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Scientific Studies and Research
Series Mathematics and Informatics
Vol. 25(2015), No. 1, 225-236

ON $\pi g\mathcal{I}$-CONTINUOUS FUNCTIONS ON IDEAL TOPOLOGICAL SPACES

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Abstract. In this paper, $\pi g\mathcal{I}$-continuous, quasi semi-$*$-normal spaces, $\pi g\mathcal{I}$ -regular spaces are introduced. Characterizations and properties of newly introduced notions are studied, as well as some connections between this notions.

1. Introduction and preliminaries

Let (X, τ) be a topological space with no separation properties assumed. For a subset A of a topological space (X, τ), $cl(A)$ and $int(A)$ denote the closure and interior of A in (X, τ) respectively.

An ideal $\mathcal{I}$ on a topological space (X, τ) is a nonempty collection of subsets of X which satisfies: (1) $A \in \mathcal{I}$ and $B \subseteq A$ implies $B \in \mathcal{I}$ (2) $A \in \mathcal{I}$ and $B \in \mathcal{I}$ implies $A \cup B \in \mathcal{I}$.

If (X, τ) is a topological space and $\mathcal{I}$ is an ideal on X, then $(X, \tau, \mathcal{I})$ is called an ideal topological space or an ideal space.

Keywords and phrases: $\pi g\mathcal{I}$-continuous, quasi semi -$*$- normal spaces, $\pi g\mathcal{I}$-regular spaces.
(2010) Mathematics Subject Classification: 54A05, 54A10.

Let $\mathcal{P}(X)$ be the power set of X. Then the operator $()^* : \mathcal{P}(X) \to \mathcal{P}(X)$ is called a local function [7] of A with respect to τ and $\mathcal{I}$, is defined as follows: for $A \subseteq X$, $A^*(\mathcal{I}, \tau) = \{x \in X : U \cap A \notin \mathcal{I}$ for every open set U containing $x\}$. We simply write A^* instead of $A^*(\mathcal{I}, \tau)$ in case there is no confusion. For every ideal topological space $(X, \tau, \mathcal{I})$ there exists a topology τ^* finer than τ, generated by $\beta(\mathcal{I}, \tau) = \{U \setminus J : U \in \tau$ and $J \in \mathcal{I}\}$ but in general $\beta(\mathcal{I}, \tau)$ is not always a topology.

Additionally $cl^*(A) = A \cup A^*$ defines a Kuratowski closure operator for the topology τ^*. Throughout this paper X denotes the ideal topological space $(X, \tau, \mathcal{I})$ and $int^*(A)$ denotes the interior of A with respect to τ^*.

DEFINITION 1.1. Let (X, τ) be a topological space. A subset A of X is said to be semi-open [8] if there exists an open set U in X such that $U \subseteq A \subseteq cl(U)$. The complement of a semi-open set is said to be semi-closed. The collection of all semi-open (resp. semi-closed) sets in X is denoted by $SO(X)$ (resp. $SC(X)$). The semi-closure of A in (X, τ) is denoted by the intersection of all semi-closed sets containing A and is denoted by $scl(A)$.

DEFINITION 1.2. For $A \subseteq X, A_*(\mathcal{I}, \tau) = \{\ x \in X/U \cap A \notin \mathcal{I}$for every $U \in SO(X)\}$ is called the semi-local function [5] of A with respect to $\mathcal{I}$ and τ, where $SO(X, x) = \{U \in SO(X) : x \in U\}$. We simply write A_* instead of $A_*(\mathcal{I}, \tau)$ in this case there is no ambiguity.

It is given in [1] that $\tau^{*s}(\mathcal{I})$ is a topology on X, generated by the sub basis $\{U - E : U \in SO(X)$ and $E \in I\}$ or equivalently $\tau^{*s}(\mathcal{I}) = \{U \subseteq X : cl^{*s}(X - U) = X - U\}$. The closure operator cl^{*s} for a topology $\tau^{*s}(\mathcal{I})$ is defined as follows: for $A \subseteq X, cl^{*s}(A) = A \cup A_*$ and $int^{*s}(A)$ denotes the interior of the set A in $(X, \tau^{*s}, \mathcal{I})$. It is known that $\tau \subseteq \tau^*(\mathcal{I}) \subseteq \tau^{*s}(\mathcal{I})$. A subset A of $(X, \tau, \mathcal{I})$ is called semi-$*$-perfect [6] if $A = A_*$. $A \subseteq (X, \tau, \mathcal{I})$ is called $*$-semi dense in-itself [6](resp. semi-$*$-closed [6]) if $A \subset A_*$ (resp. $A_* \subseteq A$). A subset A of an ideal topological space $(X, \tau, \mathcal{I})$ is said to be gI-closed [6] if $A_* \subseteq U$ whenever is open and $A \subseteq U$. A subset A of an ideal topological space $(X, \tau, \mathcal{I})$ is said to be gI-open [6] if $X - A$ is gI-closed.

