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PRESIC TYPE FIXED POINT THEOREM FOR FOUR MAPS IN PARTIAL b-METRIC-LIKE SPACES

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Abstract. In this paper, we obtain a Presic type fixed point theorem for two pairs of jointly $2k$-weakly compatible maps in partial b-metric-like spaces.We also give an example to illustrate our main theorem. We obtain three corollaries, for three and two maps respectively,which are variations of theorems from the papers [1, 2] and [8].

1. Introduction and Preliminaries

One of the generalizations of Banach contraction principle,for mappings $f : X^k \to X$ with X a complete metric space,was given by S.B.Presic [6] in 1965.

Throughout this paper, $\mathcal{R}^+$, $\mathcal{N}$ and k denote the set of all non-negative real numbers , the set of all positive integers and a positive integer respectively.

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Theorem 1.1 (6)**.** *Let (X, d) be a complete metric space and $f : X^k \to X$. Suppose that*

$$d(f(x_1, x_2, \cdots, x_k), f(x_2, x_3, \cdots, x_{k+1})) \leq \sum_{i=1}^{k} q_i \ d(x_i, x_{i+1})$$

holds for all $x_1, x_2, \cdots, x_k, x_{k+1} \in X$, where $q_i \geq 0$ for $i = 1, 2, ..., n$ and $\sum_{i=1}^{k} q_i \in [0, 1)$.
Then f has a unique fixed point x^*. Moreover, for any arbitrary points $x_1, x_2, \cdots, x_{k+1}$ in X, the sequence $\{x_n\}$ defined by $x_{n+k} = f(x_n, x_{n+1}, \cdots, x_{n+k-1})$, for all $n \in \mathcal{N}$ converges to x^*.

Later Ciric and Presic [3] generalized the above theorem as follows.

Theorem 1.2. *(*[3]*). Let (X, d) be a complete metric space and $f : X^k \to X$. Suppose that*

$$d(f(x_1, x_2, \cdots, x_k), f(x_2, x_3, \cdots, x_{k+1})) \leq \lambda \max_{1 \leq i \leq k} \{d(x_i, x_{i+1})\}$$

holds for all $x_1, x_2, \cdots, x_k, x_{k+1}$ in X, where $\lambda \in [0, 1)$.
Then f has a fixed point $x^* \in X$. Moreover, for any arbitrary points $x_1, x_2, \cdots, x_{k+1}$ in X, the sequence $\{x_n\}$ defined by $x_{n+k} = f(x_n, x_{n+1}, \cdots, x_{n+k-1})$, for all $n \in \mathcal{N}$ converges to x^*.
Moreover, if $d(f(u, u, \cdots, u), f(v, v, \cdots, v)) < d(u, v)$ holds for all $u, v \in X$ with $u \neq v$, then x^* is the unique fixed point of f.

Recently Rao et al.[1, 2] obtained some Presic type theorems for two and three maps in metric spaces.Now we give the following definition of [1, 2].

Definition 1.3. *Let X be a non empty set and $T : X^{2k} \to X$ and $f : X \to X$.The pair (f, T) is said to be $2k$-weakly compatible if $f(T(x, x, ..., x, x)) = T(fx, fx, ..., fx, fx)$ whenever $x \in X$ such that $fx = T(x, x, ..., x, x)$.*

Using this definition, Rao et al. [1] proved the following

Theorem 1.4. *(*[1]*). Let (X, d) be a metric space and $S, T : X^{2k} \to X, f : X \to X$ be mappings satisfying*

(1.4.1) $d(S(x_1, x_2, \cdots, x_{2k}), T(x_2, x_3, \cdots, x_{2k+1})) \leq \lambda \max_{1 \leq i \leq 2k} \{d(fx_i, fx_{i+1})\}$

for all $x_1, x_2, \cdots, x_{2k}, x_{2k+1}$ in X,

(1.4.2) $d(T(y_1, y_2, \cdots, y_{2k}), S(y_2, y_3, \cdots, y_{2k+1})) \leq \lambda \max_{1 \leq i \leq 2k} \{d(fy_i, fy_{i+1})\}$

for all $y_1, y_2, \cdots, y_{2k}, y_{2k+1}$ in X, where $0 \leq \lambda < 1$.
(1.4.3) *$d(S(u, \cdots, u), T(v, \cdots, v)) < d(fu, fv)$, for all $u, v \in X$ with*

