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COMMON FIXED POINTS FOR TWO PAIRS OF WEAKLY COMPATIBLE MAPPINGS IN G - METRIC SPACES

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Abstract. In this paper a general fixed point theorem for two pairs of weakly compatible mappings satisfying implicit relations in G - metric spaces, theorem which generalize and improve main results from [11] is proved.

1. Introduction

Let (X,d) be a metric space and $S,T:(X,d)\to(X,d)$ be two mappings. In 1994, Pant [23] introduced the notion of pointwise R - weakly commuting mappings. It is proved in [24] that pointwise R - weakly commutativity is equivalent to commutativity in coincidence points.

Jungck [10] defined S and T to be weakly compatible if $Sx=Tx$ implies $STx=TSx$. Thus, S and T are weakly compatible if and only if S and T are pointwise R - weakly commuting.

In [7] and [8], Dhage introduced a new class of generalized metric space, named D - metric spaces. Mustafa and Sims [15], [16] proved that most of the claims concerning the fundamental topological structures on D - metric spaces are incorrect and introduced an

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appropriate notion of generalized metric space, named G - metric space. In fact, Mustafa, Sims and other authors studied many fixed point results for self mappings in G - metric spaces under certain conditions [17], [18], [19], [20], [21], [22], [33] and other papers. Quite recently, new results are obtained in [3], [4], [5], [6], [9], [13], [31], [32].

Several classical fixed point theorems and common fixed point theorems have been recently unified by considering a general condition by an implicit relation in [25], [26] and other papers. Actually, the method is used in the study of fixed points in metric spaces, symmetric spaces, quasi - metric spaces, ultra - metric spaces, convex metric spaces, reflexive spaces, compact metric spaces, paracompact metric spaces, in two or three metric spaces, for single valued mappings, hybrid pairs of mappings and set valued mappings.

Quite recently, this method is used in the study of fixed points for mappings satisfying an contractive condition of integral type, in fuzzy metric spaces, probabilistic metric spaces and intuitionistic metric spaces.

The study of fixed points satisfying implicit relation in G - metric spaces is initiated in [27], [28], [29], [30] and in other papers.

2. Preliminaries

Definition 2.1 ([16]). Let X be a nonempty set and $G : X^3 \to \mathbb{R}_+$ be a function satisfying the following properties:

$(G_1) : G(x, y, z) = 0$ if $x = y = z$,

$(G_2) : 0 < G(x, x, y)$ for all $x, y \in X$ with $x \neq y$,

$(G_3) : G(x, x, y) \leq G(x, y, z)$ for all $x, y, z \in X$ with $z \neq y$,

$(G_4) : G(x, y, z) = G(y, z, x) = ...$ (symmetry in all three variables),

$(G_5) : G(x, y, z) \leq G(x, a, a) + G(a, y, z)$ for all $x, y, z, a \in X$ (rectangle inequality).

The function G is called a G - metric on X and the pair (X, G) is called a G - metric space.

Note that if $G(x, y, z) = 0$ then $x = y = z$.

Definition 2.2 ([16]). Let (X, G) be a G - metric space. A sequence (x_n) in (X, G) is said to be:

a) G - convergent if for $\varepsilon > 0$, there is an $x \in X$ and $k \in \mathbb{N}$ such that for all $n, m \in \mathbb{N}, n, m \geq k$, $G(x, x_n, x_m) < \varepsilon$.

b) G - Cauchy if for $\varepsilon > 0$, there is $k \in \mathbb{N}$ such that for all $n, m, p \in \mathbb{N}$, with $n, m, p \geq k$, $G(x_n, x_m, x_p) < \varepsilon$, that is $G(x_n, x_m, x_p) \to 0$ as $n, m, p \to \infty$.

A G - metric space (X, G) is said to be G - complete if every G - Cauchy sequence is G - convergent.

Lemma 2.3 ([16]). *Let (X, G) be a G - metric space. Then, the following properties are equivalent:*

1) (x_n) is G - convergent to x;

2) $G(x_n, x_n, x) \to 0$ as $n \to \infty$;

3) $G(x_n, x, x) \to 0$ as $n \to \infty$;

4) $G(x_n, x_m, x) \to 0$ as $n, m \to \infty$.

