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RICCI SOLITONS IN A HYPER GENERALIZED PSEUDO SYMMETRIC D-HOMOTHETICALLY DEFORMED KENMOTSU MANIFOLD

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Abstract. In this paper we study the nature of Ricci solitons in a hyper generalized pseudosymmetric D-homothetically deformed Kenmotsu manifold.

1. Introduction

Let the symbols ∇ and ∇^d stand for the Riemann connection and the D-homothetically deformed connection respectively. Also, let R, S, Q, r and R^d, S^d, Q^d, r^d respectively stands for curvature tensor, Ricci tensor, Ricci operator, scalar curvature with respect to ∇ and ∇^d respectively. In this study, we consider an almost contact metric manifold (M^{2n+1}, ϕ, ξ, η, g) that consists of a $(1,1)$-tensor field ϕ, a vector field ξ and a 1-form η called respectively the structure endomorphism, the characteristic vector field and the contact form.

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In a recent paper [2], K K Baishya, F. Ozen Zengin and J Mike have introduced a new type of space called *hyper generalized pseudo symmetric manifold.* In Section 3 of this paper we extend this concept to a D-homothetically deformed structure of a $(2n+1)$-dimensional Kenmotsu manifold.

A Kenmotsu manifold is said to be hyper generalized pseudo symmetric [2] (which will be abbreviated hereafter as $[H(GPS)_n, \nabla]$) if it admits the equation

$$\begin{aligned} &(\nabla_X \bar{R})(Y,U,V,\ W) \\ = \;& 2A_1(X)\bar{R}(Y,U,V,\ W) + A_1(Y)\bar{R}(X,U,V,\ W) \\ &+A_1(U)\bar{R}(Y,X,V,\ W) + A_1(V)\bar{R}(Y,U,X,\ W) \\ &+A_1(W)\bar{R}(Y,U,V,\ X) + 2A_2(X)(g\wedge S)(Y,U,V,W) \\ &+A_2(Y)(g\wedge S)(X,U,V,W) + A_2(U)\ (g\wedge S)(Y,X,V,W) \\ &+A_2(V)\ (g\wedge S)(Y,U,X,W) + A_2(W)\ (g\wedge S)(Y,U,V,X) \end{aligned} \tag{1.1}$$

where

$$\begin{aligned} (g\wedge S)(Y,U,V,W) &= g(Y,W)S(U,V) + g(U,V)S(Y,W) \\ &- g(Y,V)S(U,W) - g(U,W)S(Y,V), \end{aligned} \tag{1.2}$$

and A_1, A_2 being non-zero 1-forms defined as $A_1(X) = g(X,\theta_1)$ and $A_2(X) = g(X,\theta_2)$.

Ricci solitons were introduced by Hamilton [14]. An important topic in contact metric geometry is the study of Ricci flow and Ricci solitons. A Riemannian manifold admits a Ricci soliton [15] if there exists a smooth vector field V (called the potential vector field) such that

$$\mathcal{L}_V g + 2S + 2\lambda g = 0,$$

where $\mathcal{L}_V$ denotes the Lie derivative along V and λ is a real number. A Ricci soliton is said to be expanding, steady or shrinking according to λ is positive, zero and negative, respectively. Ricci solitons has now become a popular topic for many mathematicians, for details we refer ([13], [18], [3]). An η-Ricci soliton $(V,\ \lambda,\ \mu)$ is a generalization of a Ricci soliton defined as ([7], [10], [4])

$$\mathcal{L}_V g + 2S + 2\lambda g + 2\mu\eta\otimes\eta = 0,$$

where $\mathcal{L}_V$ denotes the Lie derivative along V and λ and μ are real numbers. An η-Ricci soliton is said to be expanding, steady or shrinking according as λ is positive, zero and negative respectively.

The paper is organized as follows: After Introduction, in Section 2, we briefly recall some known results for Kenmotsu manifolds and D-homothetic deformations on a Kenmotsu manifold and we establish some properties of the deformed Kenmotsu manifold. In Section 3, we discuss the properties of a D-homothetically deformed Kenmotsu manifold under hyper generalized pseudo symmetric curvature condition equipped with Ricci solitons and η^d-Ricci solitons. We determine a necessary condition for which the solitons of these types in such a manifold are shrinking, steady and expanding.