$u \neq v$
(1.4.4) *Suppose that* $f(X)$ *is complete and either* (f, S) *or* (f, T) *is a* $2k-$ *weakly compatible pair.*
Then there exists a unique point $p \in X$ *such that* $fp = p = S(p, \cdots, p) = T(p, \cdots, p)$.

Very recently Nazir and Abbas [8] obtained the following Presic type theorem for a pair of maps in partial metric spaces as follows

Theorem 1.5. *([8]). Let* (X, p) *be a complete partial metric space. Suppose that* $f, g : X^k \to X$ *are two mappings satisfying*

(1.5.1) $p(f(x_1, x_2, \cdots, x_k), g(x_2, x_3, \cdots, x_{k+1})) \leq \lambda \max_{1 \leq i \leq k} \{p(x_i, x_{i+1})\}$

for all $x_1, x_2, \cdots, x_k, x_{k+1}$ *in* X,*where* $\lambda \in [0, 1)$.
Then f *has a unique fixed point* x^*. *Moreover, for any arbitrary points* $x_1, x_2, \cdots, x_{k+1}$ *in* X, *the sequence* $\{x_n\}$ *defined by* $x_{n+k} = f(x_n, x_{n+1}, \cdots, x_{n+k-1})$, *converges to* x^*.

We observed that the conclusion is not clear and the proof given by Nazir and Abbas [8] to the Theorem is not correct. They wrongly used the condition (1.3.1)(Here (1.5.1)) three times in page 53 of [8] (see the line 5 from 4^{th} line, line 12 from line 11 and line 19 from line 18 from the above).

In this paper, we obtain a Presic type theorem for four mappings satisfying a slight different contractive condition in partial b-metric-like spaces. We also give an example and three corollaries to our main theorem. One of our corollary is a probable modification of main theorem of [8](Theorem 1.5 of this section).

Recently Khan et al.[5] introduced partial b-metric-like spaces by combining the concepts of b-metric-like spaces given by Alghamdi [4] and partial metric spaces given by Mathews [7].

Now we recall some basic definitions and remarks which play a crucial role in the theory of partial b-metric-like spaces.

Definition 1.6. *([5]) A partial b-metric-like on a nonempty set* X *is a function* $p : X \times X \to \mathcal{R}^+$ *such that for all* $x, y, z \in X$ *and a constant* $s \geq 1$ *the following are satisfied:*

(p_1) $p(x, y) = 0 \Rightarrow x = y$,
(p_2) $p(x, x) \leq p(x, y)$,
(p_3) $p(x, y) = p(y, x)$,
(p_4) $p(x, y) \leq s[p(x, z) + p(z, y) - p(z, z)]$.

The triad (X, p, s) *is called a partial b-metric-like space .*

Definition 1.7. *Let (X, p, s) be a partial b-metric-like space.*

(i) *A sequence $\{x_n\}$ in X is said to be convergent to $x \in X$ if $p(x, x) = \lim_{n\to\infty} p(x, x_n)$.*
(ii) *A sequence $\{x_n\}$ in X is said to be a Cauchy sequence in X if $\lim_{n,m\to\infty} p(x_n, x_m)$ exists and is finite.*
(iii) *X is said to be complete if every Cauchy sequence $\{x_n\}$ in X converges to a point $x \in X$ such that $\lim_{n\to\infty} p(x, x_n) = p(x, x) = \lim_{n,m\to\infty} p(x_n, x_m)$.*

Remark 1.8. *Let (X, p, s) be a partial b-metric-like space and $\{x_n\}$ be a sequence in X such that $\lim_{n\to\infty} p(x, x_n) = 0$. Then*

(i) *x is unique,*
(ii) *$\frac{1}{s}\ p(x, y) \le \lim_{n\to\infty} p(x_n, y) \le s\ p(x, y)$ for all $y \in X$,whenever the limit exists,*
(iii) *$p(x_n, x_0) \le s\ p(x_0, x_1) + s^2\ p(x_1, x_2) + ... + s^{n-1}\ p(x_{n-2}, x_{n-1}) + s^n\ p(x_{n-1}, x_n)$.*

Now, we introduce the definition of jointly $2k$-weakly compatible pairs.

