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ON BICLIQUES, BICLIQUE PARTITIONS AND RELATED CLASSES OF COGRAPHS

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Abstract. In this article we will highlight the relation between attack graphs, cross associations and biclique partitions. The attack graphs are used to evaluate network security risk. Also, we will give an efficient recognition algorithm for a maximal subclass of cographs (P_4-free graphs), we will give the necessary and sufficient conditions for the existence of a biclique partition and we will determine some combinatorial optimzation numbers for some classes of graphs (maximum subclasses for P_4-free) in efficient time. Also, we will determine maximum bicliques for a maximal subclass of cographs and we give some applications of minimal unbreakable graphs in optimization problems and in chemistry. Bicliques (complete bipartite graphs) of graphs have been studied extensively, partially motivated by the large number of applications.

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1. Introduction.

The maximum biclique problem is NP-hard for general graphs [25] and problem of finding a maximum biclique in a bipartite graphs is solvabil in polynomial tyme [25]. The applications of bicliques there is in automata and language theory, graph compression, artificial intelligence and biology [1]. In [3] we present a novel technique of block cipher cryptanalysis with bicliques.

In [13], Phillips and Swiler proposed a method that used attack graph to evaluate network security risk. There are two approaches of building attack graph: graph-theory-based attack graph assessment and model-based attack graph assessment. Model checking was firstly used to analyze whether a given goal state is reachable from the initial state [1,15,16,17,18,19]. A more compact representation of the attack graph was proposed based on the graph theory [10]. One other approach employs cross associations on adjacency matrices to facilitate analysis of attacks [17]. Cross associations are useful in many areas of information security and in other disciplines as: data mining, e-commerce, information retrieval and network analysis. The problem of finding cross associations (a cross association is a grouping of the rows and columns of a matrix) is closely related to the problem of finding biclique partitions. Peeters [14] has shown that the problem of finding a biclique in G with the maximum number of edges is NP-complete. Hochbaum [10] describes approximation algorithms for several problems involving bicliques. Szeider [21] defines a total biclique cover as a collection of disjoint bicliques such that every vertex in the set is in one of the bicliques and shows that the problem of determining if a bipartite graph has a total biclique cover is NP-complete.

Given a graph and an integer k, the biclique cover problem questions whether the edge-set of the graph can be covered with at most k bicliques; the biclique partition problem is defined similarly with the additional condition that the bicliques are required to be mutually edge-disjoint. The biclique vertex-cover problem questions whether the vertex-set of the given graph can be covered with at most k bicliques, the biclique vertex-partition problem is defined similarly with the additional condition that the bicliques are required to be mutually vertex-disjoint. All these four problems are known to be NP-complete.

The content of the paper is organized as follows. In Preliminaries, we give the usual terminology in graph theory. In Section 3 we determine a (maximal) biclique of cograph, we give a characterization of $\{P_4, 2P_3\}$-free graphs ($\{P_4, 2P_3\}$-free graphs are maximal subclasses of P_4-free graphs), we construct a biclique partition, a the recognition algorithm for $\{P_4, 2P_3\}$-free graphs and we determine some combinatorial optimization numbers in efficient time.

2. Preliminaries.

Throughout this paper, $G=(V,E)$ is a connected, finite and undirected graph ([4]), without loops and multiple edges, having $V=V(G)$ as the vertex set and $E=E(G)$ as the set of edges. $\overline{G}$ (*co-G*) is the complement of G. If $U\subseteq V$, by $G(U)$ ($[U]_G$ or [U]) we denote the subgraph of G induced by U. By G-X we mean the subgraph $G(V\text{-}X)$, whenever $X\subseteq V$, but we simply write G-v, when $X=\{v\}$. If $e=xy$ is an edge of a graph G, then x and y are adjacent, while x and e are incident, as are y and e. If $xy\in E$, we also use $x\sim y$, and $x\nsim y$ whenever x, y are not adjacent in G. If A, $B\subseteq V$ are disjoint and $ab\in E$ for every $a\in A$ and $b\in B$, we say that A, B are *totally adjacent* and we denote by $A\sim B$, while by $A\nsim B$ we mean that no edge of G joins some vertex of A to a vertex from B and, in this case, we say A and B are *totally non-adjacent.*

The *neighborhood* of the vertex $v\in V$ is the set $N_G(v)=\{u\in V{:}uv\in E\}$, while $N_G[v]=N_G(v)\cup\{v\}$; we denote $N(v)$ and $N[v]$, when G appears clearly from the context. The *degree* of v in G is $d_G(v)=|N_G(v)|$. The neighborhood of the vertex v in the complement of G will be denoted by $\overline{N}(v)$.

