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A GENERAL RESULT FOR PAIRS OF WEAKLY COMPATIBLE MAPPINGS IN G - METRIC SPACES

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Abstract. In this paper a general fixed point theorem in complete G - metric space for weakly compatible mappings is proved, theorem which generalizes and unifies the results from [5].

Dedicated by the first author to Professor Valeriu Popa on the Occasion of His 80th Birthday

1. Introduction

Let (X, d) be a metric space and S, $T : (X, d) \to (X, d)$ be two mappings. In 1994, Pant [15] introduced the notion of pointwise R - weakly commuting mappings. It is proved in [16] that the notion of pointwise R - weakly commutativity is equivalent to commutativity in coincidence points. Jungck [4] defined S and T to be weakly compatible if $Sx = Tx$ implies $STx = TSx$. Thus, S and T are weakly compatible if and only if S and T are pointwise R - weakly commuting.

In [2] and [3], Dhage introduced a new class of generalized metric spaces, named D - metric space. Mustafa and Sims [6], [7] proved that most of the claims concerning the fundamental topological

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structures on D - metric spaces are incorrect and introduced appropriate notion of generalized metric space, named G - metric space. In fact, Mustafa, Sims and other authors studied many fixed point results for self mappings in G - metric spaces under certain conditions [8] - [14], [23] and other papers.

In [17] and [18], Popa initiated the study of fixed points for mappings satisfying implicit relations.

Actually, the method is used in the study of fixed points in metric spaces, symmetric spaces, quasi - metric spaces, compact metric spaces, Tychonoff spaces, reflexive metric spaces, probabilistic metric spaces, convex metric spaces, in two or three metric spaces for single valued mappings, hybrid pairs of mappings and set valued mappings.

Recently, the method is used in the study of fixed points for mappings satisfying contractive conditions of integral type and in fuzzy metric spaces. There exists a vast literature in this topic which cannot be completely cited here.

The method unified different types of contractive and extensive conditions. The proof of fixed point theorems are more simple. Also, this method allows the study of local and global properties of fixed point structures.

Recently, the present authors initiated the study of fixed points in G - metric spaces using implicit relations in [19] - [21].

The study of fixed points for pairs of weakly compatible mappings in G - metric spaces is initiated in [5], [20], [22].

In this paper a general fixed point theorem in G - metric spaces for weakly compatible mappings is proved, theorem which generalize and unified the results from [5].

2. Preliminaries

Definition 2.1 ([7]). Let X be a nonempty set and $G : X^3 \to \mathbb{R}_+$ be a function satisfying the following properties:

$(G_1) : G(x, y, z) = 0$ if $x = y = z$,

$(G_2) : 0 < G(x, x, y)$ for all $x, y \in X$ with $x \neq y$,

$(G_3) : G(x, x, y) \leq G(x, y, z)$ for all $x, y, z \in X$ with $z \neq y$,

$(G_4) : G(x, y, z) = G(y, z, x) = G(z, x, y) = ...$ (symmetry in all three variables),

$(G_5) : G(x, y, z) \leq G(x, a, a) + G(a, y, z)$ for all $x, y, z, a \in X$.

The function G is called a G - metric on X and the pair (X, G) is called a G - metric space.

Note that $G(x, y, z) = 0$, then $x = y = z$.

Definition 2.2 ([7]). Let (X, G) be a metric space. A sequence (x_n) in X is said to be

a) G - convergent if for $\varepsilon > 0$, there is an $x \in X$ and $k \in \mathbb{N}$ such that for all $n, m \in \mathbb{N}$, $m, n \geq k$, $G(x, x_n, x_m) < \varepsilon$.

b) G - Cauchy if for each $\varepsilon > 0$, there exists $k \in \mathbb{N}$ such that for all $n, m, p \in \mathbb{N}$, $n, m, p \geq k$, $G(x_n, x_m, x_p) < \varepsilon$, that is $G(x_n, x_m, x_p) \to 0$ as $m, n, n \to \infty$.

c) A G - metric space is said to be G - complete if every G - Cauchy sequence is G - convergent.

