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ON SASAKIAN MANIFOLDS SATISFYING CURVATURE RESTRICTIONS WITH RESPECT TO A QUARTER SYMMETRIC METRIC CONNECTION

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Abstract. The object of the present paper is to study a quarter symmetric connection $\overline{\nabla}$ in a Sasakian manifold admitting some curvature restrictions as $R^{\overline{\nabla}} \cdot \omega^{\overline{\nabla}} = 0$ and $\omega^{\overline{\nabla}} \cdot S^{\overline{\nabla}} = 0$, where ω is a generalized quasi-conformal curvature tensor.

1. Introduction

In 2016, Baishya and Roy Chowdhury [1] introduced and studied the notion of generalized quasi-conformal curvature tensor in the context of $N(k, \mu)$- manifold. The components of a *generalized quasi-conformal curvature tensor* ω in a Riemannian manifold (M^n, g), with $n > 1$, are given by

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$$
\begin{aligned}
\omega(X,Y)Z &= \frac{n-2}{n}[(1+(n-1)a-b)-\{1+(n-1)(a+b)\}c]C(X,Y)Z \\
&\quad +[1-b+(n-1)a]E(X,Y)Z+(n-1)(b-a)P(X,Y)Z \\
&\quad +\frac{n-2}{n}(c-1)\{1+2n(a+b)\}L(X,Y)Z \qquad (1.1)
\end{aligned}
$$

for all X, Y and $Z \in \chi(M)$, where a, b, c are real constants. Here $\chi(M)$ denotes the set of all vector fields of the manifold M. The *curvature tensor* ω has an important, unifying, role because it reduces to the Riemann curvature tensor R if the scalar triple (a, b, c) $\equiv (0,0,0)$, to the conformal curvature tensor C ([5]) if $(a,b,c) \equiv \left(-\frac{1}{n-2}, -\frac{1}{n-2}, 1\right)$, to the conharmonic curvature tensor L ([9]) if (a,b,c) $\equiv (-\frac{1}{n-2}, -\frac{1}{n-2}, 0)$, to the concircular curvature tensor E ([3], p. 84) if (a, b, c) $\equiv (0,0,1)$, to the projective curvature tensor P([3], p. 84) if $(a,b,c) \equiv (-\frac{1}{n-1}, 0, 0)$ and to the m-projective curvature tensor H ([14]) if (a, b, c) $\equiv (-\frac{1}{2n-2}, -\frac{1}{2n-2}, 0)$.

The equation (1.1) can also be written as

$$
\begin{aligned}
\omega(X,Y)Z &= R(X,Y)Z + a\left[S(Y,Z)X - S(X,Z)Y\right] \\
&\quad + b\left[g(Y,Z)QX - g(X,Z)QY\right] \\
&\quad - \frac{cr}{n}\left(\frac{1}{n-1}+a+b\right)\left[g(Y,Z)X - g(X,Z)Y\right], \qquad (1.2)
\end{aligned}
$$

where Q is the Ricci operator and r is the scalar curvature.

The notion of weakly symmetric Riemannian manifold was introduced by Tamássy and Binh ([22]). Afterwards, it became a focus of growing interest for many geometers. For details, we refer to ([4], [21], [10], [11], [6], [16], [15], [20]) and the references therein.

Generalizing the notion of semi-symmetric connection, in 1975, Golab defined and studied the notion of quarter-symmetric connection ([8]). Thereafter, the notion of quarter symmetric connection has been studied by many geometers as Rastogi ([17], [18]), Mishra et al. ([12]), Yano et al. ([24]) and others. A linear connection $\overline{\nabla}$ on an n-dimensional Riemannian manifold (M,g) is called a quarter-symmetric connection ([8]) if the torsion tensor T of the connection $\overline{\nabla}$, defined by

$$
T(X,Y) = \overline{\nabla}_X Y - \overline{\nabla}_Y X - [X,Y], \qquad (1.3)
$$

satisfies

$$T(X,Y) = \eta(Y)\phi X - \eta(X)\phi Y, \tag{1.4}$$

where η is a 1-form and ϕ is a $(1,1)$ tensor field.