Definition 1.9. *Let X be a nonempty set, k a positive integer, $S, T : X^{2k} \to X$ and $f, g : X \to X$ be mappings. Then the pairs (f, S) and (g, T) are said to be jointly $2k$-weakly compatibe if $f(S(x, x, ..., x) = S(fx, fx, ..., fx)$ and $g(T(x, x, ..., x)) = T(gx, gx, ..., gx)$ whenever there exists $x \in X$ such that $fx = S(x, x, ..., x)$ and $gx = T(x, x, ..., x)$.*

Remark 1.10. *If the two pairs (f, S) and (g, T) are $2k$-weakly compatible then the pairs (f, S) and (g, T) are jointly $2k$-weakly compatible. But the converse need not be true in view of the following example.*

Example 1.11. *Let $X = [0, 1]$ and $k = 1$. Define $S(x, y) = \frac{3x^2+2y}{72}$, $T(x, y) = \frac{2x+3y^2}{72}$, $fx = \frac{x}{6}$, $gx = \frac{x^2}{4}$ for all $x, y \in X$.*
The pair (g, T) is not 2-weakly compatible since $T(x, x) = gx$ implies $x = 0, \frac{2}{15}$ and $g(T(\frac{2}{15}, \frac{2}{15})) \ne T(g\frac{2}{15}, g\frac{2}{15})$.But the pairs (f, S) and (g, T) are jointly 2-weakly compatible.

Now we present our main result.

2. Main Result

Theorem 2.1. . *Let (X, p, s) be a complete partial b-metric-like space with $s \ge 1$ and $S, T : X^{2k} \to X$ and $f, g : X \to X$ be mappings*

satisfying
(2.1.1) $S(X^{2k}) \subseteq g(X)$, $T(X^{2k}) \subseteq f(X)$,
(2.1.2)
$$p(S(x_1, x_2, \cdots, x_{2k-1}, x_{2k}), T(y_1, y_2, \cdots, y_{2k-1}, y_{2k}))$$
$$\leq \lambda \max \left\{ \begin{array}{c} p(gx_1, fy_1), p(fx_2, gy_2), p(gx_3, fy_3), p(fx_4, gy_4), \cdots, \\ p(gx_{2k-1}, fy_{2k-1}), p(fx_{2k}, gy_{2k}) \end{array} \right\}$$
for all $x_1, x_2, \cdots, x_{2k}, y_1, y_2, \cdots, y_{2k} \in X$ *and* $\lambda \in (0, \frac{1}{s^{2k}})$,
(2.1.3) (f, S) *and* (g, T) *are jointly* $2k-$*weakly compatible pairs.*
(2.1.4) *Suppose* $z = fu = gu$ *for some* $u \in X$ *whenever there exists a sequence* $\{y_{2k+n}\}_{n=1}^{\infty}$ *in* X *such that* $\lim_{n \to \infty} y_{2k+n} = z \in X$.
Then z *is the unique point in* X *such that* $z = fz = gz = S(z, z, \cdots, z) = T(z, z, \cdots, z)$.