The neighborhood of $S\subseteq V$ is the set $N(S) = \cup_{v\in S}N(v)\text{-}S$ and $N[S]=S\cup N(S)$. A graph is complete if every pair of distinct vertices is adjacent.

By P_n, C_n, K_n we mean a chordless path on $n\geq 3$ vertices, a chordless cycle on $n\geq 3$ vertices, and a complete graph on $n\geq 1$ vertices, respectively.

Let F denote a family of graphs. A graph G is called *F-free* if none of its subgraphs are in F.

The *Zykov sum* of the graphs G_1, G_2 is the graph $G=G_1+G_2$ having: $V(G)=V(G_1)\cup V(G_2)$, $E(G)=E(G_1)\cup E(G_2)\cup\{uv{:}u\in V(G_1),v\in V(G_2)\}$.

3. The results

We remind a characterization of the weak decomposition of a graph.

Definition 1. ([22], [23]) *A set $A\subseteq V(G)$ is called a weak set of the graph G if $N_G(A)\neq V(G)\text{-}A$ and $G(A)$ is connected. If A is a weak set, maximal with respect to set inclusion, then $G(A)$ is called a weak* *component. For simplicity, the weak component G(A) will be denoted with A.*

Definition 2. ([22], [23]) *Let G=(V,E) be a connected and non-complete graph. If A is a weak set, then the partition* $\{A, N(A), V\text{-}A\cup N(A)\}$ *is called a weak decomposition of G with respect to A.*

The name of *weak component* is justified by the following result.

Theorem 1.([22], [23]) *Every connected and non-complete graph G=(V,E) admits a weak component A such that* $G(V\text{-}A)=G(N(A))+G(\overline{N}(A))$.

Theorem 2.([7], [8]) Let *G=(V,E)* be a connected and non-complete graph and $A \subseteq V$. Then A is a *weak component of G if and only if G(A) is connected and* $N(A) \sim \overline{N}(A)$.

The next result, that follows from Theorem 2, ensures the existence of a weak decomposition in a connected and non-complete graph.

Corollary 1. *If G=(V,E) is a connected and non-complete graph, then V admits a weak decomposition (A,B,C), such that G(A) is a weak component and G(V-A)=G(B)+G(C).*

Theorem 2 provides an *O(n+m)* algorithm for building a weak decomposition for a non-complete and connected graph.

Algorithm for the weak decomposition of a graph ([22])

Input: A connected graph with at least two nonadjacent vertices, $G = (V, E)$.

Output: A partition $V = (A, N, R)$ such that $G(A)$ is connected, $N = N(A)$, $A \not\sim R=\overline{N}(A)$.

Begin

A:= any set of vertices such that $A \cup N(A) \neq V$ $N := N(A)$

$R := V\text{-}A \cup N(A)$

While ($\exists n \in N$, $\exists r \in R$ *such that* $nr \notin E$) *do*

Begin

$A := A \cup \{n\}$

$N := (N\text{-}\{n\}) \cup (N(n) \cap R)$

$R := R\text{-}(N(n) \cap R)$

end

end

A biclique B of G is a maximal biclique of G if B is not properly contained in another biclique of G.

A graph G=(V,E) is called unbreakable if it has at least three vertices and neither G nor G has a star cutset. The subset $A \subset V$ *is called a cutset if G-A is not connected. If, in addition, some* $v \in A$ *is* *adjacent to every vertex in A-*$\{v\}$*, then A is called a star cutset and v is called the center of A.*

A graph G = (V,E) with at least three vertices is confidentially connected if for any three distinct vertices v,x,y $\in$ *V, there exists a path* P_{xy} *in G such that* $N_G[v] \cap V(P_{xy}) \subseteq \{x, y\}$.

Let G=(V,E) be a connected, noncomplete graph and (A,N,R) a weak decomposition with G(A) a weak component and a $\in$ *A such that* $\{a\} \sim N$.

