{i|h}\eta_{j|k}\right) + \\ f_3 \left(\eta_i \eta_j a_{hk} - \eta_h \eta_j a_{ik} + \eta_h \eta_k a_{ij} - \eta_i \eta_k a_{hj}\right). \end{aligned} \tag{23}$$

Suppose M is an odd dimensional almost contact metric manifold with almost contact metric structure (ϕ, ξ, η, a). Then we have a Riemannian metric a and a 1-form $\eta = \eta_i(x)dx^i$ on M. We can construct in a natural way a Randers metric $F = \alpha + \beta$, where $\alpha = \sqrt{a_{ij}(x)y^i y^j}$, $\beta = c\eta_i(x)y^i$, with $0 < c < 1$. Then F is a Randers metric on M which is positive definite. This is because $\|b\|^2 = c^2\|\eta\|^2 = c^2 < 1$. Further we have

(1) $\|b\| = c$, a constant.
(2) $(\nabla_{\phi Y} b)Y = c(\nabla_{\phi Y}\eta)Y = -ac\ a(\phi Y, \phi Y) \neq 0$. i.e. b is not parallel.
(3) $b_{i|j} + b_{j|i} = c(\eta_{i|j} + \eta_{j|i}) = 0$, follows from 19.

Theorem 2 (P. Alegre and A. Carriazo, 2008)**.** *If* $M(f_1, f_2, f_3)$ *is an a-Sasakian generalized Sasakian space form, then* $\xi(a) = 0$ *and* $f_1 - f_3 = a^2$ *holds. Further if* M *is connected then* a *is a constant.*

If we choose $f_2 = f_3 = -c^2a^2$, then from the above theorem, $f_1 = -a^2(1-c^2)$. The equation (23) takes the form

$$\begin{aligned} R_{hijk} &= \lambda(1-\|b\|^2)\left(a_{hj}a_{ik} - a_{hk}a_{ij}\right) \\ &+ \lambda\left(2\eta_{i|h}\eta_{j|k} - b_{i|j}b_{k|h} - b_{h|j}b_{i|k}\right) \\ &+ \lambda\left(b_ib_ka_{hj} + b_jb_ha_{ik} - b_ib_ja_{hk} - b_hb_ka_{ij}\right), with\lambda = a^2. \end{aligned} \tag{24}$$

Thus by theorem 1, we conclude that

Theorem 3. *Let $(M, F = \alpha + \beta)$ be the Randers space induced by a connected a-Sasakian generalized Sasakian space form $M(f_1, f_2, f_3)$, where $\alpha = \sqrt{a_{ij}(x)y^iy^j}, \beta = c\eta_i(x)y^i$, where $0 < c < 1$. If $f_2 = f_3 = -c^2a^2$ then the Randers space (M, F) is a Randers space of positive constant curvature $\lambda = a^2$.*

This theorem holds for $c = 1$ also. But the Randers metric is not positive definite. Randers-space of constant curvature 1. We define on M,

$$\begin{aligned} a^*_{ij} &= a^2 a_{ij} \\ b^*_i(x) &= ab_i(x) \end{aligned} \tag{25}$$

Then the function $F^*(x, y) = aF(x, y)$ is a Randers metric on TM^0. Now from $g_{ij} = \frac{1}{2}\frac{\partial F^2}{\partial y^i \partial y^j}$, we get

$$\begin{aligned} g^*_{ij} = a^2 g_{ij}, g^{ij*} = \frac{1}{a^2}g^{ij}, l^*_i = al_i, l^{i*} = \frac{1}{a}l^i, \\ h^*_{ij} = a^2h_{ij}, h^{ij*} = \frac{1}{a^2}h^{ij}, h^{i*}{}_j = h^i{}_j. \end{aligned} \tag{26}$$

