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ON $fg\gamma^*$-CLOSED SETS IN FUZZY TOPOLOGICAL SPACES

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Abstract. Starting with Chang [8], many mathematicians have engaged themselves to introduce different types of fuzzy closed-like sets in a fuzzy topological space (fts, for short). Afterwards, in [2, 3, 5, 6, 7] the notion of generalized versions of fuzzy closed set have been studied. In this paper a new type of generalized version of fuzzy γ-closed set is introduced and studied using γ-closed set as a basic tool.

1. Introduction

This paper deals with a new type of generalized version of closed set in fuzzy topological space, viz., $fg\gamma^*$-closed set using fuzzy γ-open set [4] as a basic tool. It is shown that the collection of all $fg\gamma^*$-closed sets is stronger than that of fuzzy γ-closed set [4], but weaker than that of $fg\gamma$-closed set [7]. Also the mutual relationship of this set with fgs^*-closed set [5], fsg-closed set [3], $fg\beta$-closed set [3] are established. Again we introduce a new type of closure operator, viz., $fg\gamma^*$-closure operator which is an idempotent operator. Afterwards, $fg\gamma^*$-open, $fg\gamma^*$-closed, $fg\gamma^*$-compactness and $fg\gamma^*$-irresolute functions are introduced and studied. Then establish the mutual relationship of these functions with fuzzy open function [18], fuzzy closed function [18] and fuzzy continuous function [8].

Keywords and phrases: Fuzzy γ-closed set, $fg\gamma^*$-closed set, $fg\gamma^*$-closed function, $fg\gamma^*$-open function, $fg\gamma^*$-continuous function, $fg\gamma^*$-irresolute function, fuzzy semiopen set.
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It is shown that $fg\gamma^*$-continuous image of $fg\gamma^*$-regular, $fg\gamma^*$-normal and $fg\gamma^*$-compact spaces are fuzzy regular [14], fuzzy normal [13] and fuzzy compact [8] spaces respectively. Lastly, a new type of separation axiom, viz., $fg\gamma^*$-T_2 space is introduced and shown that the inverse image of fuzzy T_2-space [14] (resp., $fg\gamma^*$-T_2 space) under $fg\gamma^*$-continuous function (resp., $fg\gamma^*$-irresolute function) is $fg\gamma^*$-T_2 space.

2. Preliminaries

Throughout this paper (X, τ) or simply by X we shall mean a fuzzy topological space (fts, for short) in the sense of Chang [8]. In [19], L.A. Zadeh introduced fuzzy set as follows: A fuzzy set A is a function from a non-empty set X into the closed interval $I = [0, 1]$, i.e., $A \in I^X$. The support [19] of a fuzzy set A, denoted by $suppA$ and is defined by $suppA = \{x \in X : A(x) \neq 0\}$. The fuzzy set with the singleton support $\{x\} \subseteq X$ and the value t $(0 < t \leq 1)$ will be denoted by x_t. 0_X and 1_X are the constant fuzzy sets taking values 0 and 1 respectively in X. The complement [19] of a fuzzy set A in X is denoted by $1_X \setminus A$ and is defined by $(1_X \setminus A)(x) = 1 - A(x)$, for each $x \in X$. For any two fuzzy sets A, B in X, $A \leq B$ means $A(x) \leq B(x)$, for all $x \in X$ [19] while AqB means A is quasi-coincident (q-coincident, for short) [17] with B, i.e., there exists $x \in X$ such that $A(x) + B(x) > 1$. The negation of these two statements will be denoted by $A \not\leq B$ and $A \not{q} B$ respectively. For a fuzzy point x_t and a fuzzy set A, $x_t \in A$ means $A(x) \geq t$, i.e., $x_t \leq A$. For a fuzzy set A, clA and $intA$ will stand for fuzzy closure [8] and fuzzy interior [8] respectively. A fuzzy set A in X is called fuzzy regular open [1] (resp., fuzzy semiopen [1], fuzzy β-open [11], fuzzy γ-open [4]) if $A = int(clA)$ (resp., $A \leq cl(intA)$, $A \leq cl(int(clA))$, $A \leq cl(intA) \bigvee int(clA)$). The complement of a fuzzy semiopen (resp., fuzzy β-open, fuzzy γ-open) set is called fuzzy semiclosed [1] (resp., fuzzy β-closed [11], fuzzy γ-closed [4]). The intersection of all fuzzy semiclosed (resp., fuzzy β-closed, fuzzy γ-closed) sets containing a fuzzy set A is called fuzzy semiclosure [1] (resp., fuzzy β-closure [11], fuzzy γ-closure [4]) of A, to be denoted by $sclA$ (resp., βclA, γclA). The union of all fuzzy γ-open sets contained in a fuzzy set A in an fts X is called fuzzy γ-interior of A, denoted by $\gamma intA$ [4]. $A(\in I^X)$ is fuzzy γ-closed (resp., fuzzy γ-open) iff $A = \gamma clA$ [4] (resp., $\gamma intA$ [4]). A fuzzy set A is called a fuzzy neighbourhood (fuzzy nbd, for short) [17] of a fuzzy point x_α if there exists a fuzzy open set U in X such that $x_\alpha \in U \leq A$. If, in addition, A is fuzzy open (resp., fuzzy γ-open), then A is called fuzzy open nbd [17] (resp., fuzzy γ-open nbd [4]) of x_α. A fuzzy set A is called a fuzzy quasi neighbourhood (fuzzy q-nbd, for short) [17] of a fuzzy point x_α in an fts X if there is a fuzzy open set U in X such that $x_\alpha qU \leq A$. If, in addition, A is fuzzy open (resp., fuzzy γ-open), then A is called fuzzy open q-nbd [17] (resp., fuzzy γ-open q-nbd [4]) of x_α. The collection of all fuzzy open (resp., fuzzy regular open, fuzzy semiopen, fuzzy β-open, fuzzy γ-open) sets in an fts X is denoted by τ (resp., $FRO(X)$, $FSO(X)$, $F\beta O(X)$, $F\gamma O(X)$). The collection of all fuzzy closed (resp., fuzzy semiclosed, fuzzy β-closed, fuzzy γ-closed) sets in an fts X is denoted by τ^c (resp., $FSC(X)$, $F\beta C(X)$, $F\gamma C(X)$).

3. $fg\gamma^*$-Closed Set: Some Properties

In this section a new type of generalized version of fuzzy closed set, viz., $fg\gamma^*$-closed set is introduced and studied. Some properties of this newly defined set are shown. Again mutual relationship of this set and the sets defined in [2, 3, 5, 6, 7] are established.

We first recall the following definitions from [2, 3, 5, 6, 7] for ready references.

Definition 3.1. Let (X, τ) be an fts and $A \in I^X$. Then A is called
(i) fuzzy generalized closed (fg-closed, for short) [2, 3] if $clA \leq U$ whenever $A \leq U \in \tau$,
(ii) fuzzy semi generalized closed (fsg-closed, for short) [3] if $sclA \leq U$ whenever $A \leq U \in FSO(X)$,
(iii) fuzzy generalized β-closed ($fg\beta$-closed, for short) [3] if $\beta clA \leq U$ whenever $A \leq U \in \tau$,
(iv) fgs^*-closed set [6] if $clA \leq U$ whenever $A \leq U \in FSO(X, \tau_1)$,
(v) fuzzy generalized γ-closed ($fg\gamma$-closed, for short) [7] if $\gamma clA \leq U$ whenever $A \leq U \in \tau$.

The complements of the above mentioned fuzzy sets are called their respective open sets.

Now we introduce the following concept.

Definition 3.2. Let (X, τ) be an fts and $A \in I^X$. Then A is called fuzzy generalized γ^*-closed ($fg\gamma^*$-closed, for short) set in X if $\gamma clA \leq U$ whenever $A \leq U \in FSO(X)$.

The complement of an $fg\gamma^*$-closed set is called fuzzy generalized

γ^*-open ($fg\gamma^*$-open, for short) set.
Remark 3.3. It is clear from definitions that
(i) $fg\gamma^*$-closed set is $fg\beta$-closed set as well as $fg\gamma$-closed set,
(ii) fsg-closed set is $fg\gamma^*$-closed set and fgs^*-closed set is $fg\gamma^*$-closed set,
(iii) fuzzy γ-closed set is $fg\gamma^*$-closed set.
But the converses are not true, in general, follow from the following examples.
(iv) $fg\gamma^*$-closed set and fg-closed set are independent concepts as follows from the next examples.
Example 3.4. None of the properties of fg-closedness, $fg\beta$-closedness, $fg\gamma$-closedness implies that of $fg\gamma*$-closedness
Let $X = \{a\}$, $\tau = \{0_X, 1_X A, B\}$ where $A(a) = 0.45, B(a) = 0.6$. Then (X, τ) is an fts. Here $FSO(X) = \{o_X, 1_X, U, V\}$ where $A \leq U \leq 1_X \backslash A$ and $V \geq B$, $F\gamma O(X) = \{0_X, 1_X, W\}$ where $W > 1_X \setminus B$ and $F\gamma C(X) = \{0_X, 1_X, 1_X \setminus W\}$ where $1_X \setminus W < B$. Consider the fuzzy set C defined by $C(a) = 0.7$. Then 1_X is the only fuzzy open set in (X, τ) containing C and so $clC \leq 1_X$, $\beta clC \leq 1_X$, $\gamma clC \leq 1_X$ imply that C is fg-closed set, $fg\beta$-closed set and $fg\gamma$-closed set. But as $C \in FSO(X)$, $C \leq C$ and $\gamma clC = 1_X \not\leq C$ implies that C is not $fg\gamma^*$-closed set in (X, τ).
Example 3.5. None of fg-closedness , fsg-closedness, fgs^*-closedness is implied by $fg\gamma^*$-closedness Consider Example 3.4 and the fuzzy set D defined by $D(a) = 0.56$. Then $D < B \in FSO(X)$. Now $\gamma clD = D < B$ implies that D is $fg\gamma^*$-closed set in (X, τ). But $sclD = 1_X \not\leq B \Rightarrow D$ is not fsg-closed set in (X, τ). Also $clD = 1_X \not\leq B \Rightarrow D$ is not fgs^*-closed set in (X, τ). Again $D < B \in \tau$. But $clD = 1_X \not\leq B \Rightarrow D$ is not fg-closed set in (X, τ).
Example 3.6. There exists an $fg\gamma^*$-closed set which is not fuzzy closed
Let $X = \{a, b\}$, $\tau = \{0_X, 1_X, A\}$ where $A(a) = 0.5, A(b) = 0.4$. Then (X, τ) is an fts. Here $FSO(X, \tau) = \{0_X, 1_X, U\}$ where $A \leq U \leq 1_X \setminus A$. Consider the fuzzy set B defined by $B(a) = 0.7, B(b) = 0.5$. Then $B \notin F\gamma C(X, \tau)$, because $(clintB) \bigvee (intclB) = 1_X \setminus A \not\leq B$. As $1_X \in FSO(X, \tau)$ only containing B, B is $fg\gamma^*$-closed set in (X, τ).
Remark 3.7. It is obvious that union of two $fg\gamma^*$-closed sets is also so. But intersection of two $fg\gamma^*$-closed sets need not be so, as it seen

from the following example.

Example 3.8. Let $X = \{a, b\}$, $\tau = \{0_X, 1_X, A, B\}$ where $A(a) = 0.5, A(b) = 0.4, B(a) = 0.4, B(b) = 0.3$. Then (X, τ) is an fts. Here $FSO(X) = \{0_X, 1_X, U\}$ where $B \leq U \leq 1_X \setminus A$. Consider two fuzzy sets C and D defined by $C(a) = 0.6, C(b) = 0.55, D(a) = 0.45, D(b) = 0.7$. Then clearly C and D are $fg\gamma^*$-closed sets in (X, τ). Let $E = C \bigwedge D$. Then $E(a) = 0.45, E(b) = 0.55$. Then $E \in FSO(X)$. So $E \leq E$. Now $(clintE) \bigwedge (intclE) = (1_X \setminus A) \bigwedge A = A \not\leq E$. So $\gamma clE \not\leq E$, i.e., $E \notin F\gamma C(X)$ and so $\gamma clE \neq E \Rightarrow E$ is not $fg\gamma^*$-closed set in X.