A subset A of a space (X, τ) is said to be regular open [14] if $A = int(cl(A))$ and A is said to be regular closed [14] if $A = cl(int(A))$. Any finite union of regular open sets in (X, τ) is π-open [16] in(X, τ). The complement of a π-open [16] set in (X, τ) is π-closed in (X, τ) A subset A of a space (X, τ) is said to be g-closed [9] if $cl(A) \subseteq U$

whenever $A \subseteq U$ and U is open. A subset A of a space (X, τ) is said to be πgs-closed [2] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is π-open.

A subset A of an ideal topological space $(X, \tau, \mathcal{I})$ is said to be $\mathcal{I}_{\pi g}$-closed [12] (resp. $\pi g\mathcal{I}$-closed [13]) if $A^* \subseteq U$ (resp. $A_* \subseteq U$) whenever U is π-open and $A \subseteq U$. A subset A of an ideal topological space $(X, \tau, \mathcal{I})$ is said to be $\mathcal{P}_{\mathcal{I}_s}$-set [13] if $A = U \cap V$, where U is a π-open and V is a semi-$*$-closed set. An ideal space $(X, \tau, \mathcal{I})$ is said to be an $\pi T_{\mathcal{I}}$-space [13] if every $\pi g\mathcal{I}$-closed set is a semi-$*$-closed set. A function $f : (X, \tau) \longrightarrow (Y, \sigma)$ is said to be m-π-closed [3] if $f(A)$ is π-closed in (Y, σ) for every π-closed set A in (X, τ).A function $f : (X, \tau, \mathcal{I}) \longrightarrow (Y, \sigma)$ is said to be $\mathcal{I}_{\pi g}$-continuous[12] if $f^{-1}(A)$ is $\mathcal{I}_{\pi g}$-closed in $(X, \tau, \mathcal{I})$ for every closed subset A of Y.

LEMMA 1.3. [5] *Let $(X, \tau, \mathcal{I})$ be an ideal topological space and A, B be subsets of X. Then for the semi-local function the following properties hold:*

(1) If $A \subseteq B$ then $A_* \subseteq B_*$.
(2) If $U \in \tau$ then $U \cap A_* \subseteq (U \cap A)_*$
(3) $A_* = scl(A_*) \subseteq scl(A)$ and A_* is semi-closed in X.
(4) $(A_*)_* \subseteq A_*$.
(5) $(A \cup B)_* = A_* \cup B_*$.
(6) If $\mathcal{I} = \{\emptyset\}$, then $A_* = scl(A)$.

2. $\pi g\mathcal{I}$-CONTINUOUS FUNCTIONS

DEFINITION 2.1. A function $f : (X, \tau, \mathcal{I}) \longrightarrow (Y, \sigma)$ is said to be

(1) $\pi g\mathcal{I}$-continuous if $f^{-1}(A)$ is $\pi g\mathcal{I}$-closed in $(X, \tau, \mathcal{I})$ for every closed set A of Y.
(2) $g\mathcal{I}$-continuous if $f^{-1}(A)$ is $g\mathcal{I}$-closed in $(X, \tau, \mathcal{I})$ for every closed set A of Y.
(3) semi-$*$-continuous if $f^{-1}(A)$ is semi-$*$-closed in $(X, \tau, \mathcal{I})$ for every closed set A of Y.
(4) almost $\pi g\mathcal{I}$-continuous if $f^{-1}(A)$ is $\pi g\mathcal{I}$-closed in $(X, \tau, \mathcal{I})$ for every regular closed set A of Y.
(5) almost $g\mathcal{I}$-continuous if $f^{-1}(A)$ is $g\mathcal{I}$-closed in $(X, \tau, \mathcal{I})$ for every regular closed set A of Y.
(6) almost semi-$*$-continuous if $f^{-1}(A)$ is semi-$*$-closed in $(X, \tau, \mathcal{I})$ for every regular closed set A of Y.
(7) $\mathcal{P}_{\mathcal{I}_s}$-continuous if $f^{-1}(A)$ is $\mathcal{P}_{\mathcal{I}_s}$-set in $(X, \tau, \mathcal{I})$ for every closed set A of Y.