2. Preliminaries

According to the definition of Blair [11], an *almost contact structure* $(\phi,\ \xi,\ \eta)$ on a $(2n+1)$-dimensional Riemannian manifold satisfies the following conditions:

$$\phi^2 = -I + \eta \otimes \xi, \tag{2.1}$$

$$\eta(\xi) = 1, \tag{2.2}$$

$$\phi\xi = 0,\ \eta \circ \phi = 0,\ \text{rank } \phi = n-1. \tag{2.3}$$

Moreover, if g is a Riemannian metric on M^{2n+1} satisfying

$$g(\phi X, \phi Y) = g(X,Y) - \eta(X)\eta(Y), \tag{2.4}$$

$$g(X,\xi) = \eta(X), \tag{2.5}$$

$$g(\phi X, Y) = -g(X, \phi Y), \tag{2.6}$$

for any vector fields X, Y on M^{2n+1}, then the manifold M^{2n+1} [11] is said to admit an *almost contact metric structure* $(\phi,\ \xi,\ \eta,\ g)$.

Definition 2.1. [16] If in an almost contact metric structure $(\phi,\ \xi,\ \eta,\ g)$ on M^{2n+1}, the Riemann connection ∇ of g satisfies $(\nabla_X \phi)Y = g(\phi X, Y)\xi - \eta(Y)\phi X$, for any vector fields X, Y on M^{2n+1}, then the structure is called Kenmotsu.

Proposition 2.2. [16] *If $(M^{2n+1},\ \phi,\ \xi,\ \eta,\ g)$ is a Kenmotsu manifold, then for any vector fields X, Y, Z on M^{2n+1}, the following relations hold:*

$$\nabla_X \xi = X - \eta(X)\xi, \tag{2.7}$$

$$(\nabla_X \eta)Y = g(X,Y) - \eta(X)\eta(Y), \tag{2.8}$$

$$S(X,\xi) = -2n\eta(X), \tag{2.9}$$

$$\eta(R(X,Y)Z) = g(X,Z)\eta(Y) - g(Y,Z)\eta(X), \tag{2.10}$$

$$R(\xi,X)Y = \eta(Y)X - g(X,Y)\xi, \tag{2.11}$$

$$R(X,Y)\xi = \eta(X)Y - \eta(Y)X. \tag{2.12}$$

Definition 2.3. [1] If a contact metric manifold M^{2n+1} with the almost contact metric structure (ϕ, ξ, η, g) is transformed into (ϕ^d, ξ^d, η^d, g^d), where

$$\phi^d = \phi, \ \ \xi^d = \frac{1}{p}\xi, \ \ \eta^d = p\eta, \ \ g^d = pg + p(p-1)\eta\otimes\eta \tag{2.13}$$

and p is a positive constant, then the transformation is called a D-homothetic deformation.

The relation between the Levi-Civita connections ∇ of g and ∇^d of g^d is given by [1]:

$$\nabla^d_X Y = \nabla_X Y + \frac{p-1}{p} g(\phi X, \phi Y)\xi, \tag{2.14}$$

for any vector fields X, Y on M^{2n+1}.

In view of (2.13), (2.14) and definition of Riemannian curvature tensor, Ricci tensor, scalar curvature, we get the following:

Proposition 2.4. [9] *If a Kenmotsu structure (ϕ, ξ, η, g) on M^{2n+1} is transformed into (ϕ^d, ξ^d, η^d, g^d) under a D-homothetic deformation, then R, R^d, S, S^d, r and r^d are related by*

$$R^d(X,Y)Z = R(X,Y)Z + \frac{p-1}{p}[g(\phi Y,\phi Z)X - g(\phi X,\phi Z)Y], \tag{2.15}$$

$$S^d(X,Y) = S(X,Y) + 2n\frac{p-1}{p} g(\phi X,\phi Y), \tag{2.16}$$

$$r^d = \frac{1}{p} r + 2n(2n+1)\frac{p-1}{p^2}, \tag{2.17}$$

for any vector fields X, Y, Z on M^{2n+1}.