Lemma 2.3 ([7]). *Let (X, G) be a G - metric space. Then, the following properties are equivalent:*

1) (x_n) is G - convergent to x;

2) $G(x_n, x_n, x) \to 0$ as $n \to \infty$;

3) $G(x_n, x, x) \to 0$ as $n \to \infty$;

4) $G(x_m, x_n, x) \to 0$ as $m, n \to \infty$.

Lemma 2.4 ([7]). *If (X, G) is a G - metric space, the following are equivalent:*

1) (x_n) is G - Cauchy;

2) For every $\varepsilon > 0$, there is $k \in \mathbb{N}$ such that $G(x_n, x_m, x_m) < \varepsilon$ for all $n, m \in \mathbb{N}$, $n, m \geq k$.

Lemma 2.5 ([7]). *Let (X, G) be a G - metric space, then the function $G(x, y, z)$ is jointly continuous in all three of its variables.*

Lemma 2.6 ([7]). *Let (X, G) be a G - metric space. Then $G(x, x, y) \leq 2G(y, y, x)$ for all $x, y \in X$.*

Definition 2.7. Let f and g be self maps of a nonempty set X. If $w = fx = gx$ for some $x \in X$, then x is called a coincidence point of f and g and w is called a point of coincidence of f and g.

Lemma 2.8 ([1]). *Let f and g be weakly compatible self mappings of a nonempty set X. If f and g have an unique point of coincidence $w = fx = gx$, then w is the unique common fixed point of f and g.*

The following theorems are proved in [5].

Theorem 2.9 (Theorem 2.1 [5]). *Let (X, G) be a G - metric space and $f, g : X \to X$ satisfying the following condition*

$$G(fx, fy, fz) \leq kM(x, y, z),$$

where

$$M(x, y, z) = \max\{G(gx, gy, gz), G(gx, fy, gz), G(gy, fx, gz), G(gx, fx, gz), G(gy, fy, gz)\} \tag{2.1}$$

for all $x, y, z \in X$, *where* $k \in \left[0, \frac{1}{2}\right)$.

If $f(X) \subset g(X)$ *and* $g(X)$ *is a* G *- complete subspace of* X, *then* f *and* g *have an unique point of coincidence in* X. *Moreover, if* f *and* g *are weakly compatible, then* f *and* g *have an unique common fixed point.*

Theorem 2.10 (Theorem 2.2 [5]). *Let* (X, G) *be a* G *- metric space and* $f, g : X \to X$ *satisfying the following condition*

$$G(fx, fy, fz) \leq kM(x, y, z),$$

where

$$\begin{aligned} M(x,y,z) = a_1G(gx,gy,gz) + a_2G(gx,gx,fx)+ \\ a_3G(gy,gy,fy) + a_4G(gz,gz,fz) + a_5G(gx,gx,fy)+ \\ a_6G(gy,gy,fz) + a_7G(gz,gz,fx), \end{aligned} \tag{2.2}$$

for all $x, y, z \in X$, *where* $0 \leq a_1 + a_2 + a_3 + a_4 + a_5 + 2a_6 + a_7 < 1$.

If $f(X) \subset g(X)$ *and* $g(X)$ *is a* G *- complete subspace of* X, *then* f *and* g *have an unique point of coincidence. Moreover, if* f *and* g *are weakly compatible, then* f *and* g *have an unique common fixed point.*

Theorem 2.11 (Theorem 2.3 [5]). *Let* (X, G) *be a* G *- metric space and* $f, g : X \to X$ *satisfying the following condition*

$$G(fx, fy, fz) \leq kM(x, y, z),$$

where

(2.3)

$$\begin{aligned} M(x,y,z) = \max\{G(gx,gx,fy) + G(gx,gy,fz) + G(gz,gz,fx), \\ G(gy,fx,fx) + G(gy,fy,fy) + G(gz,fz,fz), \\ G(gy,fx,fx) + G(gz,fy,fy) + G(gx,fz,fz)\} \end{aligned}$$

for all $x, y, z \in X$, *where* $k \in \left[0, \frac{1}{6}\right)$.

