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FIXED POINTS FOR MULTIVALUED MAPPINGS IN G - METRIC SPACES

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Abstract. In this paper a general fixed point theorem for multivalued mappings in G - metric spaces, which generalize Theorem 3.1 [38], is proved and we obtain other results similarly with the results from metric spaces.

1. Introduction

In [6], [7], Dhage introduced a new class of generalized metric space, named D - metric space. Mustafa and Sims [11], [12] proved that most of the claims concerning the fundamental topological structures on D - metric spaces are incorrect and introduced an appropriate notion of generalized metric space, named G - metric space. In fact, Mustafa and Sims and other authors studied many fixed point results for self mappings in G - metric spaces under certain conditions [1], [2], [5], [13], [14], [16], [36] and other papers.

In [18], [19] and other paper, the first author introduced the study of fixed points for mappings satisfying implicit relations. Actually, the method is used in the study of fixed points in metric spaces, symmetric spaces, quasi - metric spaces, ultra - metric spaces, probabilistic metric spaces, compact metric spaces, convex metric spaces,

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in two or three metric spaces, for single valued mappings, hybrid pairs of mappings and set valued mappings. Quite recently, the method is used in the study of fixed points for mappings satisfying contractive conditions of integral type, in fuzzy metric spaces and intuitionistic metric spaces. There exists a vast literature in this topic which cannot be completely cited here.

The method unified different types of contractive and extensive conditions. With this method, the proofs of some fixed point theorems are more simple. Also, this method allows the study of local and global properties of fixed point structures. Quite recently, the present authors initiated the study of fixed points in G - metric spaces using implicit relations in [22] - [27].

The study of fixed points for multivalued mappings has been initiated by Markin [10], Nadler [17], Rus [33], [34], [35], Reich [28], [29], [30], Rhoades [31], Rhoades and Watson [32], Berinde and Berinde [4] and other authors. Popa [20], [21] initiated the study of fixed points for multivalued mappings satisfying implicit relations.

2. Preliminaries

Definition 2.1 ([12])**.** Let X be a nonempty set and $G : X^3 \to \mathbb{R}_+$ be a function satisfying the following properties:

$(G_1) : G(x, y, z) = 0$ if $x = y = z$,

$(G_2) : 0 < G(x, x, y)$ for all $x, y \in X$ with $x \neq y$,

$(G_3) : G(x, y, y) \leq G(x, y, z)$ for all $x, y, z \in X$ with $z \neq y$,

$(G_4) : G(x, y, z) = G(y, z, x) = G(z, x, y) = ...$ (symmetry in all three variables),

$(G_5) : G(x, y, z) \leq G(x, a, a) + G(a, y, z)$ for all $x, y, z, a \in X$ (rectangle inequality).

The function G is called a G - metric on X and the pair (X, G) is called a G - metric space.

Note that if $G(x, y, z) = 0$, then $x = y = z$.

Definition 2.2 ([12])**.** Let (X, G) be a G - metric space. A sequence (x_n) in X is said to be

a) G - convergent, if for $\varepsilon > 0$, there exists an $x \in X$ and $k \in \mathbb{N}$ such that for all $m, n \in \mathbb{N}$, $m, n \geq k$, $G(x, x_n, x_m) < \varepsilon$.

b) G - Cauchy if for $\varepsilon > 0$, there exists $k \in \mathbb{N}$ such that for all $m, n, p \in \mathbb{N}$, $n, m, p \geq k$, $G(x_n, x_m, x_p) < \varepsilon$, that is $G(x_n, x_m, x_p) \to 0$ as $n, m, p \to \infty$.

A G - metric space (X, G) is said to be G - complete if every G - Cauchy sequence in X is G - convergent.