From the above discussion we can conclude that the collection of all $fg\gamma^*$-open sets does not form a fuzzy topology.

Theorem 3.9. If $A(\in I^X)$ is $fg\gamma^*$-closed set in X and $B \in I^X$ is such that $A \leq B \leq \gamma clA$, then B is also $fg\gamma^*$-closed set in X.

Proof. Let $U \in FSO(X)$ be such that $B \leq U$. Then by hypothesis, $A \leq B \leq U$. As A is $fg\gamma^*$-closed set in X, $\gamma clA \leq U$ and so $A \leq B \leq \gamma clA \leq U \Rightarrow \gamma clA \leq \gamma clB \leq \gamma cl(\gamma clA) = \gamma clA \leq U \Rightarrow \gamma clB \leq U$. Consequently, B is $fg\gamma^*$-closed set in X.

Theorem 3.10. Let (X, τ) be an fts and $A, B \in I^X$. If $\gamma intA \leq B \leq A$ and A is $fg\gamma^*$-open set in X, then B is also $fg\gamma^*$-open set in X.

Proof. $\gamma intA \leq B \leq A \Rightarrow 1_X \setminus A \leq 1_X \setminus B \leq 1_X \setminus \gamma intA = \gamma cl(1_X \setminus A)$ where $1_X \setminus A$ is $fg\gamma^*$-closed set in X. By Theorem 3.9, $1_X \setminus B$ is $fg\gamma^*$-closed set in $X \Rightarrow B$ is $fg\gamma^*$-open set in X.

Theorem 3.11. Let (X, τ) be an fts and $A \in I^X$. Then A is $fg\gamma^*$-open set in X iff $K \leq \gamma intA$ whenever $K \leq A$ and $K \in FSC(X)$.

Proof. Let $A(\in I^X)$ be $fg\gamma^*$-open set in X and $K \leq A$ where $K \in FSC(X)$. Then $1_X \setminus A \leq 1_X \setminus K$ where $1_X \setminus A$ is $fg\gamma^*$-closed set in X and $1_X \setminus K \in FSO(X)$. So $\gamma cl(1_X \setminus A) \leq 1_X \setminus K \Rightarrow 1_X \setminus \gamma intA \leq 1_X \setminus K \Rightarrow K \leq \gamma intA$.

Conversely, let $K \leq \gamma intA$ whenever $K \leq A$, $K \in FSC(X)$. Then $1_X \setminus A \leq 1_X \setminus K \in FSO(X)$. Now $1_X \setminus \gamma intA \leq 1_X \setminus K \Rightarrow \gamma cl(1_X \setminus A) \leq 1_X \setminus K \Rightarrow 1_X \setminus A$ is $fg\gamma^*$-closed set in $X \Rightarrow A$ is $fg\gamma^*$-open set in X.

Theorem 3.12. Let (X, τ) be an fts and $A(\in I^X)$. If A is fuzzy semiopen set as well as $fg\gamma^*$-closed set in X, then $A \in F\gamma C(X)$.

Proof. Now $A \leq A \in FSO(X)$. By hypothesis, $\gamma clA \leq A$ (as A is $fg\gamma^*$-closed set in X) $\Rightarrow A = \gamma clA \Rightarrow A \in F\gamma C(X)$.

Similarly we can state the following theorem easily.

Theorem 3.13. Let (X, τ) be an fts and $A(\in I^X) \in FRO(X)$ as well as A is $fg\gamma^*$-closed set in X, then $A \in F\gamma C(X)$.

Theorem 3.14. Let (X, τ) be an fts and $A(\in I^X)$ be $fg\gamma*$-closed set in X and $F \in FSC(X)$ with $A \not{q} F$. Then $\gamma cl A \not{q} F$.

Proof. Now $A \not{q} F \Rightarrow A \leq 1_X \setminus F \in FSO(X)$. By assumption, $\gamma cl A \leq 1_X \setminus F \Rightarrow \gamma cl A \not{q} F$.

Remark 3.15. The converse of Theorem 3.14 may not be true, in general, as it seen from the following example.

Example 3.16. Let $X = \{a, b\}$, $\tau = \{0_X, 1_X, A, B, C\}$ where $A(a) = A(b) = 0.4, B(a) = 0.4, B(b) = 0.6, C(a) = 0.5, C(b) = 0.6$. Then (X, τ) is an fts. Consider the fuzzy set D defined by $D(a) = 0.4, D(b) = 0.5$. Now $1_X \setminus C \in FSC(X)$ and $D \not{q}(1_X \setminus C)$, $\gamma cl D = C \not{q}(1_X \setminus C)$. But D is not $fg\gamma^*$-closed set in X. Indeed, $D < B \in FSO(X)$ and $\gamma cl D = C \not\leq B$.

Definition 3.17. Let (X, τ) be an fts and x_α, a fuzzy point in X. A fuzzy set A is called a fuzzy generalized γ^*-neighbourhood ($fg\gamma^*$-nbd, for short) of x_α, if there exists an $fg\gamma^*$-open set U in X such that $x_\alpha \leq U \leq A$. If, in addition, A is $fg\gamma^*$-open set in X, then A is called an $fg\gamma^*$-open nbd of x_α.

Definition 3.18. Let (X, τ) be an fts and x_α, a fuzzy point in X. A fuzzy set A is called a fuzzy generalized γ^*-quasi neighbourhood($fg\gamma^*$-q-nbd, for short) of x_α if there is an $fg\gamma^*$-open set U in X such that $x_\alpha q U \leq A$. If, in addition, A is $fg\gamma^*$-open set in X, then A is called an $fg\gamma^*$-open q-nbd of x_α.

Note 3.19. It is clear from definitions that every $fg\gamma^*$-open set is an $fg\gamma^*$-open nbd of each of its points. But every $fg\gamma^*$-nbd of x_α may not be an $fg\gamma^*$-open set containing x_α as follows from the next example.

Example 3.20. Consider Example 3.16 and the fuzzy set E defined by $E(a) = 0.6, E(b) = 0.5$ and the fuzzy point $a_{0.4}$. We claim that E is an $fg\gamma^*$-nbd of $a_{0.4}$ though E is not an $fg\gamma^*$-open set in X. Indeed, $(1_X \setminus E)(a) = 0.4, (1_X \setminus E)(b) = 0.5$. Then as $B \in \tau$, $B \in FSO(X)$ ad so $1_X \setminus E < B$. Now $\gamma cl(1_X \setminus E) \not\leq B$ as no fuzzy set U, $1_X \setminus E \leq U \leq B$ is fuzzy γ-closed set in $X \Rightarrow 1_X \setminus E$ is not $fg\gamma^*$-closed set in $X \Rightarrow E$ is not $fg\gamma^*$-open set in X. But as $A(a) = 0.4, a_{0.4} \in A \in \tau$ and since every fuzzy open set being fuzzy γ-open set is $fg\gamma^*$-open set in X. Also, $a_{0.4} \in A \leq E \Rightarrow E$ is an $fg\gamma^*$-nbd of $a_{0.4}$.

Note 3.21. Every fuzzy open nbd (resp., open q-nbd) of a fuzzy point x_α is an $fg\gamma^*$-open nbd (resp., $fg\gamma^*$-open q-nbd) of x_α, but converses are not true, in general, follow from the next example.

Example 3.22. Let $X = \{a, b\}$, $\tau = \{0_X, 1_X, A\}$ where $A(a) = 0.5, A(b) = 0.4$. Then (X, τ) is an fts. Consider the fuzzy point $b_{0.45}$ and the fuzzy set B defined by $B(a) = B(b) = 0.5$. Clearly B is not a fuzzy open nbd of $b_{0.45}$. But as $B \in F\gamma O(X)$, B is $fg\gamma^*$-open nbd of $b_{0.45}$. Again consider the fuzzy point $b_{0.6}$. Then B is not a fuzzy open q-nbd of $b_{0.6}$ as $b_{0.6} \not{q} A \leq B$. But as $b_{0.6} q B$ where B is an $fg\gamma^*$-open set in $X \Rightarrow B$ is an $fg\gamma^*$-open q-nbd of $b_{0.6}$.

Theorem 3.23. Let $F(\in I^X)$ be an $fg\gamma^*$-closed set in an fts X with $x_t \in 1_X \setminus F$. Then there exists an $fg\gamma^*$-nbd G of x_t such that $G \not{q} F$.

Proof. Let $x_t \in 1_X \setminus F$ where $1_X \setminus F$ be an $fg\gamma^*$-open set in X. Then $1_X \setminus F$ is an $fg\gamma^*$-open nbd of x_t. So by definition, there exists an $fg\gamma^*$-open set G in X such that $x_t \in G \leq 1_X \setminus F \Rightarrow G$ is an $fg\gamma^*$-nbd of x_t with $G \not{q} F$.

Definition 3.24. The set of all $fg\gamma^*$-nbds of a fuzzy point x_t $(0 < t \leq 1)$ in an fts (X, τ) is called the $fg\gamma^*$-nbd system at x_t, denoted by $fg\gamma^*$-$N(x_t)$.

Theorem 3.25. For a fuzzy point x_t in an fts (X, τ), the following statements hold :

(i) $fg\gamma^*$-$N(x_t) \neq \emptyset$,

(ii) $G \in fg\gamma^*$-$N(x_t) \Rightarrow x_t \in G$,

(iii) $G \in fg\gamma^*$-$N(x_t)$ and $F \geq G \Rightarrow F \in fg\gamma^*$-$N(x_t)$,

(iv) $F, G \in fg\gamma^*$-$N(x_t) \Rightarrow F \bigwedge G \in fg\gamma^*$-$N(x_t)$,

(v) $G \in fg\gamma^*$-$N(x_t) \Rightarrow$ there exists $F \in fg\gamma^*$-$N(x_t)$ such that $F \leq G$ and $F \in fg\gamma^*$-$N(y_{t'})$ for every $y_{t'} \in F$.

Proof. (i) Since 1_X being an $fg\gamma^*$-open set is an $fg\gamma^*$-nbd of x_t $(0 < t \leq 1)$, $fg\gamma^*$-$N(x_t) \neq \emptyset$.

(ii) and (iii) are obvious.

(iv) Since intersection of two $fg\gamma^*$-open sets is $fg\gamma^*$-open, (iv) is obvious.

(v) Follows from Note 3.19 and Definition 3.24.

Theorem 3.26. Let x_t be a fuzzy point in an fts (X, τ). Let $fg\gamma^*$-$N(x_t)$ be a non-empty collection of fuzzy sets in X satisfying the following conditions :

(1) $G \in fg\gamma^*$-$N(x_t) \Rightarrow x_t \in G$,

(2) $F, G \in fg\gamma^*$-$N(x_t) \Rightarrow F \bigwedge G \in fg\gamma^*$-$N(x_t)$.

Let τ consist of 0_X and all those non-empty fuzzy sets G of X having the property that $x_t \in G \Rightarrow$ there exists an $F \in fg\gamma^*$-$N(x_t)$ such that

$x_t \in F \leq G$. Then τ is a fuzzy topology on X.

Proof. (i) By hypothesis, $0_X \in \tau$.

(ii) It is clear from the given property of τ that $1_X \in \tau$ as $1_X \in fg\gamma^*$-$N(x_t)$ for any fuzzy point x_t $(0 < t \leq 1)$ in an fts X (by (1)).

(iii) Let $G_1, G_2 \in \tau$. If $G_1 \bigwedge G_2 = 0_X$, then by construction of τ, $G_1 \bigwedge G_2 \in \tau$. Suppose $G_1 \bigwedge G_2 \neq 0_X$. Let $x_t \in G_1 \bigwedge G_2$ where $0 < t \leq 1$. Then $G_1(x) \geq t, G_2(x) \geq t$. Since $G_1, G_2 \in \tau$, by definition of τ, there exist $F_1, F_2 \in fg\gamma^*$-$N(x_t)$ such that $x_t \in F_1 \leq G_1$, $x_t \in F_2 \leq G_2$. Then $x_t \in F_1 \bigwedge F_2 \leq G_1 \bigwedge G_2$. By (2), $F_1 \bigwedge F_2 \in fg\gamma^*$-$N(x_t)$ and so $G_1 \bigwedge G_2 \in \tau$ by construction of τ.