In particular, if $\phi X = X$, then the quarter-symmetric connection reduces to the semi-symmetric connection ([7]). Thus the notion of quarter-symmetric connection generalizes that of semi-symmetric connection. Furthermore, if a quarter-symmetric connection $\overline{\nabla}$ admits the condition

$$(\overline{\nabla}_X g)(Y,Z) = 0 \tag{1.5}$$

then $\overline{\nabla}$ is said to be a quarter-symmetric metric connection, otherwise it is said to be a quarter-symmetric non-metric connection([13]).

The present paper is structured as follows. After introduction, a brief account of Sasakian manifolds is given in section 2. In section 3, we have studied a Sasakian manifold admitting the curvature restriction $R^{\overline{\nabla}}(\xi,X)\cdot\omega^{\overline{\nabla}} = 0$, where $R^{\overline{\nabla}}(\xi,X)$ acts on $\omega^{\overline{\nabla}}$ as a derivation of tensor algebra with respect to a quarter symmetric metric connection $\overline{\nabla}$ and tabled the nature of the Ricci tensor for different curvature restrictions. Finally, in section 4 we have studied a Sasakian manifold satisfying $\omega^{\overline{\nabla}}(\xi,X)\cdot S^{\overline{\nabla}} = 0$ with respect to the quarter symmetric metric connection $\overline{\nabla}$.

2. Sasakian manifolds

Let M be an n-dimensional almost contact metric manifold equipped with an almost contact metric structure $(\phi,\ \xi,\ \eta,\ g)$ consisting of a $(1,1)$ tensor field ϕ, a vector field ξ, a 1-form η and a Riemannian metric g. Then

$$\phi^2 X = -X + \eta(X)\xi, \eta(\xi) = 1, \eta(\phi X) = 0,\ \phi\xi = 0, \tag{2.1}$$

$$g(\phi X, \phi Y) = g(X,Y) - \eta(X)\eta(Y), \tag{2.2}$$

$$g(X,\phi Y) = -g(\phi X, Y), \eta(X) = g(X,\xi), \text{for all } X, Y \in TM. \tag{2.3}$$

An almost contact metric manifold M is said to be

(a) a contact metric manifold if

$$g(X,\phi Y) = d\eta(X,Y), \text{for all } X, Y \in TM; \tag{2.4}$$

(b) a K-contact manifold if the vector field ξ is Killing, or equivalently

$$\nabla_X \xi = -\phi X, \tag{2.5}$$

where ∇ is the Levi-Civita Riemannian connection and

(c) a Sasakian manifold if

$$(\nabla_X\phi)Y = g(X,Y)\xi - \eta(Y)X, \text{for all } X,\ Y \in TM. \tag{2.6}$$

A K-contact manifold is a contact metric manifold, while the converse is true if the Lie derivative of ϕ in the characteristic direction ξ vanishes identically. A Sasakian manifold is always a K-contact manifold. A 3-dimensional K-contact manifold is a Sasakian manifold.It is well known that a contact metric manifold is Sasakian if and only if

$$R(X,Y)\xi = \eta(Y)X - \eta(X)Y, \text{for all } X,Y \in TM. \tag{2.7}$$

In a Sasakian manifold equipped with the structure $(\phi,\ \xi,\ \eta,\ g)$, the following relations also hold ([2], [23], [19]):

$$(\nabla_X\eta)Y = g(X,\phi Y), \tag{2.8}$$

$$R(\xi,X)Y = g(X,Y)\xi - \eta(Y)X, \tag{2.9}$$

$$S(X,\xi) = (n-1)\eta(X), \tag{2.10}$$

$$R(X,\xi)Y = \eta(Y)X - g(X,Y)\xi, \tag{2.11}$$

for all $X,Y,Z \in TM$, where S is the Ricci tensor.

The relation between the quarter-symmetric metric connection $\overline{\nabla}$ and the Levi-Civita connection ∇ of (M^n,g) has been obtained by K. Yano and T. Imai [24] and is given by

$$\overline{\nabla}_X Y = \nabla_X Y - \eta(X)\phi Y. \tag{2.12}$$

The relation between the curvature tensors $R^{\overline{\nabla}}$of M with respect to the quarter-symmetric metric connection $\overline{\nabla}$ and R of M with respect to the the Riemannian connection ∇, is given by

$$\begin{aligned} R^{\overline{\nabla}}(X,Y)Z &= R(X,Y)Z - 2g(X,\phi Y)\phi Z + \eta(X)g(Y,Z)\xi \\ &\quad -\eta(Y)g(X,Z)\xi + \eta(Z)\{\eta(Y)X - \eta(X)Y\}. \end{aligned} \tag{2.13}$$