Lemma 2.4 ([16]). *If (X, G) is a G - metric space, the following properties are equivalent:*

1) (x_n) is G - Cauchy;

2) For $\varepsilon > 0$, there exists $k \in \mathbb{N}$ such that $G(x_n, x_m, x_m) < \varepsilon$ for all $m, n \geq k$, $m, n \in \mathbb{N}$.

Lemma 2.5 ([16]). *Let (X, G) be a G - metric space. Then, the function $G(x, y, z)$ is jointly continuous in all three of its variables.*

Note that each G - metric generates a topology τ_G on X [16] whose base is a family of open G - balls $B_G(x, \varepsilon) = \{G(x, \varepsilon) : x \in X, \varepsilon > 0\}$, where $B_G(x, \varepsilon) = \{y \in X : G(x, y, y) < \varepsilon\}$ for all $x, y \in X$ and $\varepsilon > 0$.

A nonempty set $A \subset X$ is G - closed if $A = \overline{A}$.

Lemma 2.6 ([12]). *Let (X, G) be a G - metric space and A a subset of X. A is G - closed if for any G - convergent sequence in A with $\lim_{n\to\infty} x_n = x$, then $x \in A$.*

In [1], [14], [28], [29] and other papers some fixed point theorems for weakly compatible mappings in G - metric spaces are proved.

Quite recently, in [11] a common fixed point theorem for two pairs of weakly compatible mappings in G - metric spaces is proved.

Theorem 2.7 ([11]). *Let (X, G) be a G - complete metric space. Suppose that $\{f, S\}$ and $\{g, T\}$ are weakly compatible pairs of self - mappings on X satisfying*

$$\begin{aligned} G(fx, fx, gy) \leq h \max\{&G(Sx, Sx, Ty), G(fx, fx, Sx), \\ &G(gy, gy, Ty), \tfrac{1}{2}[G(fx, fx, Ty) + G(gy, gy, Sx)]\} \end{aligned} \tag{2.1}$$

and

$$\begin{aligned} G(fx, gy, gy) \leq h \max\{&G(Sx, Ty, Ty), G(fx, Sx, Sx), \\ &G(gy, Ty, Ty), \tfrac{1}{2}[G(fx, Ty, Ty) + G(gy, Sx, Sx)]\} \end{aligned} \tag{2.2}$$

for all $x, y \in X$, where $h \in \left[0, \frac{1}{2}\right)$. Suppose $f(X) \subset T(X)$ and $g(X) \subset S(X)$. If one of $T(X)$ or $S(X)$ is a G - closed subspace of X, then f, g, S and T have an unique common fixed point.

The purpose of this paper is to prove a general fixed point theorem for two pairs of weakly compatible mappings satisfying implicit relations in G - metric spaces which generalizes and improves Theorem 2.7.

3. Implicit relations

The following class of implicit relations is introduced in [29].

Definition 3.1. Let $\mathfrak{F}_G$ be the set of all continuous function $F(t_1, ..., t_6) : \mathbb{R}_+^6 \to \mathbb{R}$ such that
(F_1) : F is nonincreasing in variable t_5,
(F_2) : there exists $h \in [0,1)$ such that for all $u, v \geq 0$ with $F(u, v, v, u, u+v, 0) \leq 0$ we have $u \leq hv$,
(F_3) : there exists $k \in [0,1)$ such that for all $t, t' > 0$, $F(t, t, 0, 0, t, t') \leq 0$ we have $t \leq kt'$.

The examples 3.2 - 3.9 are presented in [29]. Examples 3.10, 3.11 are new examples.

Example 3.2. $F(t_1, ..., t_6) = t_1 - k \max\left\{t_2, t_3, t_4, \dfrac{t_5 + t_6}{2}\right\}$, *where* $k \in [0,1)$.

Example 3.3. $F(t_1, ..., t_6) = t_1 - at_2 - bt_3 - ct_4 - dt_5 - et_6$, *where* $a, b, c, d, e \geq 0$ *and* $0 < a + b + c + 2d + e < 1$.

Example 3.4. $F(t_1, ..., t_6) = t_1 - k \max\{t_2, t_3, t_4, t_5, t_6\}$, *where* $k \in \left[0, \frac{1}{2}\right)$.

Example 3.5. $F(t_1, ..., t_6) = t_1 - k \max\left\{t_2, \dfrac{t_3 + t_4}{2}, \dfrac{t_5 + t_6}{2}\right\}$, *where* $k \in [0,1)$.