Proof. Suppose $x_1, x_2, \cdots, x_{2k}$ are arbitrary points in X.
From (2.1.1), define
$$\begin{aligned} y_{2k+2n-1} &= S(x_{2n-1}, x_{2n}, \cdots x_{2k+2n-2}) = gx_{2k+2n-1} \\ y_{2k+2n} &= T(x_{2n}, x_{2n+1}, \cdots x_{2k+2n-1}) = fx_{2k+2n} \; for \; n = 1, 2, \cdots \end{aligned}$$
Let
$$\begin{aligned} \alpha_{2n} &= p(fx_{2n}, gx_{2n+1}), \\ \alpha_{2n-1} &= p(gx_{2n-1}, fx_{2n}), \; n = 1, 2, \cdots \end{aligned}$$
Let $\theta = \lambda^{\frac{1}{2k}}$ and $\mu = \max\{\frac{\alpha_1}{\theta}, \frac{\alpha_2}{\theta^2}, \cdots, \frac{\alpha_{2k}}{\theta^{2k}}\}$.
Then $0 < \theta < 1$ and by the selection of μ we have
(1) $\alpha_n \leq \mu\theta^n, for \; n = 1, 2, \cdots, 2k$.
(2) $\alpha_{2k+1} = p(gx_{2k+1}, fx_{2k+2})$
$= p(S(x_1, x_2, x_3, x_4, \cdots, x_{2k-1}, x_{2k}), T(x_2, x_3, x_4, \cdots, x_{2k}, x_{2k+1}))$
$\leq \lambda \max \left\{ \begin{array}{c} p(gx_1, fx_2), p(fx_2, gx_3), p(gx_3, fx_4), \\ p(fx_4, gx_5), \cdots, p(gx_{2k-1}, fx_{2k}), (fx_{2k}, gx_{2k+1}) \end{array} \right\}$
from (2.1.2)
$= \lambda \max\{\alpha_1, \alpha_2, \alpha_3, \alpha_4, \cdots, \alpha_{2k-1}, \alpha_{2k}\}$
$\leq \lambda \max\{\mu\theta, \mu\theta^2, \mu\theta^3, \mu\theta^4, \cdots, \mu\theta^{2k-1}, \mu\theta^{2k}\} \; from(1)$
$= \lambda\mu\theta = \theta^{2k}\mu\theta$
$= \mu\theta^{2k+1}$
and
(3)$\alpha_{2k+2} = p(fx_{2k+2}, gx_{2k+3})$
$= p(S(x_3, x_4, x_5, x_6, \cdots, x_{2k+1}, x_{2k+2}), T(x_2, x_3, x_4, \cdots, x_{2k}, x_{2k+1}))$
$\leq \lambda \max \left\{ \begin{array}{c} p(gx_3, fx_2), p(fx_4, gx_3), p(gx_5, fx_4), \\ p(fx_6, gx_5), \cdots, p(gx_{2k+1}, fx_{2k}), p(fx_{2k+2}, gx_{2k+1}) \end{array} \right\}$
from (2.1.2)
$= \lambda \max\{\alpha_2, \alpha_3, \alpha_4, \alpha_5, \cdots, \alpha_{2k}, \alpha_{2k+1}\}$
$\leq \lambda \max\{\mu\theta^2, \mu\theta^3, \mu\theta^4, \mu\theta^5, \cdots, \mu\theta^{2k}, \mu\theta^{2k+1}\} \; from \; (1), (2)$
$= \lambda\mu\theta^2 = \theta^{2k}\mu\theta^2$
$= \mu\theta^{2k+2}$.

Continuing in this way,by induction, we get

(4) $\alpha_n \leq \mu\theta^n \; for \; n = 1, 2, \cdots$

Now consider

(5)$p(y_{2k+2n-1}, y_{2k+2n})$

$= p(S(x_{2n-1}, x_{2n}, \cdots, x_{2k+2n-3}, x_{2k+2n-2}),$

$T(x_{2n}, x_{2n+1}, \cdots, x_{2k+2n-2}, x_{2k+2n-1}))$

$\leq \lambda \max\{p(gx_{2n-1}, fx_{2n}), p(fx_{2n}, gx_{2n+1}), \cdots, p(fx_{2k+2n-2}, gx_{2k+2n-1})\}$

$= \lambda \max\{\alpha_{2n-1}, \alpha_{2n}, \alpha_{2n+1}, \alpha_{2n+2} \cdots, \alpha_{2k+2n-3}, \alpha_{2k+2n-2}\}$