Lemma 2.3 ([12]). *Let (X, G) be a G - metric space. The following properties are equivalent:*

1) (x_n) is G - convergent to x;

2) $G(x_n, x_n, x) \to 0$ as $n \to \infty$;

3) $G(x_n, x, x) \to 0$ as $n \to \infty$;

4) $G(x_m, x_n, x) \to 0$ as $m, n \to \infty$.

Lemma 2.4 ([12]). *If (X, G) be a G - metric space, then the following properties are equivalent:*

1) (x_n) is G - Cauchy;

2) for every $\varepsilon > 0$, there is $k \in \mathbb{N}$ such that $G(x_n, x_m, x_n) < \varepsilon$ for all $m, n \in \mathbb{N}$, $m, n \geq k$.

Lemma 2.5 ([12]). *Let (X, G) be a G - metric space. Then the function $G(x, y, z)$ is jointly continuous in all three of its variables.*

Note that each G - metric on X generates a topology τ_G on X [12], whose base is a family of open G - balls $\{B_G(x, \varepsilon) : x \in X,\ \varepsilon > 0\}$, where $B_G(x, y) = \{y \in X :\ G(x, y, y) < \varepsilon\}$.

A nonempty set $A \subset X$ is G - closed if $A = \overline{A}$.

Lemma 2.6 ([9]). *Let (X, G) be a G - metric space and A a subset of X. A is G - closed if for any G - convergent sequence $(x_n) \in A$ with $\lim_{n\to\infty} x_n = x$, then $x \in A$*

Lemma 2.7 ([8]). *Every G - metric space (X, G) defines a metric space (X, d_G) by*

$$d_G(x, y) = G(x, y, y) + G(x, x, y),$$

for all $x, y \in X$.

Let (X, G) be a G - metric space and $CB(X)$ be the family of all nonempty closed subsets of X.

As in [8], [37], we denote by H_G the Hausdorff G - distance on $CB(X)$, i.e.

$$H_G(A, B, C) = \max\{\sup_{x\in A} G(x, B, C), \sup_{x\in B} G(x, C, A), \sup_{x\in C} G(x, A, B)\},$$

where

$$\begin{aligned} G(x, B, C) &= d_G(x, B) + d_G(B, C) + d_G(x, C), \\ d_G(x, B) &= \inf\{d_G(x, y), y \in B\}, \\ d_G(A, B) &= \inf\{d_G(a, b), a \in A, b \in B\}. \end{aligned}$$

Lemma 2.8 ([38]). *Let (X, G) be a G - metric space and $A, B \in CB(X)$. Then, for $a \in A$ and $h > 1$, there exists $b \in B$ such that*

$$G(a, b, b) \leq h \cdot H_G(A, B, B).$$

Quite recently, Wats and Kumar [38] proved the following theorem:

Theorem 2.9 ([38]). *Let (X, G) be a G - complete metric space, $T : X \to CB(X)$ be a set valued map such that*

(2.1)

$$\begin{gathered} H_G(Tx, Ty, Ty) \leq \alpha G(x, y, z) + \beta[G(x, Tx, Tx) + G(y, Ty, Ty) + \\ + G(z, Tz, Tz)] + \gamma[G(x, Ty, Ty) + G(x, Tz, Tz) + G(y, Tx, Tx) + \\ + G(y, Tz, Tz) + G(z, Tx, Tx) + G(z, Ty, Ty)] \end{gathered}$$

for all $x, y, z \in X$, where $\alpha, \beta, \gamma > 0$ and $\alpha + 3\beta + 4\gamma < 1$. Then T has a fixed point.

The purpose of this paper is to prove a general theorem for multivalued mappings satisfying an implicit relation, which generalizes Theorem 2.9 and obtain other new results similarly with known results from metric spaces.

3. Implicit relations

Definition 3.1 ([20]). Let $\mathfrak{F}_M$ be the set of all continuous functions $F(t_1, ..., t_6) : \mathbb{R}_+^6 \to \mathbb{R}$ satisfying the following conditions:

(F_1) : F is increasing in variable t_1 and nonincreasing in variables t_3, t_4, t_5, t_6.