(iv) Let $\mathcal{G} = \{G_\alpha : \alpha \in \Lambda\}$ where $G_\alpha \in \tau$, for each $\alpha \in \Lambda$. Let $x_t \in \bigvee_{\alpha \in \Lambda} G_\alpha$. Then there exists $\beta \in \Lambda$ such that $x_t \in G_\beta$. By definition of τ, there exists $F_\beta \in fg\gamma^*$-$N(x_t)$ such that $x_t \in F_\beta \leq G_\beta \leq \bigvee_{\alpha \in \Lambda} G_\alpha \Rightarrow \bigvee_{\alpha \in \Lambda} G_\alpha \in \tau$.

It follows that τ is a fuzzy topology on X.

4. $fg\gamma^*$-Closure Operator and $fg\gamma^*$-Open, $fg\gamma^*$-Closed Functions

In this section we first introduce a new type of generalized version of fuzzy closure operator which is an idempotent operator. Afterwards, two new types of functions are introduced and studied and characterized these two functions by this newly defined operator.

Definition 4.1. Let (X, τ) be an fts and $A \in I^X$. Then $fg\gamma^*$-closure and $fg\gamma^*$-interior of A, denoted by $fg\gamma^*cl(A)$ and $fg\gamma^*int(A)$, are defined as follow:

$fg\gamma^*cl(A) = \bigwedge\{F : A \leq F, F \text{ is } fg\gamma^*\text{-closed set in } X\}$,

$fg\gamma^*int(A) = \bigvee\{G : G \leq A, G \text{ is } fg\gamma^*\text{-open set in } X\}$.

Remark 4.2. It is clear from definition that for any $A \in I^X$, $A \leq fg\gamma^*cl(A) \leq clA$. If A is $fg\gamma^*$-closed set in an fts X, then $A = fg\gamma^*cl(A)$. Similarly, $intA \leq fg\gamma^*int(A) \leq A$. If A is $fg\gamma^*$-open set in an fts X, then $A = fg\gamma^*int(A)$. It follows from Remark 3.7 that $fg\gamma^*cl(A)$ (resp., $fg\gamma^*int(A)$) may not be $fg\gamma^*$-closed (resp., $fg\gamma^*$-open) set in an fts X.

Result 4.3. Let (X, τ) be an fts and $A \in I^X$. Then for a fuzzy point x_t in X, $x_t \in fg\gamma^*cl(A)$ iff every $fg\gamma^*$-open q-nbd U of x_t, UqA.

Proof. Let $x_t \in fg\gamma^*cl(A)$ for any fuzzy set A in an fts X and F

be any $fg\gamma^*$-open q-nbd of x_t. Then $x_tqF \Rightarrow x_t \notin 1_X \setminus F$ which is $fg\gamma^*$-closed set in X. Then by Definition 4.1, $A \not\leq 1_X \setminus F \Rightarrow$ there exists $y \in X$ such that $A(y) > 1 - F(y) \Rightarrow AqF$.

Conversely, let for every $fg\gamma^*$-open q-nbd F of x_t, FqA. If possible, let $x_t \notin fg\gamma^*cl(A)$. Then by Definition 4.1, there exists an $fg\gamma^*$-closed set U in X with $A \leq U$, $x_t \notin U$. Then $x_tq(1_X \setminus U)$ which being $fg\gamma^*$-open set in X is $fg\gamma^*$-open q-nbd of x_t. By assumption, $(1_X \setminus U)qA \Rightarrow (1_X \setminus A)qA$, a contradiction.

Theorem 4.4. Let (X, τ) be an fts and $A, B \in I^X$. Then the following statements are true:
(i) $fg\gamma^*cl(0_X) = 0_X$,
(ii) $fg\gamma^*cl(1_X) = 1_X$,
(iii) $A \leq B \Rightarrow fg\gamma^*cl(A) \leq fg\gamma^*cl(B)$,
(iv) $fg\gamma^*cl(A \bigvee B) = fg\gamma^*cl(A) \bigvee fg\gamma^*cl(B)$,
(v) $fg\gamma^*cl(A \wedge B) \leq fg\gamma^*cl(A) \wedge fg\gamma^*cl(B)$, equality does not hold, in general, follows from Example 3.8,
(vi) $fg\gamma^*cl(fg\gamma^*cl(A)) = fg\gamma^*cl(A)$.

Proof. (i), (ii) and (iii) are obvious.
(iv) From (iii), $fg\gamma^*cl(A) \bigvee fg\gamma^*cl(B) \leq fg\gamma^*cl(A \bigvee B)$.
To prove the converse, let $x_\alpha \in fg\gamma^*cl(A \bigvee B)$. Then by Result 4.3, for any $fg\gamma^*$-open set U in X with $x_\alpha qU$, $Uq(A \bigvee B) \Rightarrow$ there exists $y \in X$ such that $U(y) + max\{A(y), B(y)\} > 1 \Rightarrow$ either $U(y) + A(y) > 1$ or $U(y) + B(y) > 1 \Rightarrow$ either UqA or $UqB \Rightarrow$ either $x_\alpha \in fg\gamma^*cl(A)$ or $x_\alpha \in fg\gamma^*cl(B) \Rightarrow x_\alpha \in fg\gamma^*cl(A) \bigvee fg\gamma^*cl(B)$.
(v) Follows from (iii).
(vi) As $A \leq fg\gamma^*cl(A)$, for any $A \in I^X$, $fg\gamma^*cl(A) \leq fg\gamma^*cl(fg\gamma^*cl(A))$ (by (iii)).

Conversely, let $x_\alpha \in fg\gamma^*cl(fg\gamma^*cl(A)) = fg\gamma^*cl(B)$ where $B = fg\gamma^*cl(A)$. Let U be any $fg\gamma^*$-open set in X with $x_\alpha qU$. Then UqB implies that there exists $y \in X$ such that $U(y) + B(y) > 1$. Let $B(y) = t$. Then y_tqU and $y_t \in B = fg\gamma^*cl(A) \Rightarrow UqA$ $\Rightarrow x_\alpha \in fg\gamma^*cl(A) \Rightarrow fg\gamma^*cl(fg\gamma^*cl(A)) \leq fg\gamma^*cl(A)$. Consequently, $fg\gamma^*cl(fg\gamma^*cl(A)) = fg\gamma^*cl(A)$.

Theorem 4.5. Let (X, τ) be an fts and $A \in I^X$. Then the following statements hold:
(i) $fg\gamma^*cl(1_X \setminus A) = 1_X \setminus fg\gamma^*int(A)$
(ii) $fg\gamma^*int(1_X \setminus A) = 1_X \setminus fg\gamma^*cl(A)$.

Proof (i). Let $x_t \in fg\gamma^*cl(1_X \setminus A)$ for a fuzzy set A in an fts (X, τ).

If possible, let $x_t \notin 1_X \setminus fg\gamma^*int(A)$. Then $1-(fg\gamma^*int(A))(x) < t \Rightarrow [fg\gamma^*int(A)](x)+t > 1 \Rightarrow fg\gamma^*int(A)qx_t \Rightarrow$ there exists at least one $fg\gamma^*$-open set $F \leq A$ with $x_tqF \Rightarrow x_tqA$. As $x_t \in fg\gamma^*cl(1_X \setminus A), Fq(1_X \setminus A) \Rightarrow Aq(1_X \setminus A)$, a contradiction. Hence

$$fg\gamma^*cl(1_X \setminus A) \leq 1_X \setminus fg\gamma^*int(A)...(1)$$

Conversely, let $x_t \in 1_X \setminus fg\gamma^*int(A)$. Then $1-[(fg\gamma^*int(A)](x) \geq t \Rightarrow x_t \not{q}(fg\gamma^*int(A)) \Rightarrow x_t \not{q}F$ for every $fg\gamma^*$-open set F contained in A ... (2).

Let U be any $fg\gamma^*$-closed set in X such that $1_X \setminus A \leq U$. Then $1_X \setminus U \leq A$. Now $1_X \setminus U$ is $fg\gamma^*$-open set in X contained in A. By (2), $x_t \not{q}(1_X \setminus U) \Rightarrow x_t \in U \Rightarrow x_t \in fg\gamma^*cl(1_X \setminus A)$ and so

$$1_X \setminus fg\gamma^*int(A) \leq fg\gamma^*cl(1_X \setminus A)...(3).$$

Combining (1) and (3), (i) follows.

(ii) Putting $1_X \setminus A$ for A in (i), we get $fg\gamma^*cl(A) = 1_X \setminus fg\gamma^*int(1_X \setminus A) \Rightarrow fg\gamma^*int(1_X \setminus A) = 1_X \setminus fg\gamma^*cl(A)$.

Let us now recall the following definition from [18] for ready references.

Definition 4.6 [18]. A function $f: X \to Y$ is called fuzzy open (resp., fuzzy closed) if $f(U)$ is fuzzy open (resp., fuzzy closed) set in Y for every fuzzy open (resp., fuzzy closed) set U in X.

Let us now introduce the following concept.

Definition 4.7. A function $h: X \to Y$ is called fuzzy generalized γ^*-open ($fg\gamma^*$-open, for short) function if $h(U)$ is $fg\gamma^*$-open set in Y for every fuzzy open set U in X.

Remark 4.8. It is clear that fuzzy open function is $fg\gamma^*$-open function. But the converse need not be true, as it seen from the following example.

Example 4.9. $fg\gamma^*$-open function does not imply fuzzy open function

Let $X = \{a,b\}$, $\tau_1 = \{0_X, 1_X, A\}$, $\tau_2 = \{0_X, 1_X\}$ where $A(a) = 0.4, A(b) = 0.6$. Then (X,τ_1) and (X,τ_2) are fts's. Consider the identity function $i:(X,\tau_1) \to (X,\tau_2)$. Since every fuzzy set in (X,τ_2) is $fg\gamma^*$-open set in (X,τ_2), clearly i is $fg\gamma^*$-open function. But $A \in \tau_1$, $i(A) = A \notin \tau_2 \Rightarrow i$ is not a fuzzy open function.

Theorem 4.10. For a bijective function $h: X \to Y$, the following statements are equivalent:

(i) h is $fg\gamma^*$-open,

(ii) $h(intA) \leq fg\gamma^*int(h(A))$, for all $A \in I^X$,
(iii) for each fuzzy point x_α in X and each fuzzy open set U in X containing x_α, there exists an $fg\gamma^*$-open set V in Y containing $h(x_\alpha)$ such that $V \leq h(U)$.

Proof (i) $\Rightarrow$ (ii). Let $A \in I^X$. Then $intA$ is a fuzzy open set in X. By (i), $h(intA)$ is $fg\gamma^*$-open set in Y. Since $h(intA) \leq h(A)$ and $fg\gamma^*int(h(A))$ is the union of all $fg\gamma^*$-open sets contained in $h(A)$, we have $h(intA) \leq fg\gamma^*int(h(A))$.

(ii) $\Rightarrow$ (i). Let U be any fuzzy open set in X. Then $h(U) = h(intU) \leq fg\gamma^*int(h(U))$ (by (ii)) $\Rightarrow$ $h(U)$ is $fg\gamma^*$-open set in $Y \Rightarrow h$ is $fg\gamma^*$-open function.

(ii) $\Rightarrow$ (iii). Let x_α be a fuzzy point in X, and U, a fuzzy open set in X such that $x_\alpha \in U$. Then $h(x_\alpha) \in h(U) = h(intU) \leq fg\gamma^*int(h(U))$ (by (ii)). Then $h(U)$ is $fg\gamma^*$-open set in Y. Let $V = h(U)$. Then $h(x_\alpha) \in V$ and $V \leq h(U)$.