There exists a $\in$ *A such that* $\{a\} \sim N$ *?*

Two answers: one with the confidentially connected graphs (I) and other with the cographs (II).

(I)

A connected and non-complete graph G = (V,E) is unbreakable if and only if $\{\overline{N}_G(v) | v \in V\}$ *is the family of the weakly components of G, while* $\{\overline{N_{\overline{G}}}(v) | v \in V\}$ *is the family of the weakly components of* $\overline{G}$.

In (6, [22]), shows that *a graph G is unbreakable if G and* $\overline{G}$ *are confidentially connected.*

G is minimal unbreakable ([22], [24]), *if and only if G is* C_k *or* $\overline{C_k}$ *for some k =5.*

The applications of minimal unbreakable graphs

We give some applications of minimal unbreakable graphs in optimization problems and in chemistry.

The following centrality indices are defined in (12) : The eccentricity of a vertex u is $e_G(u) = max\{d(u,v) | v \in V\}$. *The radius is* $r(G) = min\{e_G(u) | u \in V\}$. *The center of a graph G is* $C(G) = \{u \in V | r(G) = e_G(u)\}$.

We denote the sum of the distances from a vertex u to any other vertex in a graph $G = (V, E)$ as the total distance $s(u) = \sum_{v \in V} d(u, v)$. If the minimum total distance of G is denoted by $s(G) = min\{s(u) | u \in \mathrm{V}\}$, the *median M(G)* of G is given by $M(G) = \{u \in V | s(G) = s(u)\}$.

The distance-counting polynomial was introduced in (11) as: $H(G, x) = \sum_k d(G, k) x^k$, with $d(G, 0) = |V(G)|$ and $d(G, 1) = |E(G)|$, where $d(G, k)$ is the number of pairs of vertices lying at distance k to each other. This polynomial was called Wiener polynomial (see 20).

If G=(V,E) is a minimal unbreakable graph, then the center and the median are equal to V.

If G=(V,E) is a minimal unbreakable graph, then the Wiener polynomial is a polynomial with degree 2 if G= $\overline{C_n}$ *and with degree n/2 if* $G = C_n$.

(II)

In ([22]) it shows that G is P_4*-free if and only if 1)* $A \sim N \sim R$ *and 2) G(A), G(N) and G(R) are* P_4*-free, for (A,N,R) a weak decomposition, with G(A) the weak component.*

We define the graph Ha (*) as follows:

V(Ha)=V-$\{a\}$*;*

E(Ha)= $\left\{xy\left|xy \in E, x, y \in N_{G(A)}(a)\right.\right\}$
$\cup\left\{xy\left|xy \in E, x, y \in \overline{N}_{G(A)}(a)\right.\right\}$
$\cup\left\{xy\left|xy \in E, x, y \in N\right.\right\}$
$\cup\left\{xy\left|xy \in E, x, y \in R\right.\right\} \cup \left\{xy\left|xy \in E, x \in N_{G(A)}(a), y \in N\right.\right\}$
$\cup\left\{xy\left|xy \notin E, x \in N_{G(A)}(a), y \in \overline{N}_{G(A)}(a) \cup R\right.\right\}$.

Theorem 3. *Let G=(V,E) be a connected, noncomplete graph. Let G be cograph and (A,N,R) a weak decomposition with G(A) a weak component. Then V(B) a subset of V is a (maximal) biclique of G if and only if V(B)-*$\{a\}$ *is a (maximal) stable of Ha.*

Proof. Let B be a (maximal) biclique of G and $a \in A \cap V(B)$. Then $V(B) \subseteq \{a\} \cup N_{G(A)}(a) \cup \overline{N}_{G(A)}(a) \cup R$ in G, where the independent sets X and Y of the biclique B satisfy $X \subseteq N_{G(A)}(v)$ and $Y \subseteq \{a\} \cup \overline{N}_{G(A)}(a) \cup R$. Since B is a biclique and by the construction of H_a, we obtain that $V(B) - \{a\}$ is an independent set.