(F_2) : There exist $k > 1$ and $h \in (0, 1)$ such that for $u, v \geq 0$, $t > 0$ and $u \leq kt$, $F(t, v, v, u, u + v, 0) \leq 0$ implies $u \leq hv$.

In the following examples, condition (F_1) is obvious.

Example 3.2. $F(t_1, ..., t_6) = t_1 - \alpha t_2 - \beta(t_3 + 2t_4) - 2\gamma(t_4 + t_5 + t_6\}$, *where $\alpha, \beta, \gamma \geq 0$ and $\alpha + 3\beta + 6\gamma < 1$.*

(F_2) : *Let $1 < k < \dfrac{1}{\alpha + 3\beta + 6\gamma}$ be such that $u, v \geq 0$, $t > 0$ and $F(t, v, v, u, u + v, 0) = t - \alpha v - \beta(2v + u) - 2\gamma(2u + v) \leq 0$. If $u \leq kt$, then $u \leq hv$, where $0 < h = \dfrac{k(2\alpha + \beta + 2\gamma)}{1 - k(2\beta + 4\gamma)} < 1$.*

Example 3.3. $F(t_1, ..., t_6) = t_1 - at_2 - bt_3 - ct_4 - dt_5 - et_6$, *where $a, b, c, d, e \geq 0$ and $0 < a + b + c + 2d < 1$.*

(F_2) : *Let $1 < k < \dfrac{1}{a + b + c + 2d}$ be such that $u, v \geq 0$, $t > 0$ and $F(t, v, v, u, u + v, 0) = t - av - bv - cu - d(u + v) \leq 0$. If $u \leq kt$,* *then* $u \leq k(av + bv + cu + d(u + v))$, *which implies* $u \leq hv$, *where* $0 < h = \dfrac{k(a+b+e)}{1-k(c+d)} < 1$.

Example 3.4. $F(t_1, ..., t_6) = t_1 - a\max\{t_2, t_3, t_4, t_5, t_6\}$, *where* $a \in \left(0, \dfrac{1}{2}\right)$.

(F_2) : *Let* $1 < k < \dfrac{1}{2a}$ *be such that* $u, v \geq 0$, $t > 0$ *and* $F(t, v, v, u, u+v, 0) = t - a(u+v) \leq 0$. *If* $u \leq kt$, *then* $u \leq ak(u+v)$, *which implies* $u \leq hv$, *where* $0 < h = \dfrac{ak}{1-ak}$.

Example 3.5. $F(t_1, ..., t_6) = t_1 - a\max\left\{t_2, t_3, t_4, \dfrac{t_5+t_6}{2}\right\}$, *where* $a \in (0, 1)$.

(F_2) : *Let* $1 < k < \dfrac{1}{a}$ *be such that* $u, v \geq 0$, $t > 0$ *and* $F(t, v, v, u, u+v, 0) = t - a\max\left\{v, u, \dfrac{u+v}{2}, 0\right\} \leq 0$. *If* $u > v$, *then* $u \leq kt$ *implies* $u \leq aku < u$, *a contradiction. Hence* $u \leq v$, *which implies* $u \leq hv$, *where* $0 < h = ak < 1$.

Example 3.6. $F(t_1, ..., t_6) = t_1 - a\max\left\{t_2, \dfrac{t_3+t_4}{2}, \dfrac{t_5+t_6}{2}\right\}$, *where* $a \in (0, 1)$.

The proof is similar as in Example 3.5.

Example 3.7. $F(t_1, ..., t_6) = t_1^2 - (at_2^2 + bt_3^2 + ct_4^2) - dt_5t_6$, *where* $a, b, c, d \geq 0$ *and* $0 < a + b + c < 1$.