If $f(X) \subset g(X)$ *and* $g(X)$ *is a* G *- complete subspace of* X, *then* f *and* g *have an unique point of coincidence. Moreover, if* f *and* g *are weakly compatible, then* f *and* g *have an unique common fixed point.*

3. Implicit relations

Definition 3.1 ([20]). Let $\mathfrak{F}_G$ be the set of all continuous functions $F(t_1, ..., t_6) : \mathbb{R}_+^6 \to \mathbb{R}$ such that

(F_1) : F is nonincreasing in variable t_5,

(F_2) : There exists $h_1 \in [0, 1)$ such that for all $u, v \geq 0$, $F(u, v, v, u, u + v, 0) \leq 0$ implies $u \leq h_1 v$.

(F_3) : There exists $h_2 \in [0,1)$ such that for all $t, t' > 0$, $F(t,t,0,0,t,t') < 0$ implies $t \leq h_2 t'$.

In all the following examples, condition (F_1) is obviously.

Example 3.2. $F(t_1, ..., t_6) = t_1 - k\max\{t_2, 2t_3 + 2t_6\} - a\max\{t_4, t_5\}$, *where* $k, a \geq 0$ *and* $0 \leq 2a + 2k < 1$.

(F_2) : *Let* $u, v \geq 0$ *be and* $F(u,v,v,u,u+v,0) = u - k\max\{v, 2v\} - a\max\{u, u+v\} = u - 2kv - a(u+v) \leq 0$, *which implies* $u \leq h_1 v$, *where* $0 \leq h_1 = \dfrac{a+2k}{1-a} < 1$.

(F_3) : *Let* $t, t' > 0$ *and* $F(t,t,0,0,t,t') = t - k\max\{t, 2t'\} - at \leq 0$. *If* $t > 2t'$, *then* $t(1-(k+a)) \leq 0$, *a contradiction. Hence,* $t \leq 2t'$ *which implies* $t \leq h_2 t'$, *where* $0 \leq h_2 = \dfrac{2k}{1-a} < 1$.

Example 3.3. $F(t_1, ..., t_6) = t_1 - a_1 t_2 - (a_2 + a_3 + a_5)t_3 - a_4 t_4 - a_6 t_5 - a_7 t_6$, *where* $a_1, a_2, ..., a_7 \geq 0$ *and* $0 < a_1 + a_2 + a_3 + a_4 + a_5 + 2a_6 + a_7 < 1$.

(F_2) : *Let* $u, v \geq 0$ *be and* $F(u,v,v,u,u+v,0) = u - a_1 v - (a_2 + a_3 + a_5)v - a_4 u - a_6(u+v) \leq 0$, *which implies* $u \leq h_1 v$, *where* $0 \leq h_1 = \dfrac{a_1 + a_2 + a_3 + a_5 + a_6}{1 - a_4 - a_6} < 1$.

(F_3) : *Let* $t, t' > 0$ *and* $F(t,t,0,0,t,t') = t - a_1 t - a_6 t - a_7 t' \leq 0$ *which implies* $t \leq h_2 t'$, *where* $0 \leq h_2 = \dfrac{a_7}{1 - a_1 - a_6} < 1$.

Example 3.4. $F(t_1, ..., t_6) = t_1 - at_2 - k\max\{t_3 + t_5 + t_6, 4t_3 + 2t_4, 2t_3 + 2t_5 + 2t_6\}$, *where* $0 \leq a + 6k < 1$.

(F_2) : *Let* $u, v \geq 0$ *be and* $F(u,v,v,u,u+v,0) = u - av - k\max\{u+2v, 2u+4v\} \leq 0$ *which implies* $u \leq h_1 v$, *where* $0 \leq h_1 = \dfrac{a+4k}{1-2k} < 1$.

(F_3) : *Let* $t, t' > 0$ *and* $F(t,t,0,0,t,t') = t - at - k\max\{t + t', 2t, 2t + 2t'\} \leq 0$ *which implies* $t \leq h_2 t'$, *where* $0 \leq h_2 = \dfrac{2k}{1-a-2k} < 1$.

The following examples are proved in [20].

Example 3.5. $F(t_1, ..., t_6) = t_1 - k\max\{t_2, t_3, t_4, t_5, t_6\}$, *where* $k \in \left[0, \dfrac{1}{2}\right)$.

Example 3.6. $F(t_1, ..., t_6) = t_1 - k\max\left\{t_2, t_3, t_4, \dfrac{t_5 + t_6}{2}\right\}$, *where* $k \in [0,1)$.