If $V(B')$ is a (maximal) independent set of H_a, for some $a \in A$, then $V(B') \cap N$ is an independent set of $G\left(N_{G(A)}(a) \cup N\right)$ and $V(B') \cap \left(\overline{N}_{G(A)}(a) \cup R\right)$ is an independent set of $G(V(B') \cap \left(\overline{N}_{G(A)}(a) \cup R\right))$. Hence B' is a biclique of $G\left(V - \{a\}\right) = G\left(N_{G(A)}(a)\right) \cup N \cup \overline{N}_{G(A)}(a) \cup R)$ and $V(B') \cup \{a\}$is a biclique of G. Finally, due to the correspondence between bicliques and independent sets, this also holds for maximality by inclusion of vertices.

Since $G(A)$ is connected and P_4-free, there $A' \subset A$ such that $[A']_{G(A)}$ is connected components in $G(A)$. Instead of a, in the above theorem (theorem 3), take A'.

So $H_{A'}$(**):

V($H_{A'}$*)=V-A';*

$E(H_{A'})= \left\{xy \left| xy \in E, x, y \in N_{G(A)}(A') \right.\right\}$

$\cup \left\{xy \left| xy \in E, x, y \in \overline{N}_{G(A)}(A') \right.\right\}$

$\cup \{xy \,|xy \in E, x, y \in N\} \cup \{xy \,|xy \in E, x, y \in R\}$

$\cup \left\{xy \left| xy \in E, x \in N_{G(A)}(A'), y \in N \right.\right\}$

$\cup \left\{xy \left| xy \notin E, x \in N_{G(A)}(A'), y \in R \right.\right\}.$

Remark 1. *Let G=(V,E) be a connected, noncomplete graph. Let G be cograph and (A,N,R) a weak decomposition with G(A) a weak component. Then V(B) a subset of V is a (maximal) biclique of G if and only if V(B)-A' is a (maximal) stable of $H_{A'}$ and*
$\alpha(H_{A'}) = \alpha(H_{A'}(\overline{N}_{G(A)}(A'))) +$
$+ \max\left\{\alpha(H_{A'}(N_{G(A)}(A'))), \alpha(H_{A'}(N)) + \alpha(H_{A'}(R))\right\},$
(ie: stability number of $H_{A'}$ is equal with stability number of non-neighbors of A' (in G) + maximum of stability number of neighbors of A' (in G) and stability number of N (in G) + stability number of R (in G).)

We give a characterization of a maximal subclas of P4-free graphs, we construct a biclique partition, a the recognition algorithm for $\{P_4, 2P_3\}$-free graphs.

A *biclique cover* of a graph G is a collection of bicliques of G such that each edge of G is in at least one of the bicliques.

A *biclique partition* of a graph G is a collection of bicliques of G such that each edge of G is in exactly one of the bicliques.

$\{P_4, 2P_3\}$-free graphs are maximal subclasses of P4-free graphs.

Theorem 4. *Let G=(V,E) be connected with at least two nonadjacent vertices and (A,N,R) a weak decomposition with A weak component. G is $\{P_4, 2P_3\}$-free graph if and only if:*

1) $A{\sim}N{\sim}R$

2) *G($A \cup N$),G($N \cup R$) are $\{P_4, 2P_3\}$-free graphs.*

Proof. If G is a $\{P_4, 2P_3\}$-free graph then *G($A \cup N$)*, *G($N \cup R$)* are $\{P_4, 2P_3\}$-free graphs and $A{\sim}N{\sim}R$. We suppose that 1) and 2) hold. Since $A{\sim}N{\sim}R$ and *G(A)*, *G(N)*, *G(R)* are P_4 -free graphs it follows that G is P_4 -free. If $G \supseteq 2P_3$ then, because $A{\sim}N{\sim}R$ and $A \not\sim R$ it follows that either $2P_3 \subseteq G(A \cup N)$ or $2P_3 \subseteq G(N \cup R)$, in contradiction with 2).

Theorem 3 provides the following recognition algorithm for $\{P_4, 2P_3\}$-free graphs.

Algorithm 1

The recognition algorithm for $\{P_4, 2P_3\}$-free graphs

Input: A connected, non-complete graph $G = (V, E)$.

Output: An answer to the question: "Is G $\{P_4, 2P_3\}$-free"?