$\leq \lambda \max\{\mu\theta^{2n-1}, \mu\theta^{2n}, \mu\theta^{2n+1}, \mu\theta^{2n+2}, \cdots, \mu\theta^{2k+2n-3}, \mu\theta^{2k+2n-2}\},$

from (4)

$= \lambda\mu\theta^{2n-1} = \theta^{2k}\mu\theta^{2n-1}$

$= \mu\theta^{2k+2n-1}$

Also

(6) $p(y_{2k+2n}, y_{2k+2n+1}) = p(S(x_{2n+1}, x_{2n+2}, \cdots, x_{2k+2n-1}, x_{2k+2n}),$

$T(x_{2n}, x_{2n+1}, \cdots, x_{2k+2n-2}, x_{2k+2n-1}))$

$\leq \lambda \max\{p(gx_{2n+1}, fx_{2n}), p(fx_{2n+2}, gx_{2n+1}), \cdots, p(fx_{2k+2n}, gx_{2k+2n-1})\}$

$= \lambda \max\{\alpha_{2n}, \alpha_{2n+1}, \alpha_{2n+2}, \alpha_{2n+3}, \cdots, \alpha_{2k+2n-2}, \alpha_{2k+2n-1}\}$

$\leq \lambda \max\{\mu\theta^{2n}, \mu\theta^{2n+1}, \mu\theta^{2n+2}, \mu\theta^{2n+3}, \cdots, \mu\theta^{2k+2n-2}, \mu\theta^{2k+2n-1}\},$

from (4)

$= \lambda\mu\theta^{2n} = \theta^{2k}\mu\theta^{2n}$

$= \mu\theta^{2k+2n}$

Thus from (5) and (6) we have

(7) $p(y_{2k+n}, y_{2k+n+1}) \leq \mu\theta^{2k+n} \; for \; n = 1, 2, 3, \cdots$

Now from $m > n$ consider

(8)$p(y_{2k+n}, y_{2k+m}) \leq s\; p(y_{2k+n}, y_{2k+n+1}) + s^2\; p(y_{2k+n+1}, y_{2k+n+2}) + \cdots$

$+ s^{m-n-1}\; p(y_{2k+m-1}, y_{2k+m})$

$\leq s\; \mu\theta^{2k+n} + s^2\; \mu\theta^{2k+n+1} + \cdots + s^{m-n-1}\; \mu\theta^{2k+m-1}$

from (7) $\leq \mu[(s\theta)^{2k+n} + (s\theta)^{2k+n+1} + \cdots + (s\theta)^{2k+m-1}]$, since $s \geq 1$

$\leq \mu(s\theta)^{2k}\frac{(s\theta)^n}{1-s\theta}$, since $s\theta = s\lambda^{\frac{1}{2k}} < s\frac{1}{s} = 1$

$\to 0$ as $n \to \infty, m \to \infty$

Hence $\{y_{2k+n}\}$ is a Cauchy sequence in (X, p, s). Since (X, p, s) is complete, there exists $z \in X$ such that

$p(z, z) = \lim_{n\to\infty} p(y_{2k+n}, z) = \lim_{n,m\to\infty} p(y_{2k+n}, y_{2k+m}).$