Now we shall bring out some properties of a D-homothetically deformed structure (ϕ^d, ξ^d, η^d, g^d) of a Kenmotsu manifold M^{2n+1} as follows:

Proposition 2.5. *Under a D-homothetic deformation of a Kenmotsu structure* (ϕ, ξ, η, g) *on* M^{2n+1} *is transformed into* $(\phi^d, \xi^d, \eta^d, g^d)$*, then for any vector fields* X, Y, Z *on* M^{2n+1}*, we have*

$$\phi^d = -I + \eta^d \otimes \xi^d, \tag{2.18}$$

$$\eta^d(\xi^d) = 1, \tag{2.19}$$

$$\phi^d \xi^d = 0, \ \eta^d \circ \phi^d = 0, \tag{2.20}$$

$$g^d(\phi^d X, \phi^d Y) = g^d(X, Y) - \eta^d(X)\eta^d(Y), \tag{2.21}$$

$$g^d(X, \xi^d) = \eta^d(X), \tag{2.22}$$

$$\nabla^d_X \xi^d = \frac{1}{p}[X - \eta^d(X)\xi^d], \tag{2.23}$$

$$(\nabla^d_X \eta^d)Y = \frac{1}{p}[g^d(X, Y) - \eta^d(X)\eta^d(Y)], \tag{2.24}$$

$$S^d(X, \xi^d) = -\frac{2n}{p^2}\eta^d(X), \tag{2.25}$$

$$\eta^d(R^d(X, Y)Z) = \frac{1}{p^2}[g^d(X, Z)\eta^d(Y) - g^d(Y, Z)\eta^d(X)], \tag{2.26}$$

$$R^d(\xi^d, X)Y = \frac{1}{p^2}[\eta^d(Y)X - g^d(X, Y)\xi^d], \tag{2.27}$$

$$R^d(X, Y)\xi^d = \frac{1}{p^2}[\eta^d(X)Y - \eta^d(Y)X]. \tag{2.28}$$

Now using (2.24) and (2.25), we obtain

$$(\nabla^d_X S^d)(Y, \xi^d) = \frac{1}{p}[-\frac{2n}{p^2}g^d(X, Y) - S^d(X, Y)], \tag{2.29}$$

for any vector fields X *and* Y *on* M^{2n+1}.

3. Solitons in a hyper generalized pseudo symmetric D-homothetically deformed Kenmotsu manifold

For the beginning, we shall define a hyper generalized pseudo symmetric space on a D-homothetically deformed structure $(\phi^d, \xi^d, \eta^d, g^d)$ of a Kenmotsu manifold M^{2n+1}.

3.1. Hyper generalized pseudo symmetric deformed Kenmotsu manifold.

Definition 3.1. A D-homothetically deformed structure (ϕ^d, ξ^d, η^d, g^d) of a Kenmotsu manifold M^{2n+1} is said to be hyper generalized pseudo symmetric if it satisfies the condition

$$\begin{aligned}
&(\nabla^d_X\bar{R}^d)(Y,U,V,W)\\
=\;& 2A^d_1(X)\bar{R}^d(Y,U,V,\;W)+A^d_1(Y)\bar{R}^d(X,U,V,\;W)\\
+\;& A^d_1(U)\bar{R}^d(Y,X,V,\;W)+A^d_1(V)\bar{R}^d(Y,U,X,\;W)\\
+\;& A^d_1(W)\bar{R}^d(Y,U,V,\;X)+2A^d_2(X)(g^d\wedge S^d)(Y,U,V,W)\\
+\;& A^d_2(Y)(g^d\wedge S^d)(X,U,V,W)+A^d_2(U)\;(g^d\wedge S^d)(Y,X,V,W)\\
+\;& A^d_2(V)\;(g^d\wedge S^d)(Y,U,X,W)+A^d_2(W)\;(g^d\wedge S^d)(Y,U,V,X),
\end{aligned}\tag{3.1}$$

where

$$\begin{aligned}
&(g^d\wedge S^d)(Y,U,V,W)\\
=\;& g^d(Y,W)S^d(U,V)+g^d(U,V)S^d(Y,W)\\
-\;& g^d(Y,V)S^d(U,W)-g^d(U,W)S^d(Y,V),
\end{aligned}\tag{3.2}$$

and A^d_i are non-zero 1-forms defined by $A^d_i(X)=g^d(X,\sigma_i)$, for $i=1,2$.