THEOREM 2.2. *For a function $f : (X, \tau, \mathcal{I}) \longrightarrow (Y, \sigma)$, the following properties hold:*

(a) If f is $\mathcal{I}_{\pi g}$-continuous, then f is $\pi g\mathcal{I}$-continuous.

(b) If f is $g\mathcal{I}$-continuous, then f is $\pi g\mathcal{I}$-continuous.

(c) If f is πg-continuous, then f is $\pi g\mathcal{I}$-continuous.

REMARK 2.3. *Converse of the Theorem 2.2 need not be true as seen from the following example.*

EXAMPLE 2.4. *(a) Let $X = \{a, b, c, d\}, \tau = \{\emptyset, \{d\}, \{a, c\}, \{a, c, d\}, X\}$, $\sigma = \{\emptyset, \{a\}, \{a, c\}, X\}$ and $\mathcal{I} = \{\emptyset, \{c\}, \{d\}, \{c, d\}\}$. Let $f : (X, \tau, \mathcal{I}) \longrightarrow (X, \sigma)$ be a function defined as follows $f(b) = a$, $f(a) = b$, $f(c) = c$, and $f(d) = d$. Then f is $\pi g\mathcal{I}$-continuous but it is not $\mathcal{I}_{\pi g}$-continuous.*

(b) Let $X = \{a, b, c, d\}$, $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{b, c\}, X\}$, $\sigma = \{\emptyset, \{a, c, d\}, X\}$ and $\mathcal{I} = \{\emptyset, \{a\}\}$. Then the identity function $f : (X, \tau, \mathcal{I}) \longrightarrow (X, \sigma)$ is $\pi g\mathcal{I}$-continuous but it is not $g\mathcal{I}$-continuous.

(c) Let $X = \{a, b, c, d, e\}$, $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}, \{a, b, c, d\}, \{b, c, d, e\}, X\}$, $\sigma = \{\emptyset, \{c\}, X\}$ and $\mathcal{I} = \{\emptyset, \{b\}, \{d\}, \{b, d\}\}$. Then the identity function $f : (X, \tau, \mathcal{I}) \longrightarrow (Y, \sigma)$ is $\pi g\mathcal{I}$-continuous but it is not πg-continuous.

REMARK 2.5. *We have the following diagram among several continuity defined above.*

$$\begin{tikzcd}
\text{semi-}*\text{-continuous} \arrow[r] \arrow[d] & \text{almost semi-}*\text{-continuous} \arrow[d] \\
g\mathcal{I}\text{-continuous} \arrow[r] \arrow[d] & \text{almost } g\mathcal{I}\text{-continuous} \arrow[d] \\
\pi g\mathcal{I}\text{-continuous} \arrow[r] & \text{almost } \pi g\mathcal{I}\text{-continuous}
\end{tikzcd}$$

EXAMPLE 2.6. *Let $X = \{a, b, c\}, \tau = \{\emptyset, \{c\}, \{b, c\}, X\}$, $\sigma = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{b, c\}, X\}$ and $\mathcal{I} = \{\emptyset, \{b\}\}$. Define a function $f : (X, \tau, \mathcal{I}) \longrightarrow (X, \sigma)$ be a function defined as follows $f(b) = b$ and $f(a) = f(c) = c$. Then f is almost $\pi g\mathcal{I}$-continuous but it is not almost $g\mathcal{I}$-continuous.*

EXAMPLE 2.7. *Let $X = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, \{a, b\}, X\}$, $\sigma = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, c\}, X\}$ and $\mathcal{I} = \{\emptyset, \{b\}\}$. Then the identity function $f : (X, \tau, \mathcal{I}) \longrightarrow (Y, \sigma)$ is almost semi-$*$-continuous but it is not semi-$*$-continuous.*

EXAMPLE 2.8. *Let* $X = \{a,\ b,\ c\}$, $\tau = \{\emptyset, \{a\}, \{a,b\}, X\}$, $\sigma = \{\emptyset, \{a\}, \{b\}, \{a,b\}, \{a,c\}, X\}$ *and* $\mathcal{I} = \{\emptyset, \{b\}\}$. *Define a function* $f : (X,\tau,\mathcal{I}) \longrightarrow (X,\sigma)$ *be a function defined as follows* $f(a) = c, f(b) = b$ *and* $f(c) = a$. *Then* f *is almost* $g\mathcal{I}$*-continuous but it is not* $g\mathcal{I}$*-continuous.*