(iii) $\Rightarrow$ (i). Let U be any fuzzy open set in X and y_α, any fuzzy point in $h(U)$, i.e., $y_\alpha \in h(U)$. Then there exists unique $x \in X$ such that $h(x) = y$ (as h is bijective). Then $[h(U)](y) \geq \alpha \Rightarrow U(h^{-1}(y)) \geq \alpha \Rightarrow U(x) \geq \alpha \Rightarrow x_\alpha \in U$. By (iii), there exists $fg\gamma^*$-open set V in Y such that $h(x_\alpha) \in V$ and $V \leq h(U)$. Then $h(x_\alpha) \in V = fg\gamma^*int(V) \leq fg\gamma^*int(h(U))$. Since y_α is taken arbitrarily and $h(U)$ is the union of all fuzzy points in $h(U)$, $h(U) \leq fg\gamma^*int(f(U)) \Rightarrow h(U)$ is $fg\gamma^*$-open set in $Y \Rightarrow h$ is an $fg\gamma^*$-open function.

Theorem 4.11. If $h : X \to Y$ is $fg\gamma^*$-open, bijective function, then the following statements are true:
(i) for each fuzzy point x_α in X and each fuzzy open q-nbd U of x_α in X, there exists an $fg\gamma^*$-open q-nbd V of $h(x_\alpha)$ in Y such that $V \leq h(U)$,
(ii) $h^{-1}(fg\gamma^*cl(B)) \leq cl(h^{-1}(B))$, for all $B \in I^Y$.

Proof (i). Let x_α be a fuzzy point in X and U be any fuzzy open q-nbd of x_α in X. Then $x_\alpha q U = intU \Rightarrow h(x_\alpha) q h(intU) \leq fg\gamma^*int(h(U))$ (by Theorem 4.10 (i)$\Rightarrow$(ii)) implies that there exists at least one $fg\gamma^*$-open q-nbd V of $h(x_\alpha)$ in Y with $V \leq h(U)$.

(ii) Let x_α be any fuzzy point in X such that $x_\alpha \notin cl(h^{-1}(B))$ for any $B \in I^Y$. Then there exists a fuzzy open q-nbd U of x_α in X such that $U \not{q} h^{-1}(B)$. Now

$$h(x_\alpha) q h(U) ...(1)$$

where $h(U)$ is $fg\gamma^*$-open set in Y. Now $h^{-1}(B) \leq 1_X \setminus U$ which is a fuzzy closed set in $X \Rightarrow B \leq h(1_X \setminus U)$ (as h is injective) $\leq 1_Y \setminus h(U) \Rightarrow B \not{q} h(U)$. Let $V = 1_Y \setminus h(U)$. Then $B \leq V$ which is $fg\gamma^*$-closed set in Y. We claim that $h(x_\alpha) \notin V$. If possible, let $h(x_\alpha) \in V = 1_Y \setminus h(U)$. Then $1 - [h(U)](h(x)) \geq \alpha \Rightarrow h(U) \not{q} h(x_\alpha)$, contradicting (1). So $h(x_\alpha) \notin V \Rightarrow h(x_\alpha) \notin fg\gamma^* cl(B) \Rightarrow x_\alpha \notin h^{-1}(fg\gamma^* cl(B)) \Rightarrow h^{-1}(fg\gamma^* cl(B)) \leq cl(h^{-1}(B))$.

Theorem 4.12. An injective function $h : X \to Y$ is $fg\gamma^*$-open if and only if for each $B \in I^Y$ and F, a fuzzy closed set in X with $h^{-1}(B) \leq F$, there exists an $fg\gamma^*$-closed set V in Y such that $B \leq V$ and $h^{-1}(V) \leq F$.

Proof. Let $B \in I^Y$ and F, a fuzzy closed set in X with $h^{-1}(B) \leq F$. Then $1_X \setminus h^{-1}(B) \geq 1_X \setminus F$ where $1_X \setminus F$ is a fuzzy open set in $X \Rightarrow h(1_X \setminus F) \leq h(1_X \setminus h^{-1}(B)) \leq 1_Y \setminus B$ (as h is injective) where $h(1_X \setminus F)$ is an $fg\gamma^*$-open set in Y. Let $V = 1_Y \setminus h(1_X \setminus F)$. Then V is $fg\gamma^*$-closed set in Y such that $B \leq V$. Now $h^{-1}(V) = h^{-1}(1_Y \setminus h(1_X \setminus F)) = 1_X \setminus h^{-1}(h(1_X \setminus F)) \leq F$.

Conversely, let F be a fuzzy open set in X. Then $1_X \setminus F$ is a fuzzy closed set in X. We have to show that $h(F)$ is an $fg\gamma^*$-open set in Y. Now $h^{-1}(1_Y \setminus h(F)) \leq 1_X \setminus F$ (as h is injective). By assumption, there exists an $fg\gamma^*$-closed set V in Y such that

$$1_Y \setminus h(F) \leq V...(1)$$

and $h^{-1}(V) \leq 1_X \setminus F$. Therefore, $F \leq 1_X \setminus h^{-1}(V)$ implies that

$$h(F) \leq h(1_X \setminus h^{-1}(V)) \leq 1_Y \setminus V...(2)$$

(as h is injective). Combining (1) and (2), $h(F) = 1_Y \setminus V$ which is an $fg\gamma^*$-open set in Y. Hence h is $fg\gamma^*$-open function.

Definition 4.13. A function $h : X \to Y$ is called fuzzy generalized γ^*-closed ($fg\gamma^*$-closed, for short) function if $h(A)$ is $fg\gamma^*$-closed set in Y for each fuzzy closed set A in X.

Remark 4.14. It is obvious that every fuzzy closed function is $fg\gamma^*$-closed function, but the converse may not be true as it follows from Example 4.9. Here $1_X \setminus A \in \tau_1^c$, but $i(1_X \setminus A) = 1_X \setminus A \notin \tau_2^c \Rightarrow i$ is not a fuzzy closed function. But since every fuzzy set in (X, τ_2) is $fg\gamma^*$-closed set in (X, τ_2), clearly i is $fg\gamma^*$-closed function.

Theorem 4.15. A bijective function $h : X \to Y$ is $fg\gamma^*$-closed if and only if $fg\gamma^* cl(h(A)) \leq h(clA)$, for all $A \in I^X$.

Proof. Let us suppose that $h : X \to Y$ be an $fg\gamma^*$-closed function and

$A \in I^X$. Then $h(cl(A))$ is $fg\gamma^*$-closed set in Y. Since $h(A) \leq h(clA)$ and $fg\gamma^*cl(h(A))$ is the intersection of all $fg\gamma^*$-closed sets in Y containing $h(A)$, we have $fg\gamma^*cl(h(A)) \leq h(clA)$.

Conversely, let for any $A \in I^X$, $fg\gamma^*cl(h(A)) \leq h(clA)$. Let U be any fuzzy closed set in X. Then $h(U) = h(clU) \geq fg\gamma^*cl(h(U)) \Rightarrow h(U)$ is an $fg\gamma^*$-closed set in $Y \Rightarrow h$ is an $fg\gamma^*$-closed function.

Theorem 4.16. If $h : X \to Y$ is an $fg\gamma^*$-closed bijective function, then the following statements hold:
(i) for each fuzzy point x_α in X and each fuzzy closed set U in X with $x_\alpha \not{q} U$, there exists an $fg\gamma^*$-closed set V in Y with $h(x_\alpha) \not{q} V$ such that $V \geq h(U)$,
(ii) $h^{-1}(fg\gamma^*int(B)) \geq int(h^{-1}(B))$, for all $B \in I^Y$.

Proof (i). Let x_α be a fuzzy point in X and U be any fuzzy closed set in X with $x_\alpha \not{q} U = clU \Rightarrow h(x_\alpha) \not{q} h(clU) \geq fg\gamma^*cl(h(U))$ (by Theorem 4.15) $\Rightarrow h(x_\alpha) \not{q} V$ for some $fg\gamma^*$-closed set V in Y with $V \geq h(U)$.

(ii). Let $B \in I^Y$ and x_α be any fuzzy point in X such that $x_\alpha \in int(h^{-1}(B))$. Then there exists a fuzzy open set U in X with $U \leq h^{-1}(B)$ such that $x_\alpha \in U$. Then $1_X \setminus U \geq 1_X \setminus h^{-1}(B) \Rightarrow h(1_X \setminus U) \geq h(1_X \setminus h^{-1}(B))$ where $h(1_X \setminus U)$ is an $fg\gamma^*$-closed set in Y. Let $V = 1_Y \setminus h(1_X \setminus U)$. Then V is an $fg\gamma^*$-open set in Y and $V = 1_Y \setminus h(1_X \setminus U) \leq 1_Y \setminus h(1_X \setminus h^{-1}(B)) \leq 1_Y \setminus (1_Y \setminus B) = B$ (as h is injective). Now $U(x) \geq \alpha \Rightarrow x_\alpha \not{q} (1_X \setminus U) \Rightarrow h(x_\alpha) \not{q} h(1_X \setminus U) \Rightarrow h(x_\alpha) \leq 1_Y \setminus h(1_X \setminus U) = V \Rightarrow h(x_\alpha) \in V = fg\gamma^*int(V) \leq fg\gamma^*int(B) \Rightarrow x_\alpha \in h^{-1}(fg\gamma^*int(B))$. Since x_α is taken arbitrarily, $int(h^{-1}(B)) \leq h^{-1}(fg\gamma^*int(B))$, for all $B \in I^Y$.

Remark 4.17. Composition of two $fg\gamma^*$-closed (resp., $fg\gamma^*$-open) functions need not be so, as it seen from the following example.

Example 4.18. Let $X = \{a, b\}$, $\tau_1 = \{0_X, 1_X, A\}$, $\tau_2 = \{0_X, 1_X\}$, $\tau_3 = \{0_X, 1_X, B, C, D\}$ where $A(a) = 0.6, A(b) = 0.5$, $B(a) = B(b) = 0.4$, $C(a) = 0.4, C(b) = 0.6$, $D(a) = 0.5, D(b) = 0.6$. Then (X, τ_1), (X, τ_2) and (X, τ_3) are fts's. Consider two identity functions $i_1 : (X, \tau_1) \to (X, \tau_2)$ and $i_2 : (X, \tau_2) \to (X, \tau_3)$. Clearly i_1 and i_2 are $fg\gamma^*$-closed functions. Let $i_3 = i_2 \circ i_1 : (X, \tau_1) \to (X, \tau_3)$. We claim that i_3 is not $fg\gamma^*$-closed function. Indeed, $1_X \setminus A \in \tau_1^c$, $i_3(1_X \setminus A) = 1_X \setminus A < C \in FSO(X, \tau_3)$. But $\gamma cl_{\tau_3}(1_X \setminus A) = D \not\leq C \Rightarrow 1_X \setminus A$ is not $fg\gamma^*$-closed set in $(X, \tau_3) \Rightarrow i_3$ is not $fg\gamma^*$-closed function.

Similarly we can show that i_3 is not $fg\gamma^*$-open function though i_1

and i_2 are $fg\gamma^*$-open functions.

Theorem 4.19. If $h_1 : X \to Y$ is fuzzy closed (resp., fuzzy open) function and $h_2 : Y \to Z$ is $fg\gamma^*$-closed (resp., $fg\gamma^*$-open) function, then $h_2 \circ h_1 : X \to Z$ is $fg\gamma^*$-closed (resp., $fg\gamma^*$-open) function.

Proof. Obvious.

We now recall the following definitions from [3, 5, 6, 7] for ready references.

Definition 4.20. Let $(X, \tau_1) \to (Y, \tau_2)$ be a function. Then h is called an

(i) fg-closed function [3] if $h(A)$ is fg-closed set in Y for every $A \in \tau_1^c$,

(ii) fsg-closed function [3] if $h(A)$ is fsg-closed set in Y for every $A \in \tau_1^c$,

(iii) $fg\beta$-closed function [6] if $h(A)$ is $fg\beta$-closed set in Y for every $A \in \tau_1^c$,

(iv) fgs^*-closed function [5] if $h(A)$ is fgs^*-closed set in Y for every $A \in \tau_1^c$,

(v) $fg\gamma$-closed function [7] if $h(A)$ is $fg\gamma$-closed set in Y for every $A \in \tau_1^c$.

Remark 4.21. It is obvious that

(i) $fg\gamma^*$-closed function is $fg\beta$-closed function as well as $fg\gamma$-closed function,

(ii) fsg-closed function is $fg\gamma^*$-closed function,

(iii) fgs^*-closed function is $fg\gamma^*$-closed function.