In this section, we consider a Kenmotsu manifold (M^{2n+1},g) $n\geq 1$ which is hyper generalized pseudo symmetric. Now, making use of (3.2) in (3.1) we find

$$\begin{aligned}
&(\nabla_X\bar{R}^d)(Y,U,V,\;W)\\
=\;& 2A^d_1(X)\bar{R}^d(Y,U,V,\;W)+A^d_1(Y)\bar{R}^d(X,U,V,\;W)\\
+\;& A^d_1(U)\bar{R}^d(Y,X,V,\;W)+A^d_1(V)\bar{R}^d(Y,U,X,\;W)\\
+\;& A^d_1(W)\bar{R}^d(Y,U,V,\;X)+2A^d_2(X)[g^d(Y,W)S^d(U,V)\\
+\;& g^d(U,V)S^d(Y,W)-g^d(Y,V)S^d(U,W)-g^d(U,W)S^d(Y,V)]\\
+\;& A^d_2(Y)[g^d(X,W)S^d(U,V)+g^d(U,V)S^d(X,W)-g^d(X,V)S^d(U,W)\\
-\;& g^d(U,W)S^d(X,V)]+A^d_2(U)\;[g^d(Y,W)S^d(X,V)+g^d(X,V)S^d(Y,W)\\
-\;& g^d(Y,V)S^d(X,W)-g^d(X,W)S^d(Y,V)]+A^d_2(V)\;[g^d(Y,W)S^d(U,X)\\
+\;& g^d(U,X)S^d(Y,W)-g^d(Y,X)S^d(U,W)-g^d(U,W)S^d(Y,X)]\\
+\;& A^d_2(W)[g^d(Y,X)S^d(U,V)+g^d(U,V)S^d(Y,X)\\
-\;& g^d(Y,V)S^d(U,X)-g^d(U,X)S^d(Y,V)].
\end{aligned}\tag{3.3}$$

Now, contracting Y over W in both sides of (3.1) and using (3.2), we get

$$
\begin{aligned}
& (\nabla^d_X S^d)(U,V) \\
= \;& 2A^d_1(X)S^d(U,V) + A^d_1(U)S^d(X,V) + A^d_1(R^d(X,U)V) \\
+ \;& A^d_1(R^d(X,V)U) + A^d_1(V)S^d(X,U) \\
+ \;& 2A^d_2(X)[(2n-1)S^d(U,V) + r^d g^d(U,V)] \\
+ \;& [A^d_2(X)S^d(U,V) + A^d_2(QX)g^d(U,V) \\
- \;& g^d(X,V)A^d_2(QU) - A^d_2(U)S^d(X,V)] \\
+ \;& A^d_2(U)\ [(2n-1)S^d(X,V) + r^d g^d(X,V)] \\
+ \;& A^d_2(V)\ [(2n-1)S^d(U,X) + r^d g^d(U,X)] \\
+ \;& [A^d_2(X)S^d(U,V) + A^d_2(QX)g^d(U,V) \\
- \;& A^d_2(V)S^d(U,X) - A^d_2(QV)g^d(U,X)].
\end{aligned} \tag{3.4}
$$