Example 3.7. *$F(t_1,...,t_6) = t_1 - t_1(at_2 + bt_3 + ct_4) - dt_5t_6$, where $a,b,c,d \geq 0$ and $0 \leq a+b+c+d < 1$.*

Example 3.8. *$F(t_1,...,t_6) = t_1 - k\max\left\{t_2, \frac{t_3+t_4}{2}, \frac{t_5+t_6}{2}\right\}$, where $k \in [0,1)$.*

Example 3.9. *$F(t_1,...,t_6) = t_1^3 - c\frac{t_3^2t_4^2 + t_5^2t_6^2}{1+t_2+t_3+t_4}$, where $c \in [0,1)$.*

Example 3.10. *$F(t_1,...,t_6) = t_1^2 - at_2^2 - b\frac{t_5t_6}{1+t_3^2+t_4^2}$, where $a,b \geq 0$ and $0 \leq a+b < 1$.*

Example 3.11. *$F(t_1,...,t_6) = t_1 - at_2 - bt_3 - c\max\{2t_4, t_5+t_6\}$, where $a,b,c \geq 0$ and $0 \leq a+b+2c < 1$.*

Example 3.12. *$F(t_1,...,t_6) = t_1 - at_2 - bt_3 - c\max\{t_4+t_5, 2t_6\}$, where $a,b,c \geq 0$ and $0 \leq a+b+3c < 1$.*

Example 3.13. *$F(t_1,...,t_6) = t_1 - k\max\left\{t_2, t_3, t_4, \frac{2t_4+t_6}{3}, \frac{2t_4+t_3}{3}, \frac{t_5+t_6}{3}\right\}$, where $k \in [0,1)$.*

4. General fixed point theorem

Lemma 4.1. *Let (X,G) be a G - metric space and let $f,g : X \to X$ be two functions such that*

$$F(G(fx,fx,fz),G(gx,gx,gz),G(gx,gx,fx),\\ G(gz,gz,fz),G(gx,gx,fz),G(gz,gz,fx)) \leq 0 \tag{4.1}$$

for all $x,z \in X$ and F satisfying property (F_3). Then, f and g have at most a point of coincidence.

Proof. Suppose that $u = fp = gp$ and $v = fq = gq$. Then by (4.1) we have successively:

$$F(G(fp,fp,fq),G(gp,gp,gq),G(gp,gp,fp),\\ G(gq,gq,fq),G(gp,gp,fq),G(gq,gq,fp)) \leq 0,$$

$$F(G(gp,gp,gq),G(gp,gp,gq),0,0,G(gp,gp,gq),G(gq,gq,gp)) \leq 0$$

which implies by (F_3) that

$$G(gp,gp,gq) \leq h_2G(gq,gq,gp).$$

Similarly, we obtain

$$G(gq,gq,gp) \leq h_2G(gp,gp,gq),$$

which implies that

$$G(gp, gp, gq)(1 - h_2^2) \leq 0.$$

Hence $G(gp, gp, gq) = 0$, i.e. $gp = gq$. Therefore $u = fp = gp = gq = fq = v$. $\square$

Theorem 4.2. *Let (X, G) be a G - metric space and $f, g : X \to X$ two functions satisfying (4.1) for all $x, z \in X$, where $F \in \mathfrak{F}_G$. If $f(X) \subset g(X)$ and $g(X)$ is a G - complete metric subspace of (X, G), then f and g have an unique point of coincidence. Moreover, if f and g are weakly compatible, then f and g have an unique common fixed point.*

Proof. Let x_0 be an arbitrary point of X and $x_1 \in X$ such that $fx_0 = gx_1$. This can be done since $f(X) \subset g(X)$. Continuing this process, having chosen x_n in X, we obtain x_{n+1} such that $fx_n = gx_{n+1}$. Then, by (4.1) we have successively

$$F(G(fx_{n-1}, fx_{n-1}, fx_n), G(gx_{n-1}, gx_{n-1}, gx_n), G(gx_{n-1}, gx_{n-1}, fx_{n-1}),$$
$$G(gx_n, gx_n, fx_n), G(gx_{n-1}, gx_{n-1}, fx_n), G(gx_n, gx_n, fx_{n-1})) \leq 0,$$

$$F(G(gx_n, gx_n, gx_{n+1}), G(gx_{n-1}, gx_{n-1}, gx_n), G(gx_{n-1}, gx_{n-1}, gx_n),$$
$$G(gx_n, gx_n, gx_{n+1}), G(gx_{n-1}, gx_{n-1}, gx_{n+1}), 0) \leq 0.$$