which yields

$$S^{\overline{\nabla}}(Y,Z) = S(Y,Z) - g(Y,Z) + n\eta(Y)\eta(Z), \tag{2.14}$$

$$r^{\overline{\nabla}} = r, \tag{2.15}$$

$$S^{\overline{\nabla}}(X,\xi) = 2(n-1)\eta(X). \tag{2.16}$$

where $S^{\overline{\nabla}}$ and S are the Ricci tensors of the connections $\overline{\nabla}$ and ∇, respectively.

The generalized quasi-conformal curvature tensor $\omega^{\overline{\nabla}}$of M with respect to the quarter-symmetric metric connection $\overline{\nabla}$ is given by

$$\begin{aligned} \omega^{\overline{\nabla}}(Y,Z)U &= R^{\overline{\nabla}}(X,Y)Z + a\left[S^{\overline{\nabla}}(Z,U)Y - S^{\overline{\nabla}}(Y,U)Z\right] \\ &\quad + b\left[g(Z,U)Q^{\overline{\nabla}}Y - g(Y,U)Q^{\overline{\nabla}}Z\right] \\ &\quad - \frac{cr^{\overline{\nabla}}}{n}\left(\frac{1}{n-1} + a + b\right)[g(Z,U)Y - g(Y,U)Z]. \end{aligned} \tag{2.17}$$

3. Sasakian manifolds satisfying $R^{\overline{\nabla}} \cdot \omega^{\overline{\nabla}} = 0$

In this section we consider a Sasakian manifold satisfying the following condition

$$R^{\overline{\nabla}}(\xi,X)\cdot\omega^{\overline{\nabla}}(Y,Z)U = 0, \tag{3.1}$$

which can also be written as

$$\begin{aligned} R^{\overline{\nabla}}(\xi,X)\omega^{\overline{\nabla}}(Y,Z)U &= \omega^{\overline{\nabla}}\left(R^{\overline{\nabla}}(\xi,X)Y,Z\right)U + \omega^{\overline{\nabla}}\left(Y,R^{\overline{\nabla}}(\xi,X)Z\right)U \\ &\quad + \omega^{\overline{\nabla}}(Y,Z)R^{\overline{\nabla}}(\xi,X)U. \end{aligned} \tag{3.2}$$

Now, making use of (2.13) we get the following

$$\begin{aligned} &R^{\overline{\nabla}}(\xi,X)\omega^{\overline{\nabla}}(Y,Z)U \\ &= 2\omega^{\overline{\nabla}}(Y,Z,U,X)\xi - 2\omega^{\overline{\nabla}}(Y,Z,U,\xi)X, \end{aligned} \tag{3.3}$$

$$\begin{aligned} &\omega^{\overline{\nabla}}\left(R^{\overline{\nabla}}(\xi,X)Y,Z\right)U \\ &= \omega^{\overline{\nabla}}(2g(X,Y)\xi - 2\eta(Y)X,Z)U \\ &= 2g(X,Y)\omega^{\overline{\nabla}}(\xi,Z)U - 2\eta(Y)\omega^{\overline{\nabla}}(X,Z)U, \end{aligned} \tag{3.4}$$

$$\begin{aligned} &\omega^{\overline{\nabla}}\left(Y,R^{\overline{\nabla}}(\xi,X)Z\right)U \\ &= \omega^{\overline{\nabla}}(Y,2g(X,Z)\xi - 2\eta(Z)X)U \\ &= 2g(X,Z)\omega^{\overline{\nabla}}(Y,\xi)U - 2\eta(Z)\omega^{\overline{\nabla}}(Y,X)U, \end{aligned} \tag{3.5}$$

$$\begin{aligned} &\omega^{\overline{\nabla}}(Y,Z)R^{\overline{\nabla}}(\xi,X)U \\ &= 2\omega^{\overline{\nabla}}(Y,Z)(g(X,U)\xi - 2\eta(U)X) \\ &= 2g(X,U)\omega^{\overline{\nabla}}(Y,Z)\xi - 2\eta(U)\omega^{\overline{\nabla}}(Y,Z)X. \end{aligned} \tag{3.6}$$