From (1),(4) and (6), it follows that the spray coefficients G^{*i} and the Riemann tensor $R^{*i}{}_j$ are given by

$$G^{i*} = G^i, R^{i*}{}_j = R^i{}_j. \tag{27}$$

If F is a Randers metric of constant curvature a^2, then $R^i{}_j = a^2F^2h^i{}_j$. Therefore $R^{*i}{}_j = R^i{}_j = a^2F^2h^i{}_j = F^{*2}h^{*i}{}_j$.
This shows that the Randers metric F^* is of constant curvature 1.
This Randers metric is positive definite also. Thus we can state

Theorem 4. *Let $(M, F = \alpha + \beta)$ be the Randers space induced by a connected a-Sasakian generalized Sasakian space form $M(f_1, f_2, f_3)$, where $\alpha = \sqrt{a_{ij}(x)y^iy^j}, \beta = \eta_i(x)y^i$. If $f_2 = f_3 = -c^2a^2$ then the Randers space $(M, F^* = aF)$ is a Randers space of curvature* 1.

In an b-Kenmotsu manifold, we have

$$\eta_{i|j} = b(a_{ij} - \eta_i\eta_j), \tag{28}$$

$$\eta_{i|j} + \eta_{j|i} = b(a_{ij} - \eta_i\eta_j), \tag{29}$$

$$\eta_{i|j} - \eta_{j|i} = 0, \tag{30}$$

$$\eta_{i|j}\xi^i = \eta_{i|j}\xi^j = 0, \tag{31}$$

$$\xi^i\left(\eta_{i|j} - \eta_{j|i}\right) = 0. \tag{32}$$

Suppose the Randers metric $F = \alpha + \beta$, where $\alpha = \sqrt{a_{ij}(x)y^iy^j}, \beta = \eta_i(x)y^i$ is induced by a b-Kenmotsu manifold. Then F is a Randers metric on M which is not positive definite. For a Randers metric $F = \alpha + \beta$ induced by a b-Kenmotsu manifold, we have $b_i = \eta_i$, and so $s_{ij} = \frac{1}{2}(b_{i|j} - b_{j|i}) = 0, s_i = b^j s_{ij} = 0$, $r_{ij} = b(a_{ij} - b_ib_j)$ and $e_{00} = 2c(a_{ij} - b_ib_j)$, where $c = \frac{b}{2}$.
Hence the spray coefficients G^i and the Riemann tensor $R^i{}_j$ of F are given by

$$G^i = G^i\alpha + \tfrac{\chi}{2F} \tag{33}$$

$$R^i{}_k = \alpha R^i{}_k + 3(\tfrac{\chi^2}{4F^2} - \tfrac{\Psi}{2F})\left(\delta^i_k - \tfrac{F_{y^k}}{F}y^i\right) + \tau_k y^i, \tag{34}$$

where $\chi = b_{i|j}y^iy^j, \Psi = b_{i|j|k}y^iy^jy^k, \tau_k = \frac{1}{F}(b_{i|j|k} - b_{i|k|j})y^iy^j = -\frac{1}{F}b_j \ \ \alpha R^j_k$.
Suppose F has constant Ricci curvature λ and α has constant curvature μ. Then

$$\begin{aligned} Ric &= (n-1)\lambda F^2, \\ \alpha R^i{}_k &= \mu\alpha^2\left(\delta^i_k - \frac{\alpha_{y^k}}{\alpha}y^i\right), \alpha Ric = (n-1)\mu\alpha^2. \end{aligned} \tag{35}$$

From (34), we obtain

$$Ric = \alpha Ric + 3(n-1)\left(\left(\frac{\chi}{2F}\right)^2 - \frac{\Psi}{2F}\right) \tag{36}$$

and $R^i{}_k = \lambda F^2\delta^i_k + \widetilde{\tau}_k y^i$, where $\widetilde{\tau}_k y^k = -\lambda F^2$.
i.e. F has constant curvature λ.
From (36), it follows that F has constant Ricci curvature λ if and only if

$$\mu\alpha^2 + 3\left(\left(\frac{\chi}{2F}\right)^2 - \frac{\Psi}{2F}\right) = \lambda F^2. \tag{37}$$