EXAMPLE 2.9. *Let* $X = \{a,b,c,d\}$, $\tau = \{\emptyset, \{c\}, \{a,b\}, \{a,b,c\}, X\}$, $\sigma = \{\emptyset, \{a\}, \{b\}, \{a,b\}, X\}$ *and* $\mathcal{I} = \{\emptyset, \{a\}\}$. *Then the identity function* $f : (X,\tau,\mathcal{I}) \longrightarrow (Y,\sigma)$ *is* $g\mathcal{I}$*-continuous but it is not semi-*$*$*-continuous.*

EXAMPLE 2.10. *Let* $X = \{a,b,c,d\}$, $\tau = \{\emptyset, \{c\}, \{a,b\}, \{a,b,c\}, X\}$, $\sigma = \{\emptyset, \{d\}, X\}$ *and* $\mathcal{I} = \{\emptyset, \{a\}\}$. *Then the identity function* $f : (X,\tau,\mathcal{I}) \longrightarrow (Y,\sigma)$ *is almost* $\pi g\mathcal{I}$*-continuous but it is not* $\pi g\mathcal{I}$*-continuous.*

EXAMPLE 2.11. *Let* $X = \{a,b,c,d\}, Y = \{a,b,c\}$, $\tau = \{\emptyset, \{c\}, \{a,b\}, \{a,b,c\}, X\}$, $\sigma = \{\emptyset, \{a\}, \{b\}, \{a,b\}, \{a,c\}, X\}$ *and* $\mathcal{I} = \{\emptyset, \{a\}\}$. *Then the identity function* $f : (X,\tau,\mathcal{I}) \longrightarrow (Y,\sigma)$ *is almost* $g\mathcal{I}$*-continuous but it is not almost semi-*$*$*-continuous.*

THEOREM 2.12. *A function* $f : (X,\tau,\mathcal{I}) \longrightarrow (Y,\sigma)$, *the following are equivalent.*

(a) f *is almost* $\pi g\mathcal{I}$*-continuous.*
(b) $f^{-1}(A) \in \pi GIC(X)$ *for every regular closed set* A *of* Y.
(c) $f^{-1}(cl(int(A))) \in \pi GIC(X)$ *for every closed set* A *of* Y.
(d) $f^{-1}(int(cl(A))) \in \pi GIO(X)$, *for every* $A \in \sigma$.

Proof. $(a) \Longleftrightarrow (b)$. Obvious

$(b) \Longleftrightarrow (c)$. Suppose A is a regular closed set in Y, we have $A = cl(int(A))$ and $f^{-1}(cl(int(A))) \in \pi GIC(X)$. Conversely, suppose A is closed set in Y, we have $cl(int(A))$ is regular closed and $f^{-1}(cl(int(A)))$ $\in \pi GIC(X)$.

$(c) \Longleftrightarrow (d)$. Let A be an open set in Y. Then $Y - A$ is closed in Y, we have $f^{-1}(cl(int(Y - A))) = f^{-1}(Y - int(cl(A))) = X - f^{-1}(int(cl(A))) \in \pi GIC(X)$. Hence $f^{-1}(int(cl(A))) \in \pi GIO(X)$. A converse can be obtained similarly.

PROPOSITION 2.13. *If a function* $f : (X,\tau,\mathcal{I}) \longrightarrow (Y,\sigma)$ *is* $\pi g\mathcal{I}$*-continuous and* X *is* $\pi T_{\mathcal{I}}$*-space then* f *is semi-*$*$*-continuous.*

Proof. Let F be any closed set in Y. Since f is $\pi g\mathcal{I}$-continuous, $f^{-1}(F)$ is $\pi g\mathcal{I}$-closed in X and hence $f^{-1}(F)$ is semi-$*$-closed in X. Therefore f is semi-$*$-continuous.

PROPOSITION 2.14. *If a function $f : (X, \tau, \mathcal{I}) \longrightarrow (Y, \sigma)$ is almost $\pi g\mathcal{I}$-continuous and X is $\pi T_{\mathcal{I}}$-space then f is almost semi-$*$-continuous.*

Proof. Let V be any regular closed set in Y. Since f is almost $\pi g\mathcal{I}$-continuous, $f^{-1}(F)$ is $\pi g\mathcal{I}$-closed in X. But X is a $\pi T_{\mathcal{I}}$-space, therefore $f^{-1}(F)$ is semi-$*$-closed in X. Hence f is almost semi-$*$-continuous.