Taking account of (3.3), (3.4), (3.5), (3.6) the relation (3.2) becomes

$$\begin{aligned} 0 &= \omega^{\overline{\nabla}}(Y,Z,U,X)\,\xi-\omega^{\overline{\nabla}}(Y,Z,U,\xi)\,X \\ &\quad -g(X,Y)\,\omega^{\overline{\nabla}}(\xi,Z)\,U+\eta(Y)\,\omega^{\overline{\nabla}}(X,Z)\,U \\ &\quad -g(X,Z)\,\omega^{\overline{\nabla}}(Y,\xi)\,U+\eta(Z)\,\omega^{\overline{\nabla}}(Y,X)\,U \\ &\quad -g(X,U)\,\omega^{\overline{\nabla}}g(Y,Z)\,\xi+\eta(U)\,\omega^{\overline{\nabla}}(Y,Z)\,X, \end{aligned} \tag{3.7}$$

which yields after contraction

$$\begin{aligned} &\sum_{i=1}^{n}\omega^{\overline{\nabla}}(e_i,Z,U,e_i) \\ &= n\omega^{\overline{\nabla}}(\xi,Z,U,\xi)+\sum_{i=1}^{n}g(e_i,Z)\,\omega^{\overline{\nabla}}(e_i,\xi,U,\xi) \\ &\quad -\eta(Z)\sum_{i=1}^{n}\omega^{\overline{\nabla}}(e_i,e_i,U,\xi)+\sum_{i=1}^{n}g(e_i,U)\,\omega^{\overline{\nabla}}(e_i,Z,\xi,\xi) \\ &\quad -\eta(U)\sum_{i=1}^{n}\omega^{\overline{\nabla}}(e_i,Z,e_i,\xi). \end{aligned} \tag{3.8}$$

In view of (2.1), (2.2), (2.3), (2.7), (2.9), (2.10) and (2.17) we obtain the following relations

$$\begin{aligned} &\sum_{i=1}^{n}\omega^{\overline{\nabla}}(e_i,Z,U,e_i) \\ &= [1+a(n-1)-b]\,S(Z,U) \\ &\quad +(-1+br+nb)\,g(Z,U)+[n+na(n-1)-nb]\,\eta(Z)\,\eta(U) \\ &\quad -(n-1)\left[a+b+\frac{cr}{n}\left(\frac{1}{n-1}+a+b\right)\right]g(Z,U), \end{aligned} \tag{3.9}$$

$$\begin{aligned} &n\omega^{\overline{\nabla}}(\xi,Z,U,\xi) \\ &= anS(Z,U)+\left[2n+bn(n-1)+n^2b\right]g(Z,U) \\ &\quad -\left[2n+an(n-1)+bn(n-1)+n^2b\right]\eta(Z)\,\eta(U) \\ &\quad -n\left[a+b+\frac{cr}{n}\left(\frac{1}{n-1}+a+b\right)\right][g(Z,U)-\eta(Z)\,\eta(U)], \end{aligned} \tag{3.10}$$

$$\begin{aligned}
&\sum_{i=1}^{n} g\left(e_i, Z\right) \omega^{\overline{\nabla}}\left(e_i, \xi, U, \xi\right) \\
= \ & [2+a(n-1)+b(n-1)+nb]\,\eta(Z)\,\eta(U) \\
&-[2+b(n-1)+nb]\,g(Z,U)-aS(Z,U) \\
&-\left[a+b \ + \ \frac{cr}{n}\left(\frac{1}{n-1}+a+b\right)\right]\left[\eta(Z)\,\eta(U)-g(Z,U)\right],
\end{aligned} \tag{3.11}$$

$$\eta(Z)\sum_{i=1}^{n} \omega^{\overline{\nabla}}\left(e_i, e_i, U, \xi\right)=0, \tag{3.12}$$

$$\sum_{i=1}^{n} g\left(e_i, U\right) \omega^{\overline{\nabla}}\left(e_i, Z, \xi, \xi\right)=0, \tag{3.13}$$

$$\begin{aligned}
&\eta(U)\sum_{i=1}^{n} \omega^{\overline{\nabla}}\left(e_i, Z, e_i, \xi\right) \\
= \ & \left[2-2n+an-a-ar+3bn-b-2bn^2\right]\eta(Z)\,\eta(U) \\
&-(1-n)\left[a+b \ + \ \frac{cr}{n}\left(\frac{1}{n-1}+a+b\right)\right]\eta(Z)\,\eta(U).
\end{aligned} \tag{3.14}$$