Example 3.6. $F(t_1, ..., t_6) = t_1^2 - t_1(at_2 + bt_3 + ct_4) - dt_5t_6$, *where* $a, b, c, d \geq 0$ *and* $0 < a + b + c + d < 1$.

Example 3.7. $F(t_1, ..., t_6) = t_1^2 - at_2^2 - b\dfrac{t_5t_6}{1 + t_3 + t_4}$, *where* $a, b \geq 0$ *and* $0 < a + b < 1$.

Example 3.8. $F(t_1, ..., t_6) = t_1 - at_2 - bt_3 - c\max\{2t_4, t_5 + t_6\}$, *where* $a, b, c \geq 0$ *and* $0 < a + b + 2c < 1$.

Example 3.9. $F(t_1, ..., t_6) = t_1 - k \max\left\{t_2, t_3, t_4, \dfrac{2t_4 + t_6}{3}, \dfrac{2t_4 + t_3}{3}, \dfrac{t_5 + t_6}{3}\right\}$, *where* $k \in [0,1)$.

Example 3.10. $F(t_1,...,t_6) = t_1 - \alpha \max\{t_2,t_3,t_4\} - (1-\alpha)(at_5 + bt_6)$, *where* $0 \le \alpha < 1$, $0 \le a < \frac{1}{2}$, $0 \le b < \frac{1}{2}$.

Example 3.11. $F(t_1,...,t_6) = t_1 - \max\{at_2, b(t_3+2t_4), b(t_4+t_5+t_6)\}$, *where* $a \in (0,1)$ *and* $k \in \left[0, \frac{1}{3}\right)$.

4. Main results

Theorem 4.1. *Let* (X,G) *be a* G *- complete metric space. Suppose that* $\{f,S\}$ *and* $\{g,T\}$ *are weakly compatible pairs of self mappings of* X *satisfying*

$$\begin{gathered}\phi_1(G(Sx,Ty,Ty), G(fx,gy,gy), G(fx,Sx,Sx),\\ G(gy,Ty,Ty), G(fx,Ty,Ty), G(gy,Sx,Sx)) \le 0,\end{gathered} \tag{4.1}$$

$$\begin{gathered}\phi_2(G(Tx,Sy,Sy), G(gx,fy,fy), G(gx,Tx,Tx),\\ G(fy,Sy,Sy), G(gx,Sy,Sy), G(fy,Tx,Tx)) \le 0,\end{gathered} \tag{4.2}$$

for all $x, y \in X$, *where* $\phi_1, \phi_2 \in \mathfrak{F}_G$.

Suppose that $S(X) \subset g(X)$ *and* $T(X) \subset f(X)$. *If one of* $g(X)$ *or* $f(X)$ *is a* G *- closed subspace of* X, *then* f, g, S *and* T *have an unique common fixed point.*

Proof. Let $x_0 \in X$ be an arbitrary point of X. Since $S(X) \subset g(X)$ and $T(X) \subset f(X)$, there exists $x_1, x_2 \in X$ such that $Sx_0 = gx_1$ and $Tx_1 = fx_2$. Again, there exists $x_3, x_4 \in X$ such that $Sx_2 = gx_3$ and $Tx_3 = fx_4$. Iteratively, for each $n = 0,1,2,...$ we can choose $x_n \in X$, $y_n \in X$ such that

$$y_{2n} = Sx_{2n} = gx_{2n+1},\ y_{2n+1} = Tx_{2n+1} = fx_{2n+2}.$$

By (4.1) $n = 1,2,...$ we have successively

$$\begin{gathered}\phi_1(G(Sx_{2n},Tx_{2n+1},Tx_{2n+1}), G(fx_{2n},gx_{2n+1},gx_{2n+1}),\\ G(fx_{2n},Sx_{2n},Sx_{2n}), G(gx_{2n+1},Tx_{2n+1},Tx_{2n+1}),\\ G(fx_{2n},Tx_{2n+1},Tx_{2n+1}), G(gx_{2n+1},Sx_{2n},Sx_{2n})) \le 0,\end{gathered}$$

$$\begin{gathered}\phi_1(G(y_{2n},y_{2n+1},y_{2n+1}), G(y_{2n-1},y_{2n},y_{2n}), G(y_{2n-1},y_{2n},y_{2n}),\\ G(y_{2n},y_{2n+1},y_{2n+1}), G(y_{2n-1},y_{2n+1},y_{2n+1}), 0) \le 0.\end{gathered}$$