DEFINITION 2.15. A function $f : (X, \tau, \mathcal{I}) \longrightarrow (Y, \sigma, \mathcal{I})$ is said to be $\pi g\mathcal{I}$-irresolute if $f^{-1}(A)$ is $\pi g\mathcal{I}$-closed in $(X, \tau, \mathcal{I})$ for every $\pi g\mathcal{I}$-closed set A of $(Y, \sigma, \mathcal{I})$.

THEOREM 2.16. *A function $f : (X, \tau, \mathcal{I}) \longrightarrow (Y, \sigma)$ is $\pi g\mathcal{I}$- continuous and m-π-closed, then f is $\pi g\mathcal{I}$-irresolute.*

Proof. Assume that A is $\pi g\mathcal{I}$-closed. Let $f^{-1}(A) \subseteq U$, where U is π-open in X. Then $X - U \subseteq f^{-1}(Y - A)$ and $f(X - U) \subseteq Y - A$. Since f is m-π-closed, $f(X - U)$ is π-closed. Then, since $Y - A$ is $\pi g\mathcal{I}$-open. By [13] Theorem 18, $f(X - U) \subseteq int^{*s}(Y - A) = Y - cl^{*s}(A)$. Thus $f^{-1}(cl^{*s}(A)) \subseteq U$. Since f is $\pi g\mathcal{I}$-continuous, $f^{-1}(cl^{*s}(A))$ is $\pi g\mathcal{I}$-closed. Therefore, $cl^{*s}(f^{-1}(cl^{*s}(A))) \subseteq U$ and hence $cl^{*s}(f^{-1}(A)) \subseteq cl^{*s}(f^{-1}(cl^{*s}(A))) \subseteq U$, which proves that $f^{-1}(A)$ is $\pi g\mathcal{I}$-closed and therefore f is $\pi g\mathcal{I}$-irresolute.

THEOREM 2.17. *A function $f : (X, \tau, \mathcal{I}) \longrightarrow (Y, \sigma)$ is semi-$*$-continuous if and only if it is $\mathcal{P}_{\mathcal{I}_s}$-continuous and $\pi g\mathcal{I}$-continuous.*

Proof. This is an immediate consequence of [13] Theorem 26.

3. Quasi semi-$*$-normal spaces

DEFINITION 3.1. A space $(X, \tau, \mathcal{I})$ is said to be quasi semi-$*$-normal if for any two disjoint π-closed sets A and B in (X, τ), there exist disjoint semi-$*$-open sets U, V such that $A \subseteq U$ and $B \subseteq V$.

DEFINITION 3.2. A space $(X, \tau, \mathcal{I})$ is said to be semi-$*$-normal if for any two disjoint closed sets A and B in (X, τ), there exist disjoint semi-$*$-open sets U, V such that $A \subseteq U$ and $B \subseteq V$.

THEOREM 3.3. *Let $(X, \tau, \mathcal{I})$ be an ideal space. Then the following are equivalent.*

1. *$(X, \tau, \mathcal{I})$ is quasi semi-$*$-normal.*
2. *For every pair of disjoint π-closed sets A and B, there exist disjoint gI-open sets U, V such that $A \subseteq U$ and $B \subseteq V$.*

(3) *For every pair of disjoint π-closed sets A and B, there exist disjoint $\pi g\mathcal{I}$-open sets U, V such that $A \subseteq U$ and $B \subseteq V$.*
(4) *For each π-closed set A and for each π-open set V containing A, there exists an $\pi g\mathcal{I}$-open set U such that $A \subseteq U \subseteq cl^{*s}(U) \subseteq V$.*
(5) *For each π-closed set A and for each π-open set V containing A, there exists an semi-$*$-open set U such that $A \subseteq U \subseteq cl^{*s}(U) \subseteq V$.*

Proof. It is obvious that (1) $\Longrightarrow$ (2) and (2) $\Longrightarrow$ (3).