(F_2) : *Let* $1 < k < \dfrac{1}{\sqrt{a+b+c}}$ *be such that* $u, v \geq 0$, $t > 0$ *and* $F(t, v, v, u, u+v, 0) = t^2 - (av^2 + bv^2 + cu^2) \leq 0$. *If* $u \leq kt$, *then* $u^2 \leq k^2(av^2 + bv^2 + cu^2) \leq 0$, *which implies* $u \leq hv$, *where* $0 < h = k\sqrt{\dfrac{a+b}{1-k^2c^2}} < 1$.

Example 3.8. $F(t_1, ..., t_6) = t_1 - c\max\{t_2, t_3, \sqrt{t_4t_5}, \sqrt{t_5t_6}\}$, *where* $c \in (0, 1)$.

(F_2) : *Let* $1 < k < \dfrac{1}{c}$ *be such that* $u, v \geq 0$, $t > 0$ *and* $F(t, v, v, u, u+v, 0) = t - cv \leq 0$. *If* $u \leq kt$, *then* $u \leq hv$, *where* $0 < h = ck < 1$.

Example 3.9. *$F(t_1, ..., t_6) = t_1 - at_2 - bt_3 - c\max\{2t_4, t_5 + t_6\}$, where $a > 0$, $b, c \geq 0$ and $0 < a + b + 2c < 1$.*

(F_2) : Let $1 < k < \dfrac{1}{a+b+2c}$ be such that $u, v \geq 0$, $t > 0$ and $F(t, v, v, u, u + v, 0) = t - av - bv - c\max\{2u, u + v\} \leq 0$. If $u > v$, then $u \leq k(a + b + 2c) < u$, a contradiction. Hence $u \leq v$, which implies $u \leq hv$, where $0 < h = k(a + b + 2c) < 1$.

4. Main result

Theorem 4.1. *Let (X, G) be a complete G - metric space and $T : (X, G) \to CB(X)$ be a self valued mapping such that*

$$\begin{aligned} F(H_G(Tx, Ty, Ty)), G(x, y, y), G(x, Tx, Tx), \\ G(y, Ty, Ty), G(x, Ty, Ty), G(y, Tx, Tx)) \leq 0 \end{aligned} \tag{4.1}$$

for all $x, y \in X$, where $F \in \mathfrak{F}_M$.

Then, T has a fixed point.

Proof. Let $x_0 \in X$ be, $x_1 \in Tx_0$ and $k > 1$. By Lemma 2.8 there exists $x_2 \in Tx_1$ such that

$$G(x_1, x_2, x_2) \leq kH_G(Tx_0, Tx_1, Tx_1). \tag{4.2}$$

Continuing this process, there exists $x_{n+1} \in Tx_n$, $n = 0, 1, 2, ...$ such that

$$G(x_n, x_{n+1}, x_{n+1}) \leq kH_G(Tx_{n-1}, Tx_n, Tx_n). \tag{4.3}$$

By (4.1) we have

$$\begin{aligned} F(H_G(Tx_{n-1}, Tx_n, Tx_n)), G(x_{n-1}, x_n, x_n), G(x_{n-1}, Tx_{n-1}, Tx_{n-1}), \\ G(x_n, Tx_n, Tx_n), G(x_{n-1}, Tx_n, Tx_n), G(x_n, Tx_{n-1}, Tx_{n-1})) \leq 0. \end{aligned}$$

By $x_n \in Tx_{n-1}$ and (F_1), for $n = 1, 2, ...$ we obtain

$$\begin{aligned} F(H_G(Tx_{n-1}, Tx_n, Tx_n)), G(x_{n-1}, x_n, x_n), G(x_{n-1}, x_n, x_n), \\ G(x_n, x_{n+1}, x_{n+1}), G(x_{n-1}, x_{n+1}, x_{n+1}), 0) \leq 0. \end{aligned}$$