Now setting

$$V = \xi^d$$

and using (2.25), (2.27), (2.28) in the foregoing equation, we obtain

$$
\begin{aligned}
& (\nabla^d_X S^d)(U,\xi^d) \\
= \;& -\frac{2n}{p^2}[\{2A^d_1(X) + 2A^d_2(X)\}\eta^d(U) + \{A^d_1(U) - A^d_2(U)\}\eta^d(X)] \\
+ \;& \frac{[\eta^d(X)A^d_1(U) - \eta^d(U)A^d_1(X)]}{p^2} + [(r^d + \frac{2n}{p^2})A^d_2(\xi^d) - \frac{A^d_1(\xi^d)}{p^2}]g^d(X,U) \\
+ \;& [2A^d_2(X)\eta^d(U) + A^d_2(U)\eta^d(X)\ ][r^d - \frac{2n(2n-1)}{p^2}] \\
+ \;& 2A^d_2(QX)\eta^d(U) - \eta^d(X)A^d_2(QU) + \frac{A^d_1(X)\eta^d(U)}{p^2} \\
+ \;& [2(n-1)A^d_2(\xi^d)\ + A^d_1(\xi^d)]S^d(U,X).
\end{aligned} \tag{3.5}
$$

which yields by using (2.29)

$$\begin{aligned}
&\frac{1}{p}\left[-\frac{2n}{p^2}g^d(X,U)-S^d(X,U)\right]\\
= &-\frac{2n}{p^2}[\{2A_1^d(X)+2A_2^d(X)\}\eta^d(U)+\{A_1^d(U)-A_2^d(U)\}\eta^d(X)]\\
+ &\frac{[\eta^d(X)A_1^d(U)-\eta^d(U)A_1^d(X)]}{p^2}\\
+ &[(r^d+\frac{2n}{p^2})A_2^d(\xi^d)-\frac{A_1^d(\xi^d)}{p^2}]g^d(X,U)\\
+ &[2A_2^d(X)\eta^d(U)+A_2^d(U)\eta^d(X)\ ][r^d-\frac{2n(2n-1)}{p^2}]\\
+ &2A_2^d(QX)\eta^d(U)-\eta^d(X)A_2^d(QU)+\frac{A_1^d(X)\eta^d(U)}{p^2}\\
+ &[2(n-1)A_2^d(\xi^d)\ +A_1^d(\xi^d)]S^d(U,X).
\end{aligned} \tag{3.6}$$

We get now putting successively $X=U=\xi^d$, $U=\xi^d$ and $X=\xi^d$ in (3.6), we get respectively that

$$\frac{2n}{p^2}A_1^d\left(\xi^d\right)=\left[r^d-\frac{2n\left(2n-1\right)}{p^2}\right]A_2^d\left(\xi^d\right), \tag{3.7}$$

$$\begin{aligned}
&\frac{8n}{p^2}A_2^d\left(\xi^d\right)\eta^d\left(X\right)\\
= &\frac{4n}{p^2}A_1^d\left(X\right)+\left(\frac{8n^2}{p^2}-2r^d\right)A_2^d\left(X\right)-2A_2^d\left(QX\right),
\end{aligned} \tag{3.8}$$

and

$$\begin{aligned}
&\frac{\left[A_1^d\left(\xi^d\right)\eta^d\left(U\right)+4nA_2^d\left(\xi^d\right)\eta^d\left(U\right)\right]}{p^2}\\
= &\left(r^d-\frac{4n\left(n-1\right)}{p^2}\right)A_2^d\left(U\right)-\frac{2n-1}{p^2}A_1^d\left(U\right)-A_2^d\left(QU\right).
\end{aligned} \tag{3.9}$$

By virtue of (3.7), (3.8) and (3.9), the equation (3.6) yields

$$\begin{aligned}
&\left[2p^2\left(n-1\right)A_2^d\left(\xi^d\right)+p^2A_1^d\left(\xi^d\right)+1\right]S^d\left(X,U\right)\\
= &[A_1^d\left(\xi^d\right)-\left(2n+p^2r^d\right)A_2^d\left(\xi^d\right)-\frac{2n}{p}]g^d(X,U)\\
+ &\left[4nA_2^d\left(\xi^d\right)-A_1^d\left(\xi^d\right)\right]\eta^d(X)\eta^d(U).
\end{aligned} \tag{3.10}$$

Thus we can state the following:

Theorem 3.2. *The Ricci curvature tensor of a hyper generalized pseudo symmetric deformed structure* $(\phi^d,\ \xi^d,\ \eta^d,\ g^d)$ *of a Kenmotsu manifold* M^{2n+1} *satisfies (3.10).*