(3) $\Longrightarrow$ (4). Suppose that A is π-closed and V is a π-open set containing A. Then $A \cap V^c = \emptyset$. By assumption, there exist $\pi g\mathcal{I}$-open sets U and W such that $A \subseteq U$, $V^c \subseteq W$. Since V^c is π-closed and W is $\pi g\mathcal{I}$-open, by [[13] Theorem 18], $V^c \subseteq int^{*s}(W)$ and so $(Int^{*s}(W))^c \subseteq V$. Again, $U \cap W = \emptyset$ implies that that $U \cap int^{*s}(W) = \emptyset$ and so $cl^{*s}(U) \subseteq (int^{*s}(W))^c \subseteq V$. Hence, U is the required $\pi g\mathcal{I}$-open set such that $A \subseteq U \subseteq cl^{*s}(U) \subseteq V$.

(4) $\Longrightarrow$ (5). Let A be a π-closed set and V be a π-open set such that $A \subseteq V$. By hypothesis, there exist $\pi g\mathcal{I}$-open set W such that $A \subseteq W \subseteq cl^{*s}(W) \subseteq V$. By [[13] Theorem 18], $A \subseteq int^{*s}(W)$. If $U = int^{*s}(W)$, then U is a semi-$*$-open set and $A \subseteq U \subseteq cl^{*s}(U) \subseteq cl^{*s}(W) \subseteq V$. Therefore, $A \subseteq U \subseteq cl^{*s}(U) \subseteq V$.

(5) $\Longrightarrow$ (1). Let A and B be disjoint π-closed sets. Then $B^c$is a π-open set containing A. By assumption, there exists an semi-$*$-open set U such that $A \subseteq U \subseteq cl^{*s}(U) \subseteq B^c$. If $V = (cl^{*s}(U))^c$, then U and V are disjoint semi-$*$-open sets such that $A \subseteq U$and $B \subseteq V$.

THEOREM 3.4. *If $f : (X, \tau, \mathcal{I}) \longrightarrow (Y, \sigma, \mathcal{I})$ is an almost $\pi g\mathcal{I}$-continuous m-π-closed injection and Y is quasi semi-$*$-normal, then X is quasi semi-$*$-normal.*

Proof. Let A and B be any disjoint π-closed sets of X. Since f is an m-π-closed injection, $f(A)$ and $f(B)$ are disjoint π-closed sets of Y. By the quasi semi-$*$-normality of Y, there exists disjoint semi-$*$-open sets U and V such that $f(A) \subseteq U$ and $f(B) \subseteq V$. Now, if $G = int(cl(U))$ and $H = int(cl(V))$, then G and H are disjoint regular open sets such that $f(A) \subseteq G$ and $f(B) \subseteq H$. Since f is almost $\pi g\mathcal{I}$-continuous, $f^{-1}(G)$ and $f^{-1}(H)$ are disjoint $\pi g\mathcal{I}$-sets containing A and B respectively. It follows from Theorem 3.3 that X is quasi semi-$*$-normal.

DEFINITION 3.5. A function $f : (X, \tau, \mathcal{I}) \longrightarrow (Y, \sigma, \mathcal{I})$ is said to be $\pi g\mathcal{I}^{*s}$-continuous if $f^{-1}(A)$ is $\pi g\mathcal{I}$-closed in $(X, \tau, \mathcal{I})$ for every semi-$*$-closed subset A of $(Y, \sigma, \mathcal{I})$.

DEFINITION 3.6. A function $f : (X, \tau, \mathcal{I}) \longrightarrow (Y, \sigma, \mathcal{I})$ is said to be $g\mathcal{I}^{*s}$-closed if $f(A)$ is $g\mathcal{I}$-closed in $(Y, \sigma, \mathcal{I})$ for every semi-$*$-closed subset A of $(X, \tau, \mathcal{I})$.

THEOREM 3.7. *Let $f : (X, \tau, \mathcal{I}) \longrightarrow (Y, \sigma, \mathcal{I})$ be $\pi g\mathcal{I}^{*s}$-continuous m-π-closed injection and Y is quasi semi-$*$-normal, then X is quasi semi-$*$-normal.*

Proof. Let A and B are disjoint π-closed subsets of X. Since f is an m-π-closed injection, $f(A)$ and $f(B)$ are disjoint π-closed subsets of Y. By the quasi semi-$*$-normality of Y, there exists disjoint semi-$*$-open subsets U and V of Y such that $f(A) \subseteq U$ and $f(B) \subseteq V$. Since f is $\pi g\mathcal{I}^{*s}$-continuous, $f^{-1}(U)$ and $f^{-1}(V)$ are disjoint $\pi g\mathcal{I}$-open subsets containing A and B respectively. It follows from Theorem 3.3 that X is quasi semi-$*$-normal.