In view of (3.9) , (3.10) ,(3.11) ,(3.12) ,(3.13) ,(3.14) we get from (3.8)

$$\begin{aligned}
S(Z,U) \ = \ & \left[\frac{\{2n-1+b(1-r-4n+2n^2)\}}{(1-b)}\right] g(Z,U) \\
&+\left[\frac{a}{(1-b)}\{r-2n(n-1)\}-n\right]\eta(Z)\,\eta(U)
\end{aligned} \tag{3.15}$$

This leads to the following results:

Theorem 3.1. *An n-dimensional Sasakian manifold admitting a quarter symmetric metric connection $\overline{\nabla}$ satisfying the curvature restriction $R^{\overline{\nabla}}(\xi, X)\cdot\omega^{\overline{\nabla}}=0$ is an η- Einstein manifold.*

Theorem 3.2. *For a Sasakian manifold satisfying the following curvature conditions the corresponding expressions for the Ricci tensor are as follows:*

Curvature condition	Expression for Ricci Tensor
$R^{\overline{\nabla}}(\xi, X) \cdot R^{\overline{\nabla}} = 0$	$S = (2n-1) - n\eta \otimes \eta$
$R^{\overline{\nabla}}(\xi, X) \cdot C^{\overline{\nabla}} = 0$	$S = \frac{1-n+r}{n-1} g + \left(n - \frac{r}{n-1}\right) \eta \otimes \eta$
$R^{\overline{\nabla}}(\xi, X) \cdot L^{\overline{\nabla}} = 0$	$S = \frac{1-n+r}{n-1} g + \left(n - \frac{r}{n-1}\right) \eta \otimes \eta$
$R^{\overline{\nabla}}(\xi, X) \cdot E^{\overline{\nabla}} = 0$	$S = (2n-1) g - n\eta \otimes \eta$
$R^{\overline{\nabla}}(\xi, X) \cdot P^{\overline{\nabla}} = 0$	$S = (2n-1) g + \left(n - \frac{r}{n-1}\right) \eta \otimes \eta$
$R^{\overline{\nabla}}(\xi, X) \cdot H^{\overline{\nabla}} = 0$	$S = \left(\frac{2n^2+r}{2n-1} - 1\right) g - \left(\frac{r+n}{2n-1}\right) \eta \otimes \eta$

4. Sasakian manifolds with $\omega^{\overline{\nabla}}(\xi, X) \cdot S^{\overline{\nabla}} = 0$.

Let $M^n(\phi, \xi, \eta, g)(n > 1)$, be a Sasakian manifold admitting a quarter symmetric metric connection $\overline{\nabla}$ and satisfying the condition

$$\omega^{\overline{\nabla}}(\xi, X) \cdot S^{\overline{\nabla}}(Y, Z) = 0, \tag{4.1}$$

which can be written as

$$\begin{aligned} &\omega^{\overline{\nabla}}(\xi, X) S^{\overline{\nabla}}(Y, Z) \\ = \;& S^{\overline{\nabla}}\left(\omega^{\overline{\nabla}}(\xi, X) Y, Z\right) + S^{\overline{\nabla}}\left(Y, \omega^{\overline{\nabla}}(\xi, X) Z\right). \end{aligned} \tag{4.2}$$

Putting $Z = \xi$ in (4.2) and then using (2.16), we get

$$2(n-1)\left(\omega^{\overline{\nabla}}(\xi, X, Y, \xi)\right) + S^{\overline{\nabla}}\left(Y, \omega^{\overline{\nabla}}(\xi, X)\xi\right) = 0. \tag{4.3}$$