From (8), we have

(9) $p(z, z) = 0$

From (2.1.4), there exists $u \in X$ such that

(10) $z = fu = gu$

Now consider

(11) $p(S(u, u, \cdots, u, u), y_{2k+2n})$

$= p(S(u, u, \cdots, u), T(x_{2n}, x_{2n+1}, \cdots, x_{2k+2n-3})$

$\leq \lambda \max\{p(gu, fx_{2n}), p(fu, gx_{2n+1}), \cdots, p(gu, fx_{2k+2n-2}), p(fu, gx_{2k+2n-1})\}$
Letting $n \to \infty$ and using (9),(10)and Remark 1.8(ii) ,we get
$\frac{1}{s}\, p(S(u, u, \cdots, u, u), fu) \leq 0$ so that $fu = S(u, u, \cdots, u, u)$
Similarly we have
(12) $gu = T(u, u, \cdots, u, u)$
Since (f, S) and (g, T) are jointly $2k-$weakly compatible pairs, we have
(13) $fz = f(fu) = f(S(u, u, \cdots, u, u)) = S(fu, fu, \cdots, fu, fu)) = S(z, z, \cdots, z, z)$
and
(14) $gz = T(z, z, \cdots, z, z)$
Now consider
$p(fz, z) = p(fz, gu)$
$= p(S(z, z, z, z, \cdots, z, z), T(u, u, u, u, \cdots, u, u))$, from (13),(12)
$\leq \lambda \max\{p(gz, fu), p(fz, gu), \cdots, p(gz, fu), p(fz, gu)\}$
$= \lambda \max\{p(gz, z), d(fz, z)\}$, from (10)
Similarly we have $p(gz, z) \leq \lambda \max\{p(gz, z), p(fz, z)\}$.
Thus we have $\max\{p(fz, z), d(gz, z)\} \leq \lambda \max\{p(fz, z), p(gz, z)\}$
which in turn yields that $z = fz = gz$.
From (13) and (14), it follows that
(15) $z = fz = gz = S(z, z, \cdots, z, z) = T(z, z, \cdots, z, z)$
Suppose there exists $z' \in X$ such that
$z' = fz' = gz' = S(z', z', \cdots, z') = T(z', z', \cdots, z')$.
Then from (2.1.2) we have
$p(z, z')$
$= p(S(z, z, \cdots, z), T(z', z', \cdots, z'))$
$\leq \lambda \max\{p(gz, fz'), p(fz, gz'), \cdots, p(gz, fz'), p(fz, gz')\} = \lambda p(z, z')$
This implies that $z' = z$.
Thus z is unique point in X satisfying (15).
Now we give an example to illustrate our main Theorem 2.1.

Example 2.2. *Let* $X = [0, 1]$ *,* $p(x, y) = \max\{x^2, y^2\}$ *and* $k = 1$*.Then* (X, p, s) *is a complete partial b-metric-like space with* $s = 2$*.Define* $S(x, y) = [\frac{3x^2+2y}{72}]^{\frac{1}{2}}, T(x, y) = [\frac{2x+3y^2}{72}]^{\frac{1}{2}}, fx = [\frac{x}{6}]^{\frac{1}{2}}$ *and* $gx = \frac{x}{2}$ *for all* $x, y \in X$*.*

Then for all $x_1, x_2, y_1, y_2 \in X$, *we have*

$$\begin{aligned}
&p(S(x_1,x_2),T(y_1,y_2))\\
&= \max\{\tfrac{3x_1^2+2x_2}{72}, \tfrac{2y_1+3y_2^2}{72}\}\\
&\le \tfrac{1}{72}[\max\{3x_1^2, 2y_1\} + \max\{2x_2, 3y_2^2\}]\\
&\le \tfrac{1}{36}\max\{\max\{3x_1^2, 2y_1\}, \max\{2x_2, 3y_2^2\}\}\\
&= \tfrac{1}{3}\max\{\max\{\tfrac{x_1^2}{4}, \tfrac{y_1}{6}\}, \max\{\tfrac{x_2}{6}, \tfrac{y_2^2}{4}\}\}\\
&= \tfrac{1}{3}\max\{p(gx_1, fy_1), p(fx_2, gy_2)\}.
\end{aligned}$$

Thus the condition (2.1.2) *of Theorem* 2.1 *is satisfied. One can easily verify the remaining conditions of Theorem* 2.1. *Clearly,0 is the unique point in* X *such that* $0 = f0 = g0 = S(0,0,...,0,0) = T(0,0,...,0,0)$.