Remark 3.3. *If* $p=1$*, then the Ricci curvature tensor of a hyper generalized pseudo symmetric Kenmotsu manifold* M^{2n+1} *is of the form*

$$\begin{aligned} S^d(X,U) &= \frac{\left[A_1^d\left(\xi^d\right)-\left(r^d+2n\right)A_2^d\left(\xi^d\right)-2n\right]}{\left[1+A_1^d\left(\xi^d\right)+2\left(n-1\right)A_2^d\left(\xi^d\right)\right]}g(X,U) \\ &+ \frac{\left[4nA_2^d\left(\xi^d\right)-A_1^d\left(\xi^d\right)\right]}{\left[1+A_1^d\left(\xi^d\right)+2\left(n-1\right)A_2^d\left(\xi^d\right)\right]}\eta(X)\eta(U). \end{aligned} \tag{3.11}$$

Corollary 3.4. *If* $A_1^d\left(\xi^d\right)=A_2^d\left(\xi^d\right)=0$ *then the Ricci curvature tensor of a symmetric deformed structure* $(\phi^d,\ \xi^d,\ \eta^d,\ g^d)$ *of a Kenmotsu manifold* M^{2n+1} *is Einstein space.*

Proposition 3.5. *The scalar curvature of a hyper generalized pseudo symmetric deformed structure* $(\phi^d,\ \xi^d,\ \eta^d,\ g^d)$ *of a Kenmotsu manifold* M^{2n+1} *is given by*

$$r^d=\frac{2n}{p^2}\frac{\left[2n+A_1^d\left(\xi^d\right)-A_2^d\left(\xi^d\right)\right]}{A_2^d\left(\xi^d\right)}. \tag{3.12}$$

Proposition 3.6. *The scalar curvature of a hyper generalized pseudo symmetric Kenmotsu manifold* M^{2n+1} *is given by*

$$r=\frac{2n[2n+A_1\left(\xi\right)-A_2\left(\xi\right)]}{A_2\left(\xi\right)}. \tag{3.13}$$

3.2. **Solitons in the deformed manifold.**
Notice that in [9] and [17], the authors have also been studied some properties of Ricci solitons on Kenmotsu manifolds under D-homothetic deformations.

Ricci solitons in the deformed manifold with potential vector field $V=\xi^d$

Assume now that in a D-homothetically deformed structure $(\phi^d,\ \xi^d,\ \eta^d,\ g^d)$ of a Kenmotsu manifold M^{2n+1} the pair $(\xi^d,\ \lambda)$ defines a Ricci soliton, that is

$$\mathcal{L}_{\xi^d}g^d+2S^d+2\lambda g^d=0. \tag{3.14}$$

for a real number λ. In view of the definition of the Lie derivative and (2.23) the above equation reduces to

$$S^d(X,U) = -(\frac{1}{p} + \lambda)g^d(X,U) + \frac{1}{p}\eta^d(X)\eta^d(U). \tag{3.15}$$

for any vector fields X, Y on M^{2n+1}. Then (2.25) and (3.15) imply $\lambda = \dfrac{2n}{p^2}$ and the above equation is equivalent to

$$S^d(X,U) = -(\frac{1}{p} + \frac{2n}{p^2})g^d(X,U) + \frac{1}{p}\eta^d(X)\eta^d(U). \tag{3.16}$$

Remark 3.7. *The Ricci soliton on a D-homothetically deformed structure* $(\phi^d, \xi^d, \eta^d, g^d)$ *of a Kenmotsu manifold is expanding.*

Comparing (3.10) and (3.16), we obtain

$$4nA_2^d\left(\xi^d\right) - A_1^d\left(\xi^d\right) = \frac{1}{p}. \tag{3.17}$$

This leads to the following:

Theorem 3.8. *Assume that a Kenmotsu structure* (ϕ, ξ, η, g) *on* M^{2n+1} *is transformed into* $(\phi^d, \xi^d, \eta^d, g^d)$ *under a D-homothetic deformation which is a hyper generalized pseudo symmetric space. If the pair* $(\xi^d, \lambda = \dfrac{2n}{p^2})$ *defines a Ricci soliton on the deformed structure, then the* 1*-forms are related (3.17).*

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