In view of (2.1), (2.2), (2.3), (2.7), (2.9), (2.10) and (2.17) we obtain

$$\begin{aligned} &S^{\overline{\nabla}}\left(Y, \omega^{\overline{\nabla}}(\xi, X)\xi\right) \\ = \;& (2n-4)\eta(X)\eta(Y) - 2S(Y, X) + 2g(Y, X) \\ &+2a(n-1)(n-2)\eta(X)\eta(Y) \\ &-2a(n-1)S(Y, X) + 2a(n-1)g(Y, X) \\ &+b\left[(n-2)^2\eta(X)\eta(Y) - S^2(Y, X) + 2S(Y, X) - g(Y, X)\right] \\ &-\frac{cr}{n}\left(\frac{1}{n-1} + a + b\right)(n-2)\eta(X)\eta(Y) \\ &+\frac{cr}{n}\left(\frac{1}{n-1} + a + b\right)\left[S(Y, X) - g(Y, X)\right], \end{aligned} \tag{4.4}$$

$$\begin{aligned}
& 2(n-1)\left(\omega^{\overline{\nabla}}(\xi, X, Y, \xi)\right) \\
= \; & 2a(n-1)aS(Z,U) + 2(n-1)[2+b(n-1)+nb]\,g(Z,U) \\
& -2(n-1)[2+a(n-1)+b(n-1)+nb]\,\eta(Z)\eta(U) \\
& -2(n-1)\left[a+b \quad + \quad \frac{cr}{n}\left(\frac{1}{n-1}+a+b\right)\right] \\
& [g(Z,U)-\eta(Z)\eta(U)]. \qquad (4.5)
\end{aligned}$$

Making use of (4.4) and (4.5) in (4.3), we have

$$\begin{aligned}
& 2a(n-1)S(X,Y) - 2S(Y,X) \\
& +[4(n-1)+2b(n-1)(n-1)+2nb(n-1)]\,g(X,Y) \\
& -bS^2(Y,X) + (2n-4)\eta(X)\eta(Y) + 2g(Y,X) \\
& +2a(n-1)(n-2)\eta(X)\eta(Y) - 2a(n-1)S(Y,X) \\
& +2a(n-1)g(Y,X) + b\left[(n-2)^2\eta(X)\eta(Y) + 2S(Y,X) - g(Y,X)\right] \\
& -2(n-1)[2+a(n-1)+b(n-1)+nb]\,\eta(X)\eta(Y) \\
& -2(n-1)\left[a+b \quad + \quad \frac{cr}{n}\left(\frac{1}{n-1}+a+b\right)\right][g(X,Y)-\eta(X)\eta(Y)] \\
& -\frac{cr}{n}\left(\frac{1}{n-1}+a+b\right)(n-2)\eta(X)\eta(Y) \\
& +\frac{cr}{n}\left(\frac{1}{n-1}+a+b\right)[S(Y,X)-g(Y,X)] \\
= \; & 0.
\end{aligned}$$

Simplifying the foregoing equation, we get

$$\begin{aligned}
bS^2(X,Y) \; = \; & 2(b-1)S(X,Y) \\
& +\left(4bn^2-8bn+3b+4n-2\right)g(X,Y) \\
& +\left(4bn-3bn^2-2n\right)\eta(X)\eta(Y) \\
& -\frac{cr}{n}\left(\frac{1}{n-1}+a+b\right)(n-2)\eta(X)\eta(Y) \\
& +\frac{cr}{n}\left(\frac{1}{n-1}+a+b\right)[S(X,Y)-g(X,Y)] \\
& -\frac{cr}{n}\left(\frac{1}{n-1}+a+b\right)2(n-1)[g(X,Y)-\eta(X)\eta(Y)]. \qquad (4.6)
\end{aligned}$$

for all X, Y, provided that $b \neq 0$, where $S^2(X,Y) = S(QX,Y)$.

Theorem 4.1. *For every n-dimensional Sasakian manifold admitting a quarter symmetric metric connection $\overline{\nabla}$ satisfying $\omega^{\overline{\nabla}}(\xi, X)\cdot S^{\overline{\nabla}} = 0$,the Ricci tensor S satisfies the relation (4.6) provided that $b \neq 0$.*

But for $b = 0$,(4.6) reduces to

$$S(X,Y) = (2n-1)\, g(X,Y) - n\eta(X)\,\eta(Y).$$

This leads to the following:

Theorem 4.2. *Every Sasakian manifold admitting a quarter symmetric metric connection and satisfying at least one of the conditions $R^{\overline{\nabla}} \cdot S^{\overline{\nabla}} = 0$, $R^{\overline{\nabla}} \cdot E^{\overline{\nabla}} = 0$, $R^{\overline{\nabla}} \cdot P^{\overline{\nabla}} = 0$, is an η-Einstein manifold.*

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