But the converses are not true, in general, follow from the following examples.

Also (iv) fg-closed function and $fg\gamma^*$-closed function are independent concepts follow from the following examples.

Example 4.22. None of fg-closed function, $fg\beta$-closed function, $fg\gamma$-closed function implies $fg\gamma^*$-closed function

Let $X = \{a\}$, $\tau_1 = \{0_X, 1_X, C\}$, $\tau_2 = \{0_X, 1_X, A, B\}$ where $A(a) = 0.45, B(a) = 0.6, C(a) = 0.3$. Then (X, τ_1) and (X, τ_2) are fts's. Consider the identity function $i : (X, \tau_1) \to (X, \tau_2)$. Now $1_X \setminus C \in \tau c_1$, $i(1_X \setminus C) = 1_X \setminus C \in FSO(X, \tau_2)$ as $FSO(X, \tau_2) = \{0_X, 1_X, U, V\}$ where $A \leq U \leq 1_X \setminus A$, $V \geq B$ and $F\gamma O(X, \tau_2) = \{0_X, 1_X, W\}$ where $W > 1_X \setminus B$ and $F\gamma C(X, \tau_2) = \{0_X, 1_X, 1_X \setminus W\}$ where $1_X \setminus W < B$. So $\gamma cl_{\tau_2}(1_X \setminus C) = 1_X \not\leq 1_X \setminus C \Rightarrow 1_X \setminus C$ is not $fg\gamma^*$-closed set in $(X, \tau_2) \Rightarrow i$ is not $fg\gamma^*$-closed function. Now $1_X \setminus C < 1_X \in \tau_2$ only and so $cl_{\tau_2}(1_X \setminus C) \leq 1_X \Rightarrow 1_X \setminus C$ is fg-closed set in $(X, \tau_2) \Rightarrow i$ is fg-closed function. Also $\gamma cl_{\tau_2}(1_X \setminus C) = 1_X \Rightarrow 1_X \setminus C$ is $fg\gamma$-closed set in $(X, \tau_2) \Rightarrow i$ is $fg\gamma$-closed function. Again $\beta cl_{\tau_2}(1_X \setminus C) \leq 1_X \Rightarrow 1_X \setminus C$ is $fg\beta$-closed set in $(X, \tau_2) \Rightarrow i$ is $fg\beta$-closed function.

Example 4.23. None of fg-closed function, fsg-closed function, fgs^*-closed function is implied by $fg\gamma^*$-closed function

Let $X = \{a\}$, $\tau_1 = \{0_X, 1_X, C\}$, $\tau_2 = \{0_X, 1_X, A, B\}$ where $A(a) = 0.45, B(a) = 0.6, C(a) = 0.44$. Then (X, τ_1) and (X, τ_2) are fts's. Consider the identity function $i : (X, \tau_1) \to (X, \tau_2)$. Now $FSO(X, \tau_2) = \{0_X, 1_X, U, V\}$ where $A \leq U \leq 1_X \setminus A$, $V \geq B$. Here $1_X \setminus C \in \tau_1^c$, $i(1_X \setminus C) = 1_X \setminus C < V \in FSO(X, \tau_2)$. Then $\gamma cl_{\tau_2}(1_X \setminus C) = 1_X \setminus C < V \Rightarrow 1_X \setminus C$ is $fg\gamma^*$-closed set in $(X, \tau_2) \Rightarrow i$ is $fg\gamma^*$-closed function. But $scl_{\tau_2}(1_X \setminus C) = 1_X \not\leq V \Rightarrow 1_X \setminus C$ is not fsg-closed set in $(X, \tau_2) \Rightarrow i$ is not fsg-closed function. Again $cl_{\tau_2}(1_X \setminus C) = 1_X \not\leq V \Rightarrow 1_X \setminus C$ is not fgs^*-closed set in $(X, \tau_2) \Rightarrow i$ is not fgs^*-closed function.. Also $1_X \setminus C < B \in \tau_2$, but $cl_{\tau_2}(1_X \setminus C) = 1_X \not\leq B \Rightarrow 1_X \setminus C$ is not fg-closed set in $(X, \tau_2) \Rightarrow i$ is not fg-closed function.

5. $fg\gamma^*$-Regular, $fg\gamma^*$-Normal and $fg\gamma^*$-Compact Spaces

In this section two new types of separation axioms are introduced and characterized by $fg\gamma^*$-closed set. Also, a new type of fuzzy compactness is introduced.

Definition 5.1. An fts (X, τ) is said to be $fg\gamma^*$-regular space if for any fuzzy point x_t in X and each $fg\gamma^*$-closed set F in X with $x_t \notin F$, there exist $U, V \in F\gamma O(X)$ such that $x_t \in U, F \leq V$ and $U \not{q} V$.

Theorem 5.2. In an fts (X, τ), the following statements are equivalent:

(i) X is $fg\gamma^*$-regular,

(ii) for each fuzzy point x_t in X and any $fg\gamma^*$-open q-nbd U of x_t, there exists $V \in F\gamma O(X)$ such that $x_t \in V$ and $\gamma cl V \leq U$,

(iii) for each fuzzy point x_t in X and each $fg\gamma^*$-closed set A of X with $x_t \notin A$, there exists $U \in F\gamma O(X)$ with $x_t \in U$ such that $\gamma cl U \not{q} A$.

Proof (i) $\Rightarrow$ (ii). Let x_t be a fuzzy point in X and U, any $fg\gamma^*$-open q-nbd of x_t. Then $x_t q U \Rightarrow U(x) + t > 1 \Rightarrow x_t \notin 1_X \setminus U$ which is an $fg\gamma^*$-closed set in X. By (i), there exist $V, W \in F\gamma O(X)$ such that $x_t \in V, 1_X \setminus U \leq W$ and $V \not{q} W$. Then $V \leq 1_X \setminus W \Rightarrow \gamma cl V \leq \gamma cl(1_X \setminus W) = 1_X \setminus W \leq U$.

(ii) $\Rightarrow$ (iii). Let x_t be a fuzzy point in X and A, an $fg\gamma^*$-closed set in X with $x_t \notin A$. Then $A(x) < t \Rightarrow x_t q(1_X \setminus A)$ which being $fg\gamma^*$-open set in X is $fg\gamma^*$-open q-nbd of x_t. So by (ii), there exists $V \in F\gamma O(X)$ such that $x_t \in V$ and $\gamma clV \leq 1_X \setminus A$. Then $\gamma clV \not{q} A$.
(iii) $\Rightarrow$ (i). Let x_t be a fuzzy point in X and F be any $fg\gamma^*$-closed set in X with $x_t \notin F$. Then by (iii), there exists $U \in F\gamma O(X)$ such that $x_t \in U$ and $\gamma clU \not{q} F$. Then $F \leq 1_X \setminus \gamma clU$ (=V, say). So $V \in F\gamma O(X)$ and $V \not{q} U$ as $U \not{q} (1_X \setminus \gamma clU)$. Consequently, X is $fg\gamma^*$-regular space.

Definition 5.3. An fts (X, τ) is called $fg\gamma^*$-normal space if for each pair of $fg\gamma^*$-closed sets A, B in X with $A \not{q} B$, there exist $U, V \in F\gamma O(X)$ such that $A \leq U, B \leq V$ and $U \not{q} V$.

Theorem 5.4. An fts (X, τ) is $fg\gamma^*$-normal space if and only if for every $fg\gamma^*$-closed set F and $fg\gamma^*$-open set G in X with $F \leq G$, there exists $H \in F\gamma O(X)$ such that $F \leq H \leq \gamma clH \leq G$.

Proof. Let X be $fg\gamma^*$-normal space and let F be $fg\gamma^*$-closed set and G be $fg\gamma^*$-open set in X with $F \leq G$. Then $F \not{q} (1_X \setminus G)$ where $1_X \setminus G$ is $fg\gamma^*$-closed set in X. By hypothesis, there exist $H, T \in F\gamma O(X)$ such that $F \leq H, 1_X \setminus G \leq T$ and $H \not{q} T$. Then $H \leq 1_X \setminus T \leq G$. Therefore, $F \leq H \leq \gamma clH \leq \gamma cl(1_X \setminus T) = 1_X \setminus T \leq G$.

Conversely, let A, B be two $fg\gamma^*$-closed sets in X with $A \not{q} B$. Then $A \leq 1_X \setminus B$. By hypothesis, there exists $H \in F\gamma O(X)$ such that $A \leq H \leq \gamma clH \leq 1_X \setminus B \Rightarrow A \leq H, B \leq 1_X \setminus \gamma clH$ (=V, say). Then $V \in F\gamma O(X)$ and so $B \leq V$. Also as $H \not{q} (1_X \setminus \gamma clH)$, $H \not{q} V$. Consequently, X is $fg\gamma^*$-normal space.

We first recall the following definitions from [1, 8, 12, 10, 3] for ready references.

Definition 5.5. Let (X, τ) be an fts and $A \in I^X$. A collection $\mathcal{U}$ of fuzzy sets in X is called a fuzzy cover of A if $\bigcup \mathcal{U} \geq A$ [12]. If each member of $\mathcal{U}$ is fuzzy open (resp., fuzzy regular open, fuzzy γ-open) in X, then $\mathcal{U}$ is called a fuzzy open [12] (resp., fuzzy regular open [1], fuzzy γ-open [4]) cover of A. If, in particular, $A = 1_X$, we get the definition of fuzzy cover of X as $\bigcup \mathcal{U} = 1_X$ [8].

Definition 5.6. Let (X, τ) be an fts and $A \in I^X$. Then a fuzzy cover $\mathcal{U}$ of A (resp., of X) is said to have a finite subcover $\mathcal{U}_0$ if $\mathcal{U}_0$ is a finite subcollection of $\mathcal{U}$ such that $\bigcup \mathcal{U}_0 \geq A$ [12]. If, in particular $A = 1_X$, we get $\bigcup \mathcal{U}_0 = 1_X$ [8].

Definition 5.7. Let (X, τ) be an fts and $A \in I^X$. Then A is called fuzzy compact [8] (resp., fuzzy almost compact [9], fuzzy nearly compact [15], fuzzy γ-compact [4]) set if every fuzzy open (resp., fuzzy open, fuzzy regular open, fuzzy γ-open) cover $\mathcal{U}$ of A has a finite subcollection $\mathcal{U}_0$ such that $\bigcup \mathcal{U}_0 \geq A$ (resp., $\bigcup_{U \in \mathcal{U}_0} clU \geq A$, $\bigcup \mathcal{U}_0 \geq A$, $\bigcup \mathcal{U}_0 \geq A$). If, in particular, $A = 1_X$, we get the definition of fuzzy compact [8] (resp., fuzzy almost compact [9], fuzzy nearly compact [10], fuzzy γ-compact [4]) space as $\bigcup \mathcal{U}_0 = 1_X$ (resp., $\bigcup_{U \in \mathcal{U}_0} clU = 1_X$, $\bigcup \mathcal{U}_0 = 1_X$, $\bigcup \mathcal{U}_0 = 1_X$).

Let us now introduce the following concept.

Definition 5.8. Let (X, τ) be an fts and $A \in I^X$. Then A is called $fg\gamma^*$-compact if every fuzzy cover $\mathcal{U}$ of A by $fg\gamma^*$-open sets of X has a finite subcover. If, in particular, $A = 1_X$, we get the definition of $fg\gamma^*$-compact space X.

Theorem 5.9. Every $fg\gamma^*$-closed set in an $fg\gamma^*$-compact space X is $fg\gamma^*$-compact.

Proof. Let $A (\in I^X)$ be an $fg\gamma^*$-closed set in an $fg\gamma^*$-compact space X. Let $\mathcal{U}$ be a fuzzy cover of A by $fg\gamma^*$-open sets of X. Then $\mathcal{V} = \mathcal{U} \bigcup (1_X \setminus A)$ is a fuzzy cover of X by $fg\gamma^*$-open sets of X. As X is $fg\gamma^*$-compact space, $\mathcal{V}$ has a finite subcollection $\mathcal{V}_0$ which also covers X. If $\mathcal{V}_0$ contains $1_X \setminus A$, we omit it and get a finite subcover of A. Hence A is $fg\gamma^*$-compact set.