By (F_1) and (G_5) we obtain

$$F(G(gx_n, gx_n, gx_{n+1}), G(gx_{n-1}, gx_{n-1}, gx_n),$$
$$G(gx_{n-1}, gx_{n-1}, gx_n), G(gx_n, gx_n, gx_{n+1}),$$
$$G(gx_n, gx_n, gx_{n+1}) + G(gx_{n-1}, gx_{n-1}, gx_n), 0) \leq 0.$$

By (F_2) we obtain

$$G(gx_n, gx_n, gx_{n+1}) \leq h_1 G(gx_{n-1}, gx_{n-1}, gx_n).$$

Continuing the above process we obtain

$$G(gx_n, gx_n, gx_{n+1}) \leq h_1^n G(gx_0, gx_0, gx_1).$$

For every $m, n \in \mathbb{N}$, $m > n$ we have by repeated use of the rectangle inequality that

$$\begin{aligned} G(gx_n, gx_n, gx_m) &\leq \sum_{j=n}^{m-1} G(gx_j, gx_j, gx_{j+1}) \\ &\leq \sum_{j=n}^{m-1} h_1^j G(gx_0, gx_0, gx_1) \\ &\leq \frac{h_1^n}{1 - h_1} G(gx_0, gx_0, gx_1). \end{aligned}$$

Therefore $G(gx_n, gx_n, gx_m) \to 0$ as $n, m \to \infty$, hence, (gx_n) is a G - Cauchy sequence. Since $g(X)$ is G - complete, there exists a point q in $g(X)$ such that $gx_n \to q$ as $n \to \infty$.

Consequently, we can find a point $p \in X$ such that $gp = q$. We prove that $fp = gp$.

By (4.1) we have successively

$$\begin{gathered} F(G(fx_{n-1}, fx_{n-1}, fp), G(gx_{n-1}, gx_{n-1}, gp), G(gx_{n-1}, gx_{n-1}, fx_{n-1}), \\ G(gp, gp, fp), G(gx_{n-1}, gx_{n-1}, fp), G(gx_{n-1}, gx_{n-1}, fx_{n-1})) \leq 0, \end{gathered}$$

$$\begin{gathered} F(G(gx_n, gx_n, fp), G(gx_{n-1}, gx_{n-1}, gp), G(gx_{n-1}, gx_{n-1}, gx_n), \\ G(gp, gp, fp), G(gx_{n-1}, gx_{n-1}, fp), G(gx_{n-1}, gx_{n-1}, gx_n)) \leq 0. \end{gathered}$$

Letting n tend to infinity, we obtain

$$F(G(gp, gp, fp), 0, 0, G(gp, gp, fp), G(gp, gp, fp), 0) \leq 0.$$

By (F_2) it follows that $G(gp, gp, fp) = 0$ which implies $gp = fp$. Hence $w = gp = fp$ is a point of coincidence of f and g. By Lemma 4.1, w is the unique point of coincidence of f and g. Moreover, if f and g are weakly compatible, by Lemma 2.8, w is the unique common fixed point of f and g. □

Corollary 4.3. *Let (X, G) be a G - metric space and $f, g : X \to X$ two functions satisfying the inequality*

$$\begin{gathered} G(fx, fx, fz) \leq a \max\{G(gx, gx, fz), G(gz, gz, fz)\}+ \\ +k \max\{G(gx, gx, gz), 2G(gx, gx, fx) + 2G(gz, gz, fz)\} \end{gathered} \tag{4.2}$$

for all $x, z \in X$, where $a, k \geq 0$ and $0 \leq 2a + 2k < 1$. If $f(X) \subset g(X)$ and $g(X)$ is a G - complete metric subspace of (X, G), then f and g have an unique point of coincidence. Moreover, if f and g are weakly compatible, then f and g have an unique common fixed point.