Using (F_1) and (G_5) we obtain

$$\begin{aligned} F(H_G(Tx_{n-1}, Tx_n, Tx_n)), G(x_{n-1}, x_n, x_n), G(x_{n-1}, x_n, x_n), \\ G(x_n, x_{n+1}, x_{n+1}), G(x_{n-1}, x_n, x_n) + G(x_n, x_{n+1}, x_{n+1}), 0) \leq 0. \end{aligned} \tag{4.4}$$

By (4.3) and (F_2) we obtain

$$G(x_n, x_{n+1}, x_{n+1}) \leq hG(x_{n-1}, x_n, x_n)$$

which implies

$$G(x_n, x_{n+1}, x_{n+1}) \leq h^n G(x_0, x_1, x_1).$$

Using (G_5) we obtain for $m > n$ that

$$\begin{aligned} G(x_n, x_m, x_m) &\leq G(x_n, x_{n+1}, x_{n+1}) + G(x_{n+1}, x_{n+2}, x_{n+2}) + ... + \\ &+ G(x_{m-1}, x_m, x_m) \\ &\leq (h^n + h^{n+1} + ... + h^{m-1})G(x_0, x_1, x_1) \\ &\leq \frac{h^n}{1-h}G(x_0, x_1, x_1). \end{aligned}$$

By Lemma 2.4, (x_n) is a G - Cauchy sequence. Since (X, G) is G - complete, there exists $x \in X$ such that $\lim_{n\to\infty} x_n = x$.

By (4.1) we have

$$\begin{gathered} F(H_G(Tx_n, Tx, Tx)), G(x_n, x, x), G(x_n, Tx_n, Tx_n), \\ G(x, Tx, Tx), G(x_n, Tx, Tx), G(x, Tx_n, Tx_n)) \leq 0. \end{gathered}$$

Since $G(x_{n+1}, Tx, Tx) \leq H_G(Tx_n, Tx, Tx)$ and (F_1) we obtain

$$\begin{gathered} F(G(x_{n+1}, Tx, Tx)), G(x_n, x, x), G(x_n, x_{n+1}, x_{n+1}), \\ G(x, Tx, Tx), G(x_n, Tx, Tx), G(x, x_{n+1}, x_{n+1})) \leq 0. \end{gathered}$$

Letting n tends to infinity we obtain

$$F(G(x, Tx, Tx)), 0, 0, G(x, Tx, Tx), G(x, Tx, Tx), 0) \leq 0.$$

Since $G(x, Tx, Tx) \leq kG(x, Tx, Tx)$, by (F_2) we obtain

$$G(x, Tx, Tx) \leq 0,$$

which implies $x \in Tx$, hence x is a fixed point of T. □

Corollary 4.2 (Corrected form of Theorem 2.9)**.**

Proof. For $z = y$, by relation (2.1) we obtain

$$\begin{aligned} H_G(Tx, Ty, Ty) \leq\ & \alpha G(x, y, y) + \beta[G(x, Tx, Tx) + 2G(y, Ty, Ty)] \\ & +2\gamma[G(x, Ty, Ty) + G(y, Tx, Tx) + G(y, Ty, Ty)]. \end{aligned}$$

By Example 3.2 and Theorem 4.1 for $\alpha + 3\beta + 6\gamma < 1$, corrected form of Theorem 2.9 follows. □

Remark 4.3. *The condition $\alpha + 3\beta + 4\gamma < 1$ in Theorem 2.9 is not correct because there exists a written mistake in the proof of this theorem (*[38]*, page 65, line 7 from top).*

Remark 4.4. *1) By Theorem 4.1 and Example 3.3 for $b = c = d = e = 0$, we obtain a similar result with the theorem of Avramescu, Markin, Nadler* [3], [10], [17].

2) By Theorem 4.1 and Example 3.3 for $d = e = 0$, we obtain G - metric space a similar result with the result for Reich type, see [28] - [30] *and* [34], [35].

3) By Theorem 4.1 and Examples 3.4 - 3.9 we obtain new particular results in G - metric spaces.

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