Remark 5.10. It is clear from definitions that $fg\gamma^*$-compact space is fuzzy compact (resp., fuzzy almost compact, fuzzy nearly compact, fuzzy γ-compact) space.

6. $fg\gamma^*$-Continuous and $fg\gamma^*$-Irresolute Functions

After the introduction of fuzzy continuity [8] different types of generalized version of fuzzy continuous-like functions have been introduced and studied in [3, 5, 6, 7]. Here a new type of generalized version of fuzzy continuous-like function is introduced which is more general than the notion of fuzzy continuous function. Then it is proved that $fg\gamma^*$-continuous image of an $fg\gamma^*$-regular (resp., $fg\gamma^*$-normal, $fg\gamma^*$-compact) space is fuzzy regular [14] (resp., fuzzy normal [13], fuzzy compact [8], fuzzy almost compact [9], fuzzy nearly compact [10]) space. Again a new type of generalized version of fuzzy irresolute function, viz., $fg\gamma^*$-irresolute function is introduced and studied which is strictly weaker than that of $fg\gamma$-continuous function and independent concept of fuzzy continuous function. But it is shown that under $fg\gamma^*$-irresolute function $fg\gamma^*$-regularity, $fg\gamma^*$-normality, $fg\gamma^*$-compactness remain invariant.

Let us first recall the following definitions from [4, 8, 14, 13] for ready references.

Definition 6.1 [8]. A function $h : X \to Y$ is said to be fuzzy continuous function if $h^{-1}(V)$ is fuzzy open set in X for every fuzzy open set V in Y.

Definition 6.2 [4]. A function $f : X \to Y$ is said to be fuzzy γ-open if $f(U)$ is fuzzy γ-open set in Y for every fuzzy γ-open set U in X.

Definition 6.3 [4]. An fts (X, τ) is called fT_γ-space if every fuzzy γ-open set in X is fuzzy open set in X.

Definition 6.4 [14]. An fts (X, τ) is called fuzzy regular space if for any fuzzy point x_α in X and any fuzzy closed set F in X with $x_\alpha \notin F$, there exist fuzzy open sets U, V in X such that $x_\alpha \in U, F \leq V$ and $U \not q V$.

Definition 6.5 [13]. An fts (X, τ) is called fuzzy normal space if for each pair of fuzzy closed sets A, B in X with $A \not q B$, there exist fuzzy open sets U, V in X such that $A \leq U, B \leq V$ and $U \not q V$.

Now we introduce the following concept.

Definition 6.6. A function $h : X \to Y$ is said to be $fg\gamma^*$-continuous function if $h^{-1}(V)$ is $fg\gamma^*$-closed set in X for every fuzzy closed set V in Y.

Remark 6.7. Since every fuzzy closed set is fuzzy γ-closed set, it is clear that fuzzy continuous function is $fg\gamma^*$-continuous, but the converse need not be true, in general, as it seen from the following example.

Example 6.8. $fg\gamma^*$-continuity does not imply fuzzy continuity.
Let $X = \{a, b\}$, $\tau_1 = \{0_X, 1_X\}$, $\tau_2 = \{0_X, 1_X, A\}$ where $A(a) = A(b) = 0.5$. Then (X, τ_1) and (X, τ_2) are fts's. Consider the identity function $i : (X, \tau_1) \to (X, \tau_2)$. Since every fuzzy set in (X, τ_1) is $fg\gamma^*$-closed set in (X, τ_1), so clearly i is $fg\gamma^*$-continuous function. But $A \in \tau_2$, $i^{-1}(A) = A \notin \tau_1 \Rightarrow i$ is not a fuzzy continuous function.

Theorem 6.9. Let $h : (X, \tau) \to (Y, \sigma)$ be a function. Then the following statements are equivalent:
(i) h is $fg\gamma^*$-continuous function,
(ii) for each fuzzy point x_α in X and each fuzzy open nbd V of $h(x_\alpha)$ in

Y , there exists an $fg\gamma^*$-open nbd U of x_α in X such that $h(U) \leq V$,
(iii) $h(fg\gamma^*cl(A)) \leq cl(h(A))$, for all $A \in I^X$,
(iv) $fg\gamma^*cl(h^{-1}(B)) \leq h^{-1}(clB)$, for all $B \in I^Y$.
Proof (i) $\Rightarrow$ (ii). Let x_α be a fuzzy point in X and V, any fuzzy open nbd of $h(x_\alpha)$ in Y. Then $x_\alpha \in h^{-1}(V)$ which is $fg\gamma^*$-open in X (by (i)). Let $U = h^{-1}(V)$. Then $h(U) = h(h^{-1}(V)) \leq V$.
(ii) $\Rightarrow$ (i). Let A be any fuzzy open set in Y and x_α, a fuzzy point in X such that $x_\alpha \in h^{-1}(A)$. Then $h(x_\alpha) \in A$ where A is a fuzzy open nbd of $h(x_\alpha)$ in Y. By (ii), there exists an $fg\gamma^*$-open nbd U of x_α in X such that $h(U) \leq A$. Then $x_\alpha \in U \leq h^{-1}(A) \Rightarrow x_\alpha \in U = fg\gamma^*int(U) \leq fg\gamma^*int(h^{-1}(A))$. Since x_α is taken arbitrarily and $h^{-1}(A)$ is the union of all fuzzy points in $h^{-1}(A)$, $h^{-1}(A) \leq fg\gamma^*int(h^{-1}(A)) \Rightarrow h^{-1}(A)$ is an $fg\gamma^*$-open set in $X \Rightarrow h$ is an $fg\gamma^*$-continuous function.
(i) $\Rightarrow$ (iii). Let $A \in I^X$. Then $cl(h(A))$ is a fuzzy closed set in Y. By (i), $h^{-1}(cl(h(A)))$ is $fg\gamma^*$-closed set in X. Now $A \leq h^{-1}(h(A)) \leq h^{-1}(cl(h(A)))$ and so $fg\gamma^*cl(A) \leq fg\gamma^*cl(h^{-1}(cl(h(A)))) = h^{-1}(cl(h(A))) \Rightarrow h(fg\gamma^*cl(A)) \leq cl(h(A))$.
(iii) $\Rightarrow$ (i). Let V be a fuzzy closed set in Y. Put $U = h^{-1}(V)$. Then $U \in I^X$. By (iii), $h(fg\gamma^*cl(U)) \leq cl(h(U)) = cl(h(h^{-1}(V))) \leq clV = V \Rightarrow fg\gamma^*cl(U) \leq h^{-1}(V) = U \Rightarrow U$ is $fg\gamma^*$-closed set in $X \Rightarrow h$ is $fg\gamma^*$-continuous function.
(iii) $\Rightarrow$ (iv). Let $B \in I^Y$ and $A = h^{-1}(B)$. Then $A \in I^X$. By (iii), $h(fg\gamma^*cl(A)) \leq cl(h(A)) \Rightarrow h(fg\gamma^*cl(h^{-1}(B))) \leq cl(h(h^{-1}(B))) \leq clB \Rightarrow fg\gamma^*cl(h^{-1}(B)) \leq h^{-1}(clB)$.
(iv) $\Rightarrow$ (iii). Let $A \in I^X$. Then $h(A) \in I^Y$. By (iv), $fg\gamma^*cl(h^{-1}(h(A))) \leq h^{-1}(cl(h(A))) \Rightarrow fg\gamma^*cl(A) \leq fg\gamma^*cl(h^{-1}(h(A))) \leq h^{-1}(cl(h(A))) \Rightarrow h(fg\gamma^*cl(A)) \leq cl(h(A))$.
Remark 6.10. A composition of two $fg\gamma^*$-continuous functions need not be so, as it seen from the following example.
Example 6.11. Let $X = \{a, b\}$, $\tau_1 = \{0_X, 1_X, A\}$, $\tau_2 = \{0_X, 1_X\}$, $\tau_3 = \{0_X, 1_X, B\}$ where $A(a) = 0.4, A(b) = 0.7, B(a) = 0.6, B(b) = 0.3$. Then (X, τ_1), (X, τ_2) and (X, τ_3) are fts's. Consider two identity functions $i_1 : (X, \tau_1) \to (X, \tau_2)$ and $i_2 : (X, \tau_2) \to (X, \tau_3)$. Then clearly i_1 and i_2 are $fg\gamma^*$-continuous functions. Now $1_X \setminus B \in \tau_3^c$. So $(i_2 \circ i_1)^{-1}(1_X \setminus B) = 1_X \setminus B \leq A \in FSO(X, \tau_1)$. But $\gamma cl_{\tau_1}(1_X \setminus B) = 1_X \not\leq A \Rightarrow 1_X \setminus B$ is not $fg\gamma^*$-closed set in $(X, \tau_1) \Rightarrow i_2 \circ i_1$ is not an $fg\gamma^*$-continuous function.
Theorem 6.12. If $h_1 : X \to Y$ is $fg\gamma^*$-continuous function and

$h_2 : Y \to Z$ is fuzzy continuous function, then $h_2 \circ h_1 : X \to Z$ is $fg\gamma^*$-continuous function.

Proof. Obvious.

Theorem 6.13. If a bijective function $h : X \to Y$ is $fg\gamma^*$-continuous, fuzzy open function from an $fg\gamma^*$-regular, fT_γ-space X onto an fts Y, then Y is fuzzy regular space.

Proof. Let y_α be a fuzzy point in Y and F, a fuzzy closed set in Y with $y_\alpha \notin F$. As h is bijective, there exists unique $x \in X$ such that $h(x) = y$. So $h(x_\alpha) \notin F \Rightarrow x_\alpha \notin h^{-1}(F)$ where $h^{-1}(F)$ is $fg\gamma^*$-closed set in X (as h is an $fg\gamma^*$-continuous function). By hypothesis, there exist $U, V \in F\gamma O(X)$ such that $x_\alpha \in U, h^{-1}(F) \leq V$ and $U \not{q} V$. Then $h(x_\alpha) \in h(U)$, $F = h(h^{-1}(F))$ (as h is bijective)$\leq h(V)$ and $h(U) \not{q} h(V)$. Since X is fT_γ-space, U, V are fuzzy open sets in X. Now as h is a fuzzy open function, $h(U), h(V)$ are fuzzy open sets in Y with $y_\alpha \in h(U), F \leq h(V)$ and $h(U) \not{q} h(V)$ (Indeed, $h(U)qh(V) \Rightarrow$ there exists $z \in Y$ such that $[h(U)](z) + [h(V)](z) > 1 \Rightarrow U(h^{-1}(z)) + V(h^{-1}(z)) > 1$ as h is bijective $\Rightarrow UqV$, a contradiction). Hence Y is a fuzzy regular space.

In a similar manner we can prove the following theorems easily.

Theorem 6.14. If a bijective function $h : X \to Y$ is $fg\gamma^*$-continuous, fuzzy open function from an $fg\gamma^*$-normal, fT_γ-space X onto an fts Y, then Y is fuzzy normal space.

Theorem 6.15. If a bijective function $h : X \to Y$ is $fg\gamma^*$-continuous, fuzzy γ-open function from an $fg\gamma^*$-regular (resp., $fg\gamma^*$-normal) space X onto an fT_γ-space Y, then Y is fuzzy regular (resp., fuzzy normal) space.

Definition 6.16. A function $h : X \to Y$ is called fuzzy generalized γ^*-irresolute ($fg\gamma^*$-irresolute, for short) function if $h^{-1}(U)$ is an $fg\gamma^*$-open set in X for every $fg\gamma^*$-open set U in Y.

Now we state the following two theorems for which the proofs are very similar to that of Theorem 6.13.

Theorem 6.17. If a bijective function $h : X \to Y$ is $fg\gamma^*$-irresolute, fuzzy γ-open function from an $fg\gamma^*$-regular (resp., $fg\gamma^*$-normal) space X onto an fts Y, then Y is an $fg\gamma^*$-regular (resp., $fg\gamma^*$-normal) space.

Theorem 6.18. If a bijective function $h : X \to Y$ is $fg\gamma^*$-irresolute, fuzzy open function from an $fg\gamma^*$-regular (resp., $fg\gamma^*$-normal), fT_γ-space X onto an fts Y, then Y is $fg\gamma^*$-regular (resp., $fg\gamma^*$-normal) space.