Begin

1. $L_G \leftarrow \{G\}$
2. *while* $L_G \neq \phi$ *do*
3. extracts an element H from L_G
4. determine the weak decomposition (A, N, R) with $[A]H$ weak component
5. *if* ($\exists a \in A$,$\exists n \in N$ such that $an \notin E$) *then* G is not $\{P_4, 2P_3\}$-free *else*
6. introduce in L_G subgraphs $[V - R], [V - A]$ incomplete and of at least order 4
7. *Return*: G is $\{P_4, 2P_3\}$-free
8. *end*

EndRecognition

The complexity of the algorithm 1

Because step 4 takes $O(n+m)$ time, and the other steps of the cycle while take less time, it

results that the algorithm is executed in an overall time of $O(n(n+m))$.

Theorem 5. *Let G=(V,E) a connected and non-complete graph. Let (A,N,R) be a weak decomposition, with G(A) as weak component. If G=(V,E) is $\{P_4, 2P_3\}$-free and k= min $\{|N|, |A|+|R|\}$ then there is a biclique partition of length 2k of a graph G'=(V$\cup$V', E), where V' is a copy of V, and E'=$\{x\ y', x'\ y\ |xy \in E\}$.*

Proof. Because $G = (V, E)$ is $\{P_4, 2P_3\}$-free then either N or A $\cup$R is a dominating set for G. Let $k = min\{|N|, |A|+|R|\}$ be. Let $D = \{v_1, ..., v_k\}$ be a dominating set for G. Clearly D can be either N or $A \cup R$, let $D = N$. We construct $G' = (V \cup V', E)$ from G as follows:

$V' = \{v'|v \in V\}$;

$E' = \{xy', x'y|xy \in E\}$.

Note that G' is a bipartite graph, since all edges connect a vertex in V with some vertex in V'. Let $f : V \to V'$ by $f(v) = v', \forall v \in V$

function one on one. For $\forall X \subseteq V, X' = \{f(x)|x \in X\} = f(X)$. Let $D* = D \cup D'$, where $D' = f(D)$.

Let $X = A \cup R$.

For $i = 1$ to $k - 1$

let $w_i \in X$;

$X \to X - \{w_i\}$.

$w_k \to A \cup R - \cup_{i=1}^{k-1} \{w_i\}$.

Let $N_i = \{w_i\}, i = 1, ..., k$. Let $N_i' = f(N\_i), i = 1, ..., k$.

Clearly, $V = \cup_{i=1}^{k} (\{v_i\} \cup N_i)$ and $|N_i| \geq 1, i = 1, ..., k$. V is partitioned by the sets $(\{v_i\} \cup N_i)$ for all $w_i \in D = N, i = 1, ..., k$.

For each $v \in D*$ we define $N'(v)$ as follows:

if $v \in D$ then $N'(w_i) = f(N_i), i = 1, ..., k$;

if $v \in D'$ then $N'(w_i^{'}) = f^{-1}(N_i^{'}), i = 1, ..., k$.

$D*$ is a dominating set for G' . Because, for all $v \in D \cup D'$ (that is v is $v_1, \cdots, v_{k,} v_1^{'}, \cdots, v_k^{'}$, v is adjacent to every vertex in $N'(v)$ it follows that the sets $\{v\} \cup N'(v)$ form bicliques. For $v \in V'$, the sets $N'(v)$ are disjoint since the sets $N(v)$ are disjoint for $v \in V$. Because $D*$ is a dominating set for G', every vertex in G' will appear in one of these sets. Since $|D*| = 2k$, the sets $\{v\} \cup N'(v)$ for all $v \in D*$ form a biclique partition of G' of size $2k$.

Corollary 2. *If G is a connected graph, $\{P_4, 2P_3\}$-free and (A,N,R) a weak decomposition with A the weak component then the following holds:*

$\alpha(G)$=$max\{\alpha(G(N)), \alpha(G(A)) + \alpha(G(R))\}$

$\omega(G)$=$\omega(G(N))$+$max\{\omega(G(A)), \omega G((R))\}$.

We notice that the determination of α and ω takes $O(n(n + m))$ time.

4. Conclusions

In this paper we determine a (maximal) biclique of cograph, we give a a characterization of are maximal subclasses of P4-free graphs, we construct a biclique partition, a the recognition algorithm for $\{P_4, 2P_3\}$-free graphs.

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