By (F_1) and (G_5) we have that

$$\begin{gathered}\phi_1(G(y_{2n},y_{2n+1},y_{2n+1}), G(y_{2n-1},y_{2n},y_{2n}),\\ G(y_{2n-1},y_{2n},y_{2n}), G(y_{2n},y_{2n+1},y_{2n+1}),\\ G(y_{2n-1},y_{2n},y_{2n}) + G(y_{2n},y_{2n+1},y_{2n+1}), 0) \le 0.\end{gathered}$$

By (F_2) we have

$$G(y_{2n},y_{2n+1},y_{2n+1}) \le hG(y_{2n-1},y_{2n},y_{2n}),$$

where $h = \max\{h_1, h_2\}$.

Again, by (4.2) we have successively

$$\phi_2(G(Tx_{2n+1}, Sx_{2n+2}, Sx_{2n+2}), G(gx_{2n+1}, fx_{2n+2}, fx_{2n+2}),$$
$$G(gx_{2n+1}, Tx_{2n+1}, Tx_{2n+1}), G(fx_{2n+1}, Sx_{2n+2}, Sx_{2n+2}),$$
$$G(gx_{2n+1}, Sx_{2n+2}, Sx_{2n+2}), G(fx_{2n+2}, Tx_{2n+1}, Tx_{2n+1})) \leq 0,$$

$$\phi_2(G(y_{2n+1}, y_{2n+2}, y_{2n+2}), G(y_{2n}, y_{2n+1}, y_{2n+1}), G(y_{2n}, y_{2n+1}, y_{2n+1}),$$
$$G(y_{2n+1}, y_{2n+2}, y_{2n+2}), G(y_{2n}, y_{2n+2}, y_{2n+2}), 0) \leq 0.$$

By (F_1) and (G_5) we have that

$$\phi_2(G(y_{2n+1}, y_{2n+2}, y_{2n+2}), G(y_{2n}, y_{2n+1}, y_{2n+1}),$$
$$G(y_{2n}, y_{2n+1}, y_{2n+1}), G(y_{2n+1}, y_{2n+2}, y_{2n+2}),$$
$$G(y_{2n}, y_{2n+1}, y_{2n+1}) + G(y_{2n+1}, y_{2n+2}, y_{2n+2}), 0) \leq 0.$$

By (F_2) we have

$$G(y_{2n+1}, y_{2n+2}, y_{2n+2}) \leq hG(y_{2n}, y_{2n+1}, y_{2n+1}),$$

which implies

$$G(y_n, y_{n+1}, y_{n+1}) \leq hG(y_{n-1}, y_n, y_n), n = 1, 2, ...$$

Then

$$G(y_n, y_{n+1}, y_{n+1}) \leq h^n G(y_0, y_1, y_1).$$

We will prove that $\{y_n\}$ is a G - Cauchy sequence in X. For $n, m \in \mathbb{N}$ with $m > n$ we have repeating (G_5) that

$$\begin{aligned} G(y_n, y_m, y_m) &\leq G(y_n, y_{n+1}, y_{n+1}) + G(y_{n+1}, y_{n+2}, y_{n+2}) + \\ &+ ... + G(y_{m-1}, y_m, y_m) \\ &\leq [h^n + h^{n+1} + ... + h^{m-n}]G(y_0, y_1, y_1) \\ &\leq \frac{h^n}{1-h} G(y_0, y_1, y_1). \end{aligned}$$

Letting n tends to infinity we obtain $G(y_n, y_m, y_m) \to 0$ as $n, m \to \infty$. This implies that $\{y_n\}$ is a G - Cauchy sequence in X. Since (X, G) is G - complete, there exists $z \in X$ such that $y_n \to z$ as $n \to \infty$. This implies that $\lim_{n\to\infty} y_{2n} = \lim_{n\to\infty} y_{2n+1} = z$.