THEOREM 3.8. *Let $f : (X, \tau, \mathcal{I}) \longrightarrow (Y, \sigma, \mathcal{I})$ be a $\pi g\mathcal{I}^{*s}$-continuous closed surjection. If X is quasi semi-$*$-normal, then Y is semi-$*$-normal*

Proof. Let A and B be disjoint closed subsets of Y. Since f is π-continuous, $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint π-closed subsets of X. By the quasi semi-$*$-normality of X, there exists disjoint semi-$*$-open subsets U and V of Y such that $f^{-1}(A) \subseteq U$ and $f^{-1}(B) \subseteq V$. Now we set $K = Y - f(X - U)$ and $L = Y - f(X - V)$. Then K and L are $g\mathcal{I}$-open subsets of Y such that $A \subseteq K, B \subseteq L$. Since A and B are disjoint closed sets, K and L are $g\mathcal{I}$-open. We have $A \subseteq int^{*s}(K)$ and $B \subseteq int^{*s}(L)$ and $int^{*s}(K) \cap int^{*s}(L) = \emptyset$. Hence Y is semi-$*$-normal.

4. $\pi g\mathcal{I}$-Regular Spaces

DEFINITION 4.1. A space (X, τ) with an ideal $\mathcal{I}$ is said to be s-regular [10] (resp. $\mathcal{I}_g$-regular [11]) if for each closed set F and each point $x \notin F$, there exist disjoint semi-open (resp. $\mathcal{I}_g$-open) sets U and V such that $x \in U$ and $F \subseteq V$.

DEFINITION 4.2. An ideal space $(X, \tau, \mathcal{I})$ is called an $\pi g\mathcal{I}$-regular space if for each pair consisting of a point x and closed set B not containing x, there exist disjoint $\pi g\mathcal{I}$-open sets U and V such that $x \in U$ and $B \subseteq V$.

REMARK 4.3. *(1) Every regular space is $\mathcal{I}_g$-regular.*
(2) Every $\mathcal{I}_g$regular space is $\pi g\mathcal{I}$-regular.

EXAMPLE 4.4. *Let $X = \{a, b, c, d\}$, $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, d\}, \{a, b, d\}, X\}$ and $\mathcal{I} = \{\emptyset, \{b\}, \{c\}, \{b, c\}\}$. Then $(X, \tau, \mathcal{I})$ is $\pi g\mathcal{I}$- regular but it is not $\mathcal{I}_g$-regular since we can not find disjoint $\mathcal{I}_g$-open sets for $\{a\}$ and a closed set $\{b,\ c,\ d\}$.*

THEOREM 4.5. *Let $(X, \tau, \mathcal{I})$ be an ideal space. Then the following statements are equivalent:*

(1) X is $\pi g\mathcal{I}$-regular.
(2) For every closed set B not containing $x \in X$, there exist disjoint $\pi g\mathcal{I}$-open sets U and V such that $x \in U$ and $B \subseteq V$.
(3) For every π-open set V containing $x \in X$, there exists an $\pi g\mathcal{I}$-open set U of X such that $x \in U \subseteq cl^{*s}(U) \subseteq V$.
(4) For every π-closed set A, the intersection of all the $\pi g\mathcal{I}$-closed neighborhoods of A is A.
(5) For every set A and an π-open set B such that $A \cap B \neq \emptyset$, there exists an $\pi g\mathcal{I}$-open set F such that $A \cap F \neq \emptyset$ and $cl^{*s}(F) \subseteq B$.
(6) For every non empty set A and π-closed set B such that $A \cap B = \emptyset$, there exist disjoint $\pi g\mathcal{I}$-open sets U and V such that $A \cap U \neq \emptyset$ and $B \subseteq V$.

Proof. (1) and (2) are equivalent by Definition 4.2.

(2) $\Longrightarrow$ (3) Let V be a π-open set such that $x \in V$. Then $X - V$ is a π-closed set not containing x. Therefore, there exist disjoint $\pi g\mathcal{I}$-open subsets U and W of X such that $x \in U$ and $X - V \subseteq W$. Now $X - V \subseteq W$ implies $X - V \subseteq int^{*s}(W)$ and so $X - int^{*s}(W) \subseteq V$. Again $U \cap W = \emptyset$ implies that $U \cap int^{*s}(W) = \emptyset$ and hence $cl^{*s}(U) \subseteq X - int^{*s}(W)$. Therefore $x \in U \subseteq cl^{*s}(U) \subseteq V$. This proves (3).