The above equation is equivalent to following two equations:

$$3\chi^2 = 2\beta\Psi + 4(\lambda - \mu)\alpha^4 + 4(6\lambda - \mu)\alpha^2\beta^2 + 4\lambda\beta^2. \tag{38}$$

$$\Psi = 4(\mu - 2\lambda)\alpha^2\beta - 8\lambda\beta^3. \tag{39}$$

The equation (39) in (38) gives

$$3\chi^2 = 4(\lambda - \mu)\alpha^4 + 4(2\lambda + \mu)\alpha^2\beta^2 - 12\lambda\beta^4. \tag{40}$$

Differentiating covariantly and transvecting the resulting by y^k, we obtain

$$3\Psi = 4(\mu + 2\lambda)\alpha^2\beta - 24\lambda\beta^3. \tag{41}$$

From (39) and (41), we obtain $\mu = 4\lambda$. Putting this in (40), we obtain $\chi^2 = -4\lambda(\alpha^2 - \beta^2)$.
This shows that λ is negative. Thus we have

Theorem 5. *Let $(M, F = \alpha + \beta)$ be the Randers space induced by an b-Kenmotsu manifold, where $\alpha = \sqrt{a_{ij}(x)y^i y^j}, \beta = \eta_i(x)y^i$. If F has constant Ricci curvature λ and α has constant curvature μ, then $\lambda = \frac{1}{4}\mu$ and this λ is negative.*

We next see when α has constant curvature. We have,

(1) the ϕ-sectional curvature of a generalized Sasakian space form $M(f_1, f_2, f_3)$ is $f_1 + 3f_2$ [P. Alegre and A. Carriazo, 2008].
(2) a Kenmotsu manifold of constant ϕ-sectional curvature is a Riemannian space form of constant sectional curvature -1 [Kenmotsu 1972].

Thus if f_1 and f_2 are constants, then b-Kenmotsu generalized Sasakian space form is a Riemannian space form of constant sectional curvature -1. i.e. α has constant curvature -1. From the above discussions, we have

Theorem 6. *Let $(M, F = \alpha + \beta)$ be the Randers space induced by an b-Kenmotsu generalized Sasakian space form with f_1, f_2 as constants. If F has constant Ricci curvature λ ,then α has constant curvature -1 and F has constant curvature $\lambda = -\frac{1}{4}$.*

Let $F = \alpha + \beta$ be a Randers metric induced by b-Kenmotsu manifold.
Since $b_i = \eta_i$, from (29) and (30), we have
$s_{ij} = \frac{1}{2}(b_{i|j} - b_{j|i}) = 0$ and $e_{00} = 2c(a_{ij} - b_i b_j)$, where $c = \frac{b}{2}$. We have the following theorem due to [Zhongmin Shen,2003]:

Theorem 7. *Let $F = \alpha + \beta$ be a Randers metric of constant curvature $K = \lambda$ on a manifold M. Suppose that F satisfies $e_{00} = 2c(\alpha^2 - \beta^2)$ for* *some scalar function $c(x)$ on M and β is closed. Then $\lambda = -c^2 \leq 0$. F is either locally Minkowskian $\lambda = 0$ or curvature in the form $\lambda = -c^2$.*

Thus it follows that if $b = 0$, then $\lambda = 0$ and the Randers space (M, F) reduces to a locally Minkowskian space.
If $b \neq 0$, then the Randers space (M, F) is of constant negative curvature $\lambda = -\frac{b^2}{4}$. Thus we state that

Theorem 8. *If M is an b-Kenmotsu generalized Sasakian space form with f_1 and f_2 are constants and if the induced Randers metric$F = \alpha + \beta$ has constant Ricci curvature, then b must be equal to one.*

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Department of Mathematics, Bangalore University,
Central College Campus, Bangalore 560001, INDIA.
e-mail:hgnraj@yahoo.com.