Suppose that $g(X)$ is G - closed. It follows that $z = gu$ for some $u \in X$. Using (4.1) we have successively

$$\phi_1(G(Sx_{2n}, Tu, Tu), G(fx_{2n}, gu, gu), G(fx_{2n}, Sx_{2n}, Sx_{2n}),$$
$$G(gu, Tu, Tu), G(fx_{2n}, Tu, Tu), G(gu, Sx_{2n}, Sx_{2n})) \leq 0,$$

$$\phi_1(G(y_{2n}, Tu, Tu), G(y_{2n-1}, gu, gu), G(y_{2n-1}, y_{2n}, y_{2n}),$$
$$G(gu, Tu, Tu), G(y_{2n-1}, Tu, Tu), G(gu, y_{2n}, y_{2n})) \leq 0.$$

Letting n tends to infinity we obtain

$$\phi_1(G(z,Tu,Tu),0,0,G(z,Tu,Tu),G(z,Tu,Tu),0)\leq 0,$$

which implies by (F_2) that $G(z,Tu,Tu)=0$, i.e. $z=Tu=gu$.

Since $\{g,T\}$ is weakly compatible, we have $gz=gTu=Tgu=Tz$.

Next we prove that $z=gz=Tz$.

By (4.1) we have successively

$$\begin{gathered}\phi_1(G(Sx_{2n},Tz,Tz),G(fx_{2n},gz,gz),G(fx_{2n},Sx_{2n},Sx_{2n}),\\ G(gz,Tz,Tz),G(fx_{2n},Tz,Tz),G(gz,Sx_{2n},Sx_{2n}))\leq 0,\end{gathered}$$

$$\begin{gathered}\phi_1(G(y_{2n},Tz,Tz),G(y_{2n-1},gz,gz),G(y_{2n-1},y_{2n},y_{2n}),\\ G(z,Tz,Tz),G(y_{2n-1},Tz,Tz),G(gz,y_{2n},y_{2n}))\leq 0.\end{gathered}$$

Letting n tend to infinity we obtain

$$\phi_1(G(z,gz,gz),G(z,gz,gz),0,0,G(z,gz,gz),G(gz,z,z))\leq 0.$$

If $z\neq gz$ we obtain by (F_3) that

$$G(z,gz,gz)\leq kG(z,z,gz),$$

where $k=\max\{k_1,k_2\}$.

By (4.2) we have successively

$$\begin{gathered}\phi_2(G(Tz,Sx_{2n},Sx_{2n}),G(gz,fx_{2n},fx_{2n}),G(gz,Tz,Tz),\\ G(fx_{2n},Sx_{2n},Sx_{2n}),G(gz,Sx_{2n},Sx_{2n}),G(fx_{2n},Tz,Tz))\leq 0,\end{gathered}$$

$$\begin{gathered}\phi_2(G(gz,y_{2n},y_{2n}),G(gz,y_{2n-1},y_{2n-1}),0,\\ 0,G(gz,y_{2n},y_{2n}),G(y_{2n-1},gz,gz))\leq 0.\end{gathered}$$

Letting n tend to infinity we obtain

$$\phi_2(G(gz,z,z),G(gz,z,z),0,0,G(gz,z,z),G(z,gz,gz))\leq 0.$$

By (F_3) we have

$$G(gz,z,z)\leq kG(z,gz,gz).$$

Hence

$$G(z,gz,gz)\leq kG(z,z,gz)\leq k^2G(z,gz,gz)$$

which implies $G(z,gz,gz)(1-k^2)\leq 0$. Hence $G(z,gz,gz)=0$, i.e. $z=gz=Tz$. Therefore, z is a common fixed point of g and T.

Since $T(X)\subset f(X)$, there exists $v\in X$ such that $gz=z=Tz=fv$. Then, by (4.2) we have successively

$$\begin{gathered}\phi_2(G(Tz,Sv,Sv),G(gz,fv,fv),G(gz,Tz,Tz),\\ G(fv,Sv,Sv),G(gz,Sv,Sv),G(fv,Tz,Tz))\leq 0,\end{gathered}$$

$$\phi_2(G(z,Sv,Sv),0,0,G(z,Sv,Sv),G(z,Sv,Sv),0)\leq 0,$$

which implies by (F_2) that $G(z,Sv,Sv)=0$, i.e. $z=Sv=fv$.

Since $Sv = fv$ and $\{f, S\}$ is weakly compatible we obtain $Sz = Sfv = fSv = fz$. Hence, $fz = Sz$.