(3) $\Longrightarrow$ (4) Let A be a π-closed set and $x \in X - A$. By (3), there exists an $\pi g\mathcal{I}$-open set U such that $x \in U \subseteq cl^{*s}(U) \subseteq X - A$. Thus $A \subseteq X - cl^{*s}(U) \subseteq X - U$. Consequently, $X - U$ is an $\pi g\mathcal{I}$-closed neighbourhood of A and $x \notin X - U$. This proves (4).

(4) $\Longrightarrow$ (5) Let $A \cap B \neq \emptyset$, where B is π-open in X. Let $x \in A \cap B$. Then $X - B$ is π-closed and $x \notin X - B$. Then by (4), there exists an $\pi g\mathcal{I}$-closed neighborhood V of $X - B$ such that $x \notin V$. Let $X - B \subseteq U \subseteq V$, where U is $\pi g\mathcal{I}$-open. Then $F = X - V$ is $\pi g\mathcal{I}$-open such that $x \in F$ and $A \cap F \neq \emptyset$. Also $X - U$ is $\pi g\mathcal{I}$-closed and $cl^{*s}(F) = (X - V) \subseteq cl^{*s}(X - U) \subseteq B$. This proves (5).

(5) $\Longrightarrow$ (6) Suppose $A \cap B = \emptyset$, where A is non-empty and B is π-closed. Then $X - B$ is π-open and $A \cap (X - B) \neq \emptyset$. By (5), there exists

an $\pi g\mathcal{I}$-open set U such that $A \cap U \neq \emptyset$ and $U \subseteq cl^{*s}(U) \subseteq X - B$. Put $V = X - cl^{*s}(U)$. Then $B \subseteq V$ and U and V are $\pi g\mathcal{I}$-open sets such that $V = X - cl^{*s}(U) \subseteq X - U$. This proves (6).

(6) $\Longrightarrow$ (1) This is obvious. This completes the proof.

References

[1] M.E. Abd El-Monsef, E.F. Lashien and A.A. Nasef, *Some topological operators via ideals*, Kyungpook Math. J., 32, No. 2 (1992), 273-284.

[2] G. Aslim, A. Caksu Guler, T. Noiri, *On πgs-closed sets in topological spaces*, Acta Math. Hungar., 112(4) (2006), 275-283.

[3] E. Ekici and C. W. Baker, *On πg-closed sets and continuity,* Kochi. J. Math., 2(2007), 35-42.

[4] D. Jankovic and T. R. Hamlett, *New topologies from old via ideals*, Amer. Math. Monthly, 97(4) (1990), 295-310.

[5] M. Khan and T. Noiri, *Semi-local functions in ideal topological spaces*, J. Adv. Res. Pure Math., 2(1) (2010), 36-42.

[6] M. Khan and T. Noiri, *On gI-closed sets in ideal topological spaces*, J. Adv. Stud. in Top., 1(2010),29-33.

[7] K. Kuratowski, *Topology*, Vol. I, Academic press, New York, 1966.

[8] N. Levine, *Semi-open sets and semi-continuity in topological spaces*, Amer. Math. Monthly, 70 (1963), 36-41.

[9] N. Levine, *Generalized closed sets in topology*, Rend. Circ. Math.Palermo, 19(2)(1970), 89-96.

[10] S. N. Maheshwari, R. Prasad, *On s-regular spaces*, Glasnik Mat., 10, No. 30(1975), 347-350.

[11] M. Navaneethakrishnan, J. Paulraj Joseph, D. Sivaraj, *$\mathcal{I}_g$-normal and $\mathcal{I}_g$-regular spaces*, Acta Math., Hungar. 125(4)(2009), 327-340.

[12] M. Rajamani, V. Inthumathi and S. Krishnaprakash, *$\mathcal{I}_{\pi g}$-closed sets and $\mathcal{I}_{\pi g}$-continuity*, J. Adv. Res. Pure. Math.,2(4) (2010), 63-72.

[13] R. Santhi and M. Rameshkumar *On $\pi g\mathcal{I}$-closed sets in ideal topological spaces*, International Journal of Computer Applications, 94(5), 2014.

[14] M. H. Stone, *Application of the theory of boolean rings to general topology*, Trans. Amer. Math. Soc., 41(1937), 374-481.

[15] R. Vaidyanathaswamy, *Set Topology*, Chelsea Publishing Company, 1960.

[16] V. Zaitsev, *On certain classes of topological spaces and their bicompactifications*, Dokl. Akad. Nauk SSSR., 178 (1968), 778-779.

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