Theorem 6.19. A function $h : X \to Y$ is $fg\gamma^*$-irresolute function if and only if for each fuzzy point x_α in X and each $fg\gamma^*$-open nbd V in Y of $h(x_\alpha)$, there exists an $fg\gamma^*$-open nbd U in X of x_α such that $h(U) \leq V$.

Proof. Let $h : X \to Y$ be an $fg\gamma^*$-irresolute function. Let x_α be a fuzzy point in X and V be any $fg\gamma^*$-open nbd of $h(x_\alpha)$ in Y. Then $h(x_\alpha) \in V \Rightarrow x_\alpha \in h^{-1}(V)$ which being an $fg\gamma^*$-open set in X is an $fg\gamma^*$-open nbd of x_α in X. Put $U = h^{-1}(V)$. Then U is an $fg\gamma^*$-open nbd of x_α in X and $h(U) = h(h^{-1}(V)) \leq V$.

Conversely, let A be an $fg\gamma^*$-open set in Y and x_α be any fuzzy point in X such that $x_\alpha \in h^{-1}(A)$. Then $h(x_\alpha) \in A$. By hypothesis, there exists an $fg\gamma^*$-open nbd U of x_α in X such that $h(U) \leq A \Rightarrow x_\alpha \in U = fg\gamma^*int(U) \leq fg\gamma^*int(h^{-1}(A))$. Since x_α is taken arbitrarily and $h^{-1}(A)$ is the union of all fuzzy points in $h^{-1}(A)$, $h^{-1}(A) \leq fg\gamma^*int(h^{-1}(A)) \Rightarrow h^{-1}(A) = fg\gamma^*int(h^{-1}(A)) \Rightarrow h^{-1}(A)$ is $fg\gamma^*$-open set in $X \Rightarrow h$ is an $fg\gamma^*$-irresolute function.

Theorem 6.20. Let $h : X \to Y$ be an $fg\gamma^*$-continuous function from X onto an fts Y and $A(\in I^X)$ be an $fg\gamma^*$-compact set in X. Then $h(A)$ is a fuzzy compact (resp., fuzzy almost compact, fuzzy nearly compact) set in Y.

Proof. Let $\mathcal{U} = \{U_\alpha : \alpha \in \Lambda\}$ be a fuzzy cover of $h(A)$ by fuzzy open (resp., fuzzy open, fuzzy regular open) sets of Y. Then $h(A) \leq \bigcup_{\alpha\in\Lambda} U_\alpha \Rightarrow A \leq h^{-1}(\bigcup_{\alpha\in\Lambda} U_\alpha) = \bigcup_{\alpha\in\Lambda} h^{-1}(U_\alpha)$. Then $\mathcal{V} = \{h^{-1}(U_\alpha) : \alpha \in \Lambda\}$ is a fuzzy cover of A by $fg\gamma^*$-open sets of X as h is an $fg\gamma^*$-continuous function. As A is $fg\gamma^*$-compact set in X, there exists a finite subcollection Λ_0 of Λ such that $A \leq \bigcup_{\alpha\in\Lambda_0} h^{-1}(U_\alpha)$ $\Rightarrow h(A) \leq h(\bigcup_{\alpha\in\Lambda_0} h^{-1}(U_\alpha) \leq \bigcup_{\alpha\in\Lambda_0} U_\alpha \Rightarrow h(A)$ is fuzzy compact (resp., fuzzy almost compact, fuzzy nearly compact) set in Y.

Since fuzzy open set $fg\gamma^*$-open, we can state the following theorems easily the proofs of which are same as that of Theorem 6.20.

Theorem 6.21. Let $h : X \to Y$ be an $fg\gamma^*$-irresolute function from X onto an fts Y and $A(\in I^X)$ be an $fg\gamma^*$-compact set in X. Then $h(A)$ is $fg\gamma^*$-compact (resp., fuzzy compact, fuzzy almost compact, fuzzy nearly compact) set in Y.

Theorem 6.22. Let $h : X \to Y$ be an $fg\gamma^*$-continuous function from an $fg\gamma^*$-compact space X onto an fts Y . Then Y is fuzzy compact (resp., fuzzy almost compact, fuzzy nearly compact) space.

Theorem 6.23. Let $h : X \to Y$ be an $fg\gamma^*$-irresolute function from an $fg\gamma^*$-compact space X onto an fts Y . Then Y is $fg\gamma^*$-compact (resp., fuzzy compact, fuzzy almost compact, fuzzy nearly compact) space.

Remark 6.24. It is clear from definitions that (i) $fg\gamma^*$-irresolute function is $fg\gamma^*$-continuous, but the converse may not be true, as it seen from the following example.

Also (ii) fuzzy continuity and $fg\gamma^*$-irresoluteness are independent concepts follow from the following examples.

Example 6.25. None of fuzzy continuous function, $fg\gamma^*$-continuous function implies that of $fg\gamma^*$-irresolute function

Let $X = \{a\}$, $\tau_1 = \{0_X, 1_X, A, B\}$, $\tau_2 = \{0_X, 1_X\}$ where $A(a) = 0.45, B(a) = 0.6$. Then $(X.\tau_1)$ and (X, τ_2) are fts's. Consider the identity function $i : (X, \tau_1) \to (X, \tau_2)$. Now every fuzzy set in (X, τ_2) is $fg\gamma^*$-closed set in (X, τ_2). Consider the fuzzy set C defined by $C(a) = 0.7$. Then C is $fg\gamma^*$-closed set in (X, τ_2). Now $i^{-1}(C) = C \leq C \in FSO(X, \tau_1)$. But $\gamma cl_{\tau_1} C = 1_X \not\leq C \Rightarrow C$ is not $fg\gamma^*$-closed set in $(X, \tau_1) \Rightarrow i$ is not $fg\gamma^*$-irresolute function. But clearly i is fuzzy continuous as well as $fg\gamma^*$-continuous function.

Example 6.26. There exists an $fg\gamma^*$-irresolute function which is not fuzzy continuous

Let $X = \{a, b\}$, $\tau_1 = \{0_X, 1_X\}$, $\tau_2 = \{0_X, 1_X, A\}$ where $A(a) = 0.4, A(b) = 0.7$. Then (X, τ_1) and (X, τ_2) are fts's. Consider the identity function $i : (X, \tau_1) \to (X, \tau_2)$. Since every fuzzy set in (X, τ_1) is $fg\gamma^*$-closed set in (X, τ_1), clearly i is $fg\gamma^*$-irresolute function. But $i^{-1}(A) = A \notin \tau_1 \Rightarrow i$ is not fuzzy continuous function.

Now to establish the mutual relationships of these newly defined types of functions with the functions defined in [3, 5, 6, 7], we have to recall the following functions from [3, 5, 6, 7] for ready references.

Definition 6.27. Let $h : (X, \tau_1) \to (Y, \tau_2)$ be a function. Then h is called

(i) fg-continuous [3] if $h^{-1}(V)$ is fg-closed set in X for every $V \in \tau_2^c$,

(ii) $fg\beta$-continuous [6] if $h^{-1}(V)$ is $fg\beta$-closed set in X for every $V \in \tau_2^c$,

(iii) fsg-continuous [3] if $h^{-1}(V)$ is fsg-closed set in X closed set in X for every $V \in \tau_2^c$,

(iv) fgs^*-continuous function [5] if $h^{-1}(V)$ is fgs^*-closed set in X for every $V \in \tau_2^c$,
(v) $fg\gamma$-continuous [7] if $h^{-1}(V)$ is $fg\gamma$-closed set in X for every $V \in \tau_2^c$.

Remark 6.28. It is clear from definitions that
(i) every $fg\gamma^*$-continuous function is $fg\gamma$-continuous as well as $fg\beta$-continuous,
(ii) fsg-continuous functions and fgs^*-continuous functions are both $fg\gamma^*$-continuous functions.
But the converses are not true, in general, follow from the following examples.
Also (iii) fg-continuity and $fg\gamma^*$-continuity are independent concepts as it seen from the following examples.

Example 6.29. None of fg-continuity, $fg\beta$-continuity, $fg\gamma$-continuity implies that of $fg\gamma^*$-continuity
Let $X = \{a\}$, $\tau_1 = \{0_X, 1_X, A, B\}$, $\tau_2 = \{0_X, 1_X, C\}$ where $A(a) = 0.45, B(a) = 0.6, C(a) = 0.3$. Then (X, τ_1) and (X, τ_2) are fts's. Consider the identity function $i : (X, \tau_1) \to (X, \tau_2)$. Now $1_X \setminus C \in \tau_2^c$, $i^{-1}(1_X \setminus C) = 1_X \setminus C \leq 1_X \setminus C \in FSO(X, \tau_1)$. But $\gamma cl_{\tau_1}(1_X \setminus C) = 1_X \not\leq 1_X \setminus C \Rightarrow 1_X \setminus C$ is not $fg\gamma^*$-closed set in $(X, \tau_1) \Rightarrow i$ is not $fg\gamma^*$-continuous function. But as 1_X is the only fuzzy open set in (X, τ_1) containing $1_X \setminus C$, clearly i is fg-continuous, $fg\beta$-continuous and $fg\gamma$-continuous function.

Example 6.30. None of fg-continuity, fsg-continuity, fgs^*-continuity is implied by $fg\gamma^*$-continuity
Let $X = \{a\}$, $\tau_1 = \{0_X, 1_X, A, B\}$, $\tau_2 = \{0_X, 1_X, C\}$ where $A(a) = 0.45, B(a) = 0.6, C(a) = 0.44$. Then (X, τ_1) and (X, τ_2) are fts's. Consider the identity function $i : (X, \tau_1) \to (X, \tau_2)$. Now $1_X \setminus C \in \tau_2^c$, $i^{-1}(1_X \setminus C) = 1_X \setminus C < B \in FSO(X, \tau_1)$. Now $\gamma cl_{\tau_1}(1_X \setminus C) = 1_X \setminus C < B \Rightarrow 1_X \setminus C$ is $fg\gamma^*$-closed set in $(X, \tau_1) \Rightarrow i$ is $fg\gamma^*$-continuous function. But $scl_{\tau_1}(1_X \setminus C) = 1_X \not\leq B \Rightarrow 1_X \setminus C$ is not fsg-closed set in $(X, \tau_1) \Rightarrow i$ is not fsg-continuous function. Also $cl_{\tau_1}(1_X \setminus C) = 1_X \not\leq B \Rightarrow 1_X \setminus C$ is not fgs^*-closed set in $(X, \tau_1) \Rightarrow i$ is not fgs^*-continuous function. Again $1_X \setminus C < B \in \tau_1$, but $cl_{\tau_1}(1_X \setminus C) = 1_X \not\leq B \Rightarrow 1_X \setminus C$ is not fg-closed set in $(X, \tau_1) \Rightarrow i$ is not fg-continuous function.

7. $fg\gamma^*$-T_2 Space

The notion of fuzzy T_2-space was introduced in [14]. Afterwards, several types of fuzzy separation axioms have been introduced and studied by many mathematicians. In this context here we introduce a new type of generalized version of separation axiom in fuzzy topology. Afterwards, a strong and weak form of the notion of $fg\gamma^*$-continuous function are introduced and also some of their applications are shown.

We first recall the following definition and theorem from [14, 15] for ready references.

Definition 7.1 [14]. An fts (X, τ) is called fuzzy T_2-space if for any two distinct fuzzy points x_α and y_β; when $x \neq y$, there exist fuzzy open sets U_1, U_2, V_1, V_2 such that $x_\alpha \in U_1, y_\beta q V_1, U_1 \not{q} V_1$ and $x_\alpha q U_2, y_\beta \in V_2, U_2 \not{q} V_2$; when $x = y$ and $\alpha < \beta$ (say), there exist fuzzy open sets U and V in X such that $x_\alpha \in U, y_\beta q V$ and $U \not{q} V$.