Corollary 2.3. *Let* (X,p,s) *be a partial b-metric-like space with* $s \ge 1$ *and* $S,T : X^{2k} \to X$ *and* $f : X \to X$ *be mappings satisfying*
(2.3.1) $S(X^{2k}) \subseteq f(X)$, $T(X^{2k}) \subseteq f(X)$,
(2.3.2) $p(S(x_1,x_2,\cdots,x_{2k-1},x_{2k}), T(y_1,y_2,\cdots,y_{2k-1},y_{2k})) \le \lambda \max\limits_{1\le i\le 2k}\{p(fx_i, fy_i)\}$ *for all* $x_1,x_2,\cdots,x_{2k},y_1,y_2,\cdots,y_{2k} \in X$ *and* $\lambda \in (0, \frac{1}{s^{2k}})$,
(2.3.3) $f(X)$ *is a complete subspace of* X,
(2.3.4) *the pairs* (f,S) *or* (f,T) *is* $2k-$*weakly compatible.*
Then there exists a unique $z \in X$ *such that* $z = fz = S(z,z,\cdots,z,z) = T(z,z,\cdots,z,z)$.

Corollary 2.4. *Let* (X,p,s) *be a complete partial b-metric-like space with* $s \ge 1$ *and* $S,T : X^{2k} \to X$ *be mappings satisfying* (2.4.1) $p(S(x_1,x_2,\cdots,x_{2k-1},x_{2k}), T(y_1,y_2,\cdots,y_{2k-1},y_{2k})) \le \lambda \max\limits_{1\le i\le 2k}\{p(x_i,y_i)\}$ *for all* $x_1,x_2,\cdots,x_{2k},y_1,y_2,\cdots,y_{2k} \in X$ *and* $\lambda \in (0, \frac{1}{s^{2k}})$
Then there exists a unique $u \in X$ *such that*
$u = S(u,u,\cdots,u,u) = T(u,u,\cdots,u,u)$.

Corollary 2.5. *Let* (X,p,s) *be a partial b-metric-like space with* $s \ge 1$*and* $S : X^k \to X$ *and* $f : X \to X$ *be mapping satisfying*
(2.5.1) $S(X^k) \subseteq f(X)$,
(2.5.2) $p(S(x_1,x_2,\cdots,x_{k-1},x_k), S(y_1,y_2,\cdots,y_{k-1},y_k)) \le \lambda \max\limits_{1\le i\le k}\{p(fx_i,fy_i)\}$ *for all* $x_1,x_2,\cdots,x_k,y_1,y_2,\cdots,y_k \in X$ *and* $\lambda \in (0, \frac{1}{s^k})$,
(2.5.3) $f(X)$ *is a complete subspace of* X,
(2.5.4) *the pair* (f,S) *is* $k-$*weakly compatible.*
Then there exists a unique $z \in X$ *such that* $z = fz = S(z,z,\cdots,z,z)$.

References

[1] K.P.R.Rao, G.N.V.Kishore and Md.Mustaq Ali,**Generalization of Banach contraction principle of Presic type for three maps**, Math.Sci. 3(3), 2009, 273 - 280.

[2] K.P.R.Rao, Md.Mustaq Ali and B.Fisher,**Some Presic type generalization of Banach contraction principle**, Math.Moravica 15(2011), 41 - 47.

[3] Lj.B.Ciric and S.B.Presic,**On Presic type generalization of Banach contraction mapping principle**, Acta.Math.Univ.Comenianae, LXXVI (2), (2007), 143-147.

[4] M.Alghamdi,N.Ali Hussain and P.Salimi,**Fixed point and coupled fixed point theorems on b-metric-like spaces**,J.Inequalities and Applications,2013,(2013),402,25 pages.

[5] M.S.Khan,K.P.R.Rao and K.V.S.Parvathi,**A unique common coupled fixed point theorem for four maps in partial b-metric-like spaces**,SQU Journal for Science,19(2),(2015),55-61.

[6] S.B.Presic, **Sur une classe d' inequations aux differences finite et sur la convergence de certaines suites**, Pub de l' inst. Math. Belgrade, 5(19), (1965), 75-78.

[7] S.G.Matthews , **Partial metric topology, in Proceedings of the 8th Summer Conference on General Topology and Applications** 1994. vol. 728, pp. 183-197, Annals of the New York Academy of Sciences, 1994.

[8] Talat Nazir and Mujahid Abbas, **Common fixed point of Presic type contraction mappings in partial metric spaces**, Journal of Nonlinear Analysis and optimization, 5(1),(2014), 49 - 55.

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