By (4.1) we have successively

$$\phi_1(G(Sz, Tz, Tz), G(fz, gz, gz), G(fz, Sz, Sz),$$
$$G(gz, Tz, Tz), G(fz, Tz, Tz), G(gz, Sz, Sz)) \leq 0,$$

$$\phi_1(G(fz, z, z), G(fz, z, z), 0, 0, G(fz, z, z), G(z, fz, fz)) \leq 0,$$

which implies by (F_3) that

$$G(fz, z, z) \leq kG(z, fz, fz).$$

By (4.2) we have successively

$$\phi_2(G(Tz, Sz, Sz), G(gz, fz, fz), G(gz, Tz, Tz),$$
$$G(fz, Sz, Sz), G(gz, Sz, Sz), G(fz, Tz, Tz)) \leq 0,$$

$$\phi_2(G(z, fz, fz), G(z, fz, fz), 0, 0, G(z, fz, fz), G(fz, z, z)) \leq 0,$$

which implies by (F_3) that

$$G(z, fz, fz) \leq kG(fz, z, z) \leq k^2 G(z, fz, fz).$$

Hence $G(z, fz, fz)(1 - k^2) \leq 0$ which implies $G(z, fz, fz) = 0$, i.e. $z = fz = Sz$. Hence, z is a common fixed point of f, g, S and T.

Suppose that w is another common fixed point of f, g, S and T.

Then by (4.1) we have successively

$$\phi_1(G(z, Tw, Tw), G(fz, gw, gw), G(fz, Sz, Sz),$$
$$G(gw, Tw, Tw), G(fz, Tw, Tw), G(gw, Sz, Sz)) \leq 0,$$

$$\phi_1(G(z, w, w), G(z, w, w), 0, 0, G(z, w, w), G(w, z, z)) \leq 0,$$

which implies

$$G(z, w, w) \leq kG(w, z, z).$$

Similarly, we have

$$G(w, z, z) \leq kG(z, w, w),$$

which implies

$$G(z, w, w)(1 - k^2) \leq 0,$$

a contradiction. Hence $z = w$.

In the case $T(X)$ is a G - closed set of $f(X)$, the proof is similarly. □

Remark 4.2. *A similar theorem with Theorem 4.1 is obtained if one of $g(X)$ and $f(X)$ is a G - complete subspace of X instead of one of $g(X)$ and $f(X)$ is G - closed.*

Corollary 4.3. *Let (X, G) be a G - complete metric space. Suppose that $\{f, S\}$ and $\{g, T\}$ are weakly compatible pairs of self mappings of X satisfying*

$$\begin{aligned} G(Sx, Ty, Ty) \leq h \max\{&G(fx, gy, gy), G(fx, Sx, Sx), \\ &G(gy, Ty, Ty), \tfrac{1}{2}[G(fx, Ty, Ty) + G(gy, Sx, Sx)]\}, \end{aligned} \tag{4.3}$$

$$\begin{aligned} G(Tx, Sy, Sy) \leq h \max\{&G(gx, fy, fy), G(gx, Tx, Tx), \\ &G(fy, Sy, Sy), \tfrac{1}{2}[G(gx, Sy, Sy) + G(fy, Tx, Tx)]\} \leq 0, \end{aligned} \tag{4.4}$$

for all $x, y \in X$ and $h \in [0, 1)$. Suppose that $S(X) \subset g(X)$ and $T(X) \subset f(X)$. If one of $g(X)$ or $f(X)$ is a G - closed subspace of X, then f, g, S and T have an unique common fixed point.

Proof. The proof it follows from Theorem 4.1 and Example 3.2 with $h_1 = h_2 = h$. □

Remark 4.4. *1. In the proof of Theorem 2.1* [2]*, page 4, lines 10 - 1 from the bottom, there exists some written mistakes and hence the proof of the fact that the sequence $\{y_n\}$ is a G - Cauchy sequence is not correct. Similarly, in the proof of Theorems 2.1 and 2.4* [11]*. For a correct form of Theorem 2.1* [11]*, I suggest the inequality*

$$\begin{aligned} G(fx, gy, gy) \leq \; & h \max\{G(Sx, Ty, Ty), G(Sx, fx, fx), \\ & G(Ty, gy, gy), \frac{1}{2}[G(Sx, gy, gy) + G(Ty, fx, fx)]\} \end{aligned}$$

instead inequality (2) [2], [11].

2. Corollary 4.3 is a generalization of correct form of Theorem 2.1 [10] *because $h \in [0, 1)$ instead $h \in \left[0, \frac{1}{2}\right)$ and the fact that the sequence $\{y_n\}$ is a Cauchy sequence is correct.*

3. By Examples 3.3 - 3.11 we obtain new particular results.

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