Theorem 7.2 [15]. An fts (X, τ) is fuzzy T_2-space if and only if for any two distinct fuzzy points x_α and y_β in X; when $x \neq y$, there exist fuzzy open sets U, V in X such that $x_\alpha q U$, $y_\beta q V$ and $U \not{q} V$; when $x = y$ and $\alpha < \beta$ (say), x_α has a fuzzy open nbd U and y_β has a fuzzy open q-nbd V such that $U \not{q} V$.

Let us introduce the following concept.

Definition 7.3. An fts (X, τ) is called $fg\gamma^*$-T_2 space, if for any two distinct fuzzy points x_α and y_β in X; when $x \neq y$, there exist $fg\gamma^*$-open sets U, V in X such that $x_\alpha q U$, $y_\beta q V$ and $U \not{q} V$; when $x = y$ and $\alpha < \beta$ (say), x_α has an $fg\gamma^*$-open nbd U and y_β has an $fg\gamma^*$-open q-nbd V such that $U \not{q} V$.

Theorem 7.4. If an injective function $h : X \to Y$ is $fg\gamma^*$-continuous function from an fts X onto a fuzzy T_2-space Y, then X is $fg\gamma^*$-T_2 space.

Proof. Let x_α and y_β be two distinct fuzzy points in X. Then $h(x_\alpha)$ $(= z_\alpha$, say) and $h(y_\beta)(= w_\beta$, say) are two distinct fuzzy points in Y.

Case I. Suppose $x \neq y$. Then $z \neq w$. Since Y is fuzzy T_2-space, there exist fuzzy open sets U, V in Y such that $z_\alpha q U, w_\beta q V$ and $U \not{q} V$. As h is $fg\gamma^*$-continuous function, $h^{-1}(U)$ and $h^{-1}(V)$ are $fg\gamma^*$-open sets in X with $x_\alpha q h^{-1}(U), y_\beta q h^{-1}(V)$ and $h^{-1}(U) \not{q} h^{-1}(V)$ [Indeed, $z_\alpha q U \Rightarrow U(z) + \alpha > 1 \Rightarrow U(h(x)) + \alpha > 1 \Rightarrow [h^{-1}(U)](x) + \alpha > 1 \Rightarrow x_\alpha q h^{-1}(U)$. Again, $h^{-1}(U) q h^{-1}(V) \Rightarrow$ there exists $t \in X$ such that $[h^{-1}(U)](t) + [h^{-1}(V)](t) > 1 \Rightarrow U(h(t)) + V(h(t)) > 1 \Rightarrow U q V$, a contradiction].

Case II. Suppose $x = y$ and $\alpha < \beta$ (say). Then $z = w$ and $\alpha < \beta$. Since Y is fuzzy T_2-space, there exist a fuzzy open nbd U of x_α and a fuzzy open q-nbd V of w_β such that $U \not{q} V$. Then $U(z) \geq \alpha \Rightarrow [h^{-1}(U)](x) \geq \alpha \Rightarrow x_\alpha \in h^{-1}(U), y_\beta q h^{-1}(V)$ and $h^{-1}(U) \not{q} h^{-1}(V)$ where $h^{-1}(U)$ and $h^{-1}(V)$ are $fg\gamma^*$-open sets in X as h is $fg\gamma^*$-continuous function. Consequently, X is $fg\gamma^*$-T_2-space.

In a similar manner, we can prove the following theorems.

Theorem 7.5. If a bijective function $h : X \to Y$ is $fg\gamma^*$-irresolute function from an fts X onto an $fg\gamma^*$-T_2 space Y, then X is $fg\gamma^*$-T_2 space.

Theorem 7.6. If a bijective function $h : X \to Y$ is $fg\gamma^*$-open function from a fuzzy T_2-space X onto an fts Y, then Y is $fg\gamma^*$-T_2-space.

Definition 7.7. A function $h : X \to Y$ is called

(i) strongly $fg\gamma^*$-continuous if $h^{-1}(V)$ is fuzzy closed set in X for every $fg\gamma^*$-closed set V in Y,

(ii) weakly $fg\gamma^*$-continuous if $h^{-1}(V) \in F\gamma C(X)$ for every $fg\gamma^*$-closed set V in Y.

Remark 7.8. It is clear from above discussion that every strongly $fg\gamma^*$-continuous function is weakly $fg\gamma^*$-continuous, $fg\gamma^*$-continuous and $fg\gamma^*$-irresolute functions. But the converses are not true, in general, as it follow from the following example.

Example 7.9. None of weakly $fg\gamma^*$-continuity, $fg\gamma^*$-continuity and $fg\gamma^*$-irresoluteness implies strongly $fg\gamma^*$-continuity

Let $X = \{a, b\}$, $\tau_1 = \{0_X, 1_X\}$, $\tau_2 = \{0_X, 1_X, A\}$ where $A(a) = 0.4, A(b) = 0.7$. Then (X, τ_1) and (X, τ_2) are fts's. Consider the identity function $i : (X, \tau_1) \to (X, \tau_2)$. Since every fuzzy set in (X, τ_1) is fuzzy γ-closed as well as $fg\gamma^*$-closed set in (X, τ_1), clearly i is weakly $fg\gamma^*$-continuous function, $fg\gamma^*$-continuous function and $fg\gamma^*$-irresolute function. Now consider the fuzzy set B defined by $B(a) = B(b) = 0.5$. Since B is fuzzy γ-closed set in (X, τ_2), B is clearly $fg\gamma^*$-closed set in (X, τ_2). But $i^{-1}(B) = B \notin \tau_1^c \Rightarrow i$ is not strongly $fg\gamma^*$-continuous function.

Remark 7.10. Every weakly $fg\gamma^*$-continuous function is $fg\gamma^*$-continuous function as well as $fg\gamma^*$-irresolute, but the converses are not true, in general, follow from the following examples.

Example 7.11. $fg\gamma^*$-continuity does not imply weakly $fg\gamma^*$-continuity

Let $X = \{a, b\}$, $\tau_1 = \{0_X, 1_X, A\}$, $\tau_2 = \{0_X, 1_X\}$ where $A(a) =$

$0.5, A(b) = 0.6$. Then (X, τ_1) and (X, τ_2) are fts's. Consider the identity function $i : (X, \tau_1) \to (X, \tau_2)$. Clearly i is $fg\gamma^*$-continuous function. Since every fuzzy set in (X, τ_2) is $fg\gamma^*$-closed set in (X, τ_2), A is also $fg\gamma^*$-closed set in (X, τ_2). Now $i^{-1}(A) = A \notin F\gamma C(X, \tau_1)$ as $(cl(intA)) \bigwedge (int(clA)) = 1_X \not\leq A \Rightarrow i$ is not weakly $fg\gamma^*$-continuous function.

Example 7.12. $fg\gamma^*$-irresoluteness does not imply weakly $fg\gamma^*$-continuity

Let $X = \{a, b\}$, $\tau_1 = \{0_X, 1_X, A\}$, $\tau_2 = \{0_X, 1_X, B\}$ where $A(a) = 0.5, A(b) = 0.4, B(a) = 0.5, B(b) = 0.6$. Then (X, τ_1) and (X, τ_2) are fts's. Consider the identity function $i : (X, \tau_1) \to (X, \tau_2)$. Now $FSO(X, \tau_1) = \{0_X, 1_X, T\}$ where $A \leq T \leq 1_X \setminus A$, $F\gamma C(X, \tau_1) = \{0_X, 1_X, M\}$ where $M \leq 1_X \setminus A$, $FSO(X, \tau_2) = \{0_X, 1_X, U\}$ where $U \geq B$, the collection of all $fg\gamma^*$-closed sets in $(X, \tau_2) = F\gamma C(X, \tau_2) = \{0_X, 1_X, V\}$ where $V \not\geq B$. Now consider the fuzzy set C such that $C \leq B$. Then clearly $C \in F\gamma C(X, \tau_1)$. Now $i^{-1}(C) = C \Rightarrow C$ is $fg\gamma^*$-closed set in (X, τ_1). But if $C \not\geq B$, then 1_X is the only fuzzy semiopen set in (X, τ_1) containing C and so $\gamma cl_{\tau_1} C \leq 1_X \Rightarrow C$ is $fg\gamma^*$-closed set in $(X, \tau_1) \Rightarrow i$ is $fg\gamma^*$-irresolute function. Next consider the fuzzy set D defined by $D(a) = 0.6, D(b) = 0.5$. Then as $D \not\geq B$, D is $fg\gamma^*$-closed set in (X, τ_2). Now $i^{-1}(D) = D$. But $(clintD) \bigwedge (intclD) = 1_X \setminus A \not\leq D \Rightarrow D \notin F\gamma C(X, \tau_1) \Rightarrow i$ is not weakly $fg\gamma^*$-continuous function.

Remark 7.13. It is clear from definitions that

(i) strongly $fg\gamma^*$-continuity implies fuzzy continuity, but not conversely as follows from the next example,

(ii) weakly $fg\gamma^*$-continuity and fuzzy continuity are independent concepts, see the following examples.

Example 7.14. Fuzzy continuity does not imply strongly $fg\gamma^*$-continuity as well as weakly $fg\gamma^*$-continuity

Consider Example 7.11. Here i is not weakly $fg\gamma^*$-continuous function and so by Remark 7.8, i is not also strongly $fg\gamma^*$-continuous function. Obviously i is fuzzy continuous function.

Example 7.15. Weakly $fg\gamma^*$-continuous function does not imply fuzzy continuous function

Consider Example 7.9. Here i is weakly $fg\gamma^*$-continuous function. But clearly i is not fuzzy continuous function as $A \in \tau_2$, but $i^{-1}(A) = A \notin \tau_1^c$.

Since fuzzy open set is fuzzy γ-open and hence $fg\gamma^*$-open, we can prove the following theorems easily.

Theorem 7.16. If a bijective function $h : X \to Y$ is strongly $fg\gamma^*$-continuous, fuzzy open function from a fuzzy regular (resp., fuzzy normal) space X onto an fts Y, then Y is $fg\gamma^*$-regular (resp., $fg\gamma^*$-normal) space.

Theorem 7.17. If a bijective function $h : X \to Y$ is weakly $fg\gamma^*$-continuous, fuzzy γ-open function from an $fg\gamma^*$-regular (resp., $fg\gamma^*$-normal) space X onto an fts Y, then Y is $fg\gamma^*$-regular (resp., $fg\gamma^*$-normal) space.

Theorem 7.18. If a bijective function $h : X \to Y$ is strongly $fg\gamma^*$-continuous, fuzzy γ-open function from an $fg\gamma^*$-regular (resp., $fg\gamma^*$-normal) space X onto an fts Y, then Y is $fg\gamma^*$-regular (resp., $fg\gamma^*$-normal) space.

Theorem 7.19. If a bijective function $h : X \to Y$ is weakly $fg\gamma^*$-continuous, fuzzy open function from a fuzzy regular (resp., fuzzy normal), fT_γ-space X onto an fts Y, then Y is $fg\gamma^*$-regular (resp., $fg\gamma^*$-normal) space.

Theorem 7.20. If a bijective function $h : X \to Y$ is strongly $fg\gamma^*$-continuous (resp., weakly $fg\gamma^*$-continuous) function from an fts X onto an $fg\gamma^*$-T_2 space Y, then X is fuzzy T_2 space (resp., $fg\gamma^*$-T_2 space).

Theorem 7.21. If a bijective function $h : X \to Y$ is strongly $fg\gamma^*$-continuous (resp., weakly $fg\gamma^*$-continuous) function from a fuzzy compact (resp., fuzzy γ-compact) space X onto an fts Y, then Y is $fg\gamma^*$-compact space.

Note 7.22. It is clear from definitions that composition of two strongly $fg\gamma^*$-continuous (resp., weakly $fg\gamma^*$-continuous) functions is also so.

Theorem 7.23. (i) If $h_1 : X \to Y$ is strongly $fg\gamma^*$-continuous and $h_2 : Y \to Z$ is weakly $fg\gamma^*$-continuous functions, then $h_2 \circ h_1 : X \to Z$ is strongly $fg\gamma^*$-continuous function.

(ii) If $h_1 : X \to Y$ is weakly $fg\gamma^*$-continuous and $h_2 : Y \to Z$ is strongly $fg\gamma^*$-continuous functions, then $h_2 \circ h_1 : X \to Z$ is weakly $fg\gamma^*$-continuous function.

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