Proof. The proof follows from Theorem 4.2 and Example 3.2. □

Remark 4.4. *If in (2.1) $x = y$, then by (G_5) and Lemma 2.6 we obtain*

$$\begin{aligned} G(fx, fx, fz) &\leq k \max\{G(gx, gx, gz), G(fx, fx, gz)\} \\ &\leq k \max\{G(gx, gx, gz), G(gx, fx, fx) + G(fx, fx, gz)\} \\ &\leq k \max\{G(gx, gx, gz), G(gx, gx, fx) + G(gz, gz, fx)\} \\ &\leq a \max\{G(gz, gz, fz), G(gx, gx, fz)\} + \\ &\quad +k \max\{G(gx, gx, gz), G(gx, gx, fx) + G(gz, gz, fx)\}. \end{aligned}$$

Then by Corollary 4.3 we obtain Theorem 2.9 for $a = 0$ *and* $k \in \left[0, \frac{1}{2}\right)$.

Corollary 4.5. *Let* (X, G) *be a* G *- metric space and* $f, g : X \to X$ *two functions satisfying the inequality*

(4.3)
$$G(fx, fx, fz) \leq a_1 G(gx, gx, fz) + (a_2 + a_3 + a_5) G(gx, gx, fx) + \\ + a_4 G(gz, gz, fz) + a_6 G(gx, gx, fz) + a_7 G(gz, gz, fx),$$

for all $x, z \in X$*, where* $a_1, ..., a_7 \geq 0$ *and* $0 \leq a_1 + a_2 + a_3 + a_4 + a_5 + 2a_6 + a_7 < 1$.

If $f(X) \subset g(X)$ *and* $g(X)$ *is a* G *- complete metric subspace of* X*, then* f *and* g *have an unique point of coincidence in* X.

Moreover, if f *and* g *are weakly compatible, then* f *and* g *have an unique common fixed point.*

Proof. The proof follows from Theorem 4.2 and Example 3.3. □

Remark 4.6. *If in (2.2)* $x = y$*, then we obtain (4.3) and by Corollary 4.5 we obtain Theorem 2.10.*

Corollary 4.7. *Let* (X, G) *be a* G *- metric space and* $f, g : X \to X$ *two functions satisfying the following inequality*

$$G(fx, fx, fz) \leq aG(gx, gx, fz) + \\ + k \max\{G(gx, gx, fx) + G(gx, gx, fz) + G(gz, gz, fx), \\ 4G(gx, gx, fx) + 2G(gz, gz, fz), \\ 2G(gx, gx, fx) + 2G(gz, gz, fx) + 2G(gx, gx, fz)\}$$

for all $x, z \in X$*, where* $a, k \geq 0$ *and* $0 \leq a + 6k < 1$.

If $f(X) \subset g(X)$ *and* $g(X)$ *is a* G *- complete metric subspace of* X*, then* f *and* g *have an unique point of coincidence in* X.

Moreover, if f *and* g *are weakly compatible, then* f *and* g *have an unique common fixed point.*

Proof. The proof follows from Theorem 4.2 and Example 3.4. □

Remark 4.8. *By (2.3) with* $x = y$ *and Lemma 2.6 we obtain*

$$G(fx, fx, fz) \leq k \max\{G(gx, gx, fx) + G(gx, gx, fz) + G(gz, gz, fx), \\ 2G(gx, fx, fx) + G(gz, fz, fz), \\ G(gx, fx, fx) + G(gz, fx, fx) + G(gx, fz, fz)\} \\ \leq aG(gx, gx, gz) + \\ + k \max\{G(gx, gx, fx) + G(gx, gx, fz) + G(gz, gz, fx), \\ 4G(gx, gx, fx) + 2G(gz, gz, fz), \\ 2G(gx, gx, fx) + 2G(gz, gz, fx) + 2G(gx, gx, fz)\},$$

for all $x, z \in X$, where $a, k \geq 0$ and $0 \leq a + 6k < 1$.

Then by Corollary 4.7, for $a = 0$ and $k \in \left[0, \dfrac{1}{6}\right)$ we obtain Theorem 2.11.

Remark 4.9. *By Theorem 4.2 and Examples 3.5 - 3.13 we